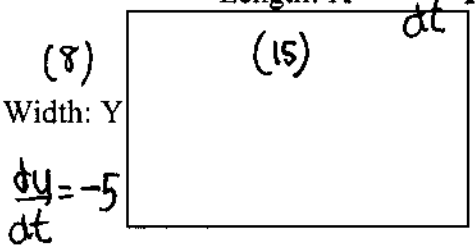
Name Answers Related Rates Quiz 2014.doc 40/40

Solve the following related rates problems. You must show work to get credit.

1) The length of a rectangle is **increasing** at a rate of 2 cm/sec while the width is **decreasing** at a rate 5 cm/sec. When the length is 15 cm and the width is 8 cm find the rates of change of the following.



a) The area

$A = xy$

$\frac{dA}{dt} = \frac{dx}{dt}y + x\frac{dy}{dt}$ ✓ ✓

$\frac{dA}{dt} = 2(8) + 15(-5)$ ✓ ✓

$\frac{dA}{dt} = -59 \text{ cm}^2/\text{sec}$ ✓

- wrong sub (-1)
- no neg. rate (-1)

[6 each]

- (-2) if incorrect product rule $-10 \text{ cm}^2/\text{sec}$
- no product rule (-3)

b) The perimeter

$P = 2x + 2y$ ✓

$\frac{dP}{dt} = 2\frac{dx}{dt} + 2\frac{dy}{dt}$ ✓ ✓

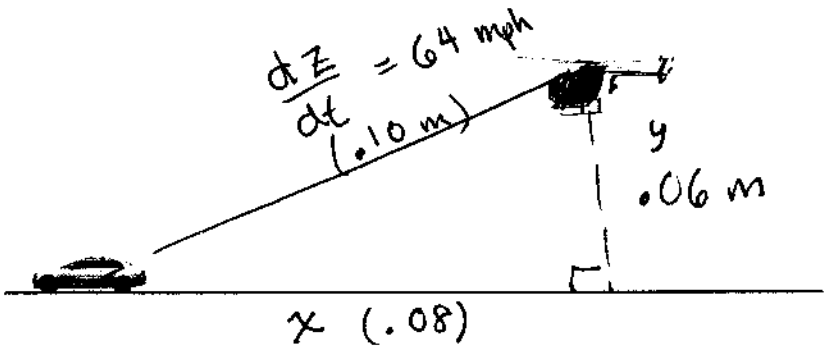
$\frac{dP}{dt} = 2(2) + 2(-5) = -6 \text{ cm/sec}$ ✓ ✓ ✓

2) A highway patrol helicopter is hovering 0.06 miles above a level, straight highway. The helicopter pilot sees a car on the highway and determines with radar that at that particular instant, the distance between the helicopter and the car is 0.10 miles and is increasing at a rate of 64 miles per hour. Find the car's speed along the highway.

(-2) if wrong sub. for 64 mph 51.2 mph

$x^2 + y^2 = z^2$ ✓

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$ ✓ ✓ (-2) if $\frac{dy}{dt} \neq 0$ ✓ ✓



$2(.08)\frac{dx}{dt} + 0 = 2(.1)(64)$

$\frac{dx}{dt} = 80$ m.p.h ✓

3) The volume of a sphere is decreasing at a rate of 8π cm^3/min. At what rate is the surface area changing when the radius is 2 cm? **Leave answer in terms of π.**

$A = 4\pi r^2$ ✓

$\frac{dA}{dt} = 8\pi r\frac{dr}{dt}$ ✓ ✓

$\frac{dA}{dt} = 8\pi(2)\left(-\frac{1}{2}\right)$ ✓

$\frac{dA}{dt} = -8\pi$ cm^2/min ✓ ✓

$V = \frac{4}{3}\pi r^3$ ✓

$\frac{dV}{dt} = 4\pi r^2\frac{dr}{dt}$ ✓ ✓

$-8\pi = 4\pi(2)^2\frac{dr}{dt}$

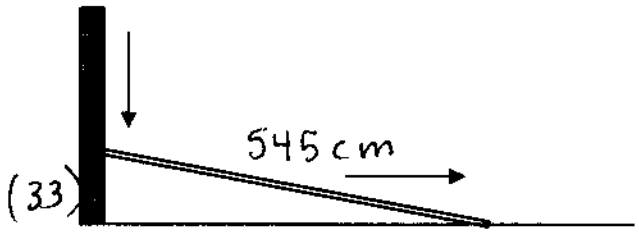
$-\frac{8\pi}{16\pi} = \frac{dr}{dt}$ [10]

$-\frac{1}{2} = \frac{dr}{dt}$ ✓

if just sub in for $\frac{dr}{dt}$ (-5)

-2 if $\frac{dz}{dt} \neq 0$ -1 for wrong sub x & y -2 ex: if put 0 for $\frac{dy}{dt}$ ans: 1090

4) The bottom of a 545 cm long ladder is sliding away from the base of a wall at a rate of 66 cm/sec. How fast is the top of the ladder moving when it is 33 cm from the ground?



(544) $\frac{dx}{dt} = 66$

$x^2 + y^2 = z^2$ ✓

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$ ✓✓

$2(544)(66) + 2(33)\frac{dy}{dt} = 0$ ✓

$\frac{dy}{dt} = -\frac{71808}{66}$ ✓

$\frac{dy}{dt} = -1088.$

≈ −1088 cm/sec

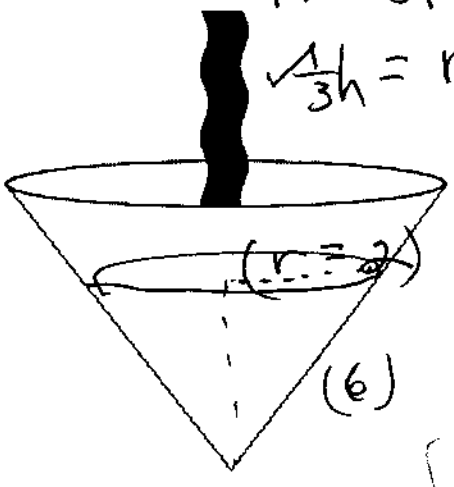
5) Water is pouring into a cone shaped tank at a rate of 8 cubic feet per minute. The height of the tank of the tank is 12 feet and the radius at the top is 4 feet. How fast is the water level rising when the water is 6 feet deep? **Leave answer in terms of π.**

$V = \frac{1}{3}\pi r^2 h$ ✓

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h = \frac{1}{27}\pi h^3$ ✓

$h = 3r$

$\frac{1}{3}h = r$



$\frac{dV}{dt} = \frac{1}{9}\pi h^2 \frac{dh}{dt}$ ✓✓

$8 = \frac{1}{9}\pi(6)^2 \frac{dh}{dt}$ ✓

$\frac{72}{36\pi} = \frac{dh}{dt}$

$\frac{2}{\pi}$ ft/min ✓

≈ .6366 f/m

Extra Credit Questions:

1) Find the following limit. $\lim_{x \to -\infty} \dfrac{x^2 - 5x + 3}{3x + 2}$ $\rightarrow \lim_{x \to -\infty} \dfrac{x}{3}$

$= -\infty$ [+1]

2) Find the derivative of: $x^2 y + 5y^2 = y - 8$

$$2xy + x^2 \frac{dy}{dx} + 10y \frac{dy}{dx} = \frac{dy}{dx}$$ [+1]

$$\frac{dy}{dx}(x^2 + 10y - 1) = -2xy$$

$$\frac{dy}{dx} = \frac{-2xy}{x^2 + 10y - 1} \quad \text{or} \quad \frac{2xy}{1 - 10y - x^2}$$

Formulas:

Volume of a cone: $v = \dfrac{1}{3}\pi r^2 h$

Surface area of a sphere: $A = 4\pi r^2$

Volume of a sphere: $V = \dfrac{4}{3}\pi r^3$

Area of a circle: $A = \pi r^2$

Volume of a cube: $V = x^3$

Surface area of a cube: $A = 6x^2$