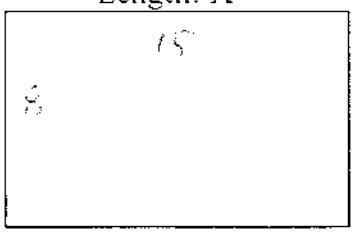
Name_____ Related Rates Quiz 2014.doc

Solve the following related rates problems. You must show work to get credit.

1) The length of a rectangle is **increasing** at a rate of 2 cm/sec while the width is **decreasing** at a rate 5 cm/sec. When the length is 15 cm and the width is 8 cm find the rates of change of the following.



a) The area

$\frac{dx}{dt} = 2 \quad \frac{dy}{dt} = -5$

$A = xy$

$\frac{dA}{dt} = \frac{dx}{dt}y + x\frac{dy}{dt}$

$\frac{dA}{dt} = (2)(8) + (15)(-5)$

$\frac{dA}{dt} = -59 \text{ cm}^2/\text{sec}$

b) The perimeter

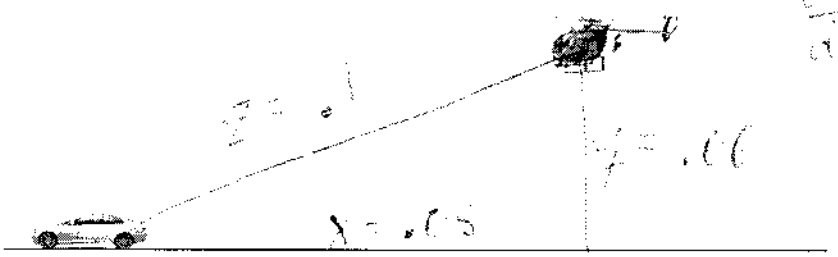
$P = 2y + 2x$

$\frac{dP}{dt} = 2\frac{dy}{dt} + 2\frac{dx}{dt}$

$\frac{dP}{dt} = 2(-5) + 2(2)$

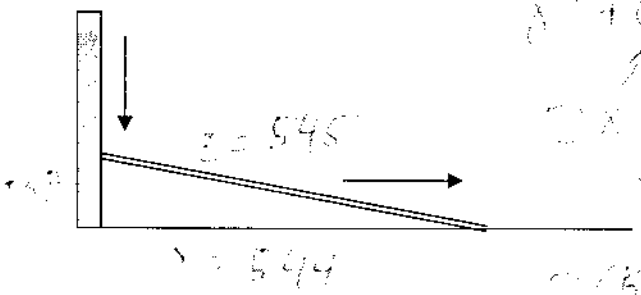
$\frac{dP}{dt} = -6 \text{ cm/sec}$

2) A highway patrol helicopter is hovering 0.06 miles above a level, straight highway. The helicopter pilot sees a car on the highway and determines with radar that at that particular instant, the distance between the helicopter and the car is 0.10 miles and is increasing at a rate of 64 miles per hour. Find the car's speed along the highway.



3) The volume of a sphere is decreasing at a rate of 8π cm^3/min. At what rate is the surface area changing when the radius is 2 cm? **Leave answer in terms of π.**

4) The bottom of a 545 cm long ladder is sliding away from the base of a wall at a rate of 66 cm/sec. How fast is the top of the ladder moving when it is 33 cm from the ground?



$x^2 + y^2 = z^2$

$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$

$2(544)(66) + 2(33)\frac{dy}{dt} = 0$

$\frac{dy}{dt} = \frac{-2(544)(66)}{2(33)}$

$\frac{dy}{dt} = -1088 \text{ cm/sec}$

5) Water is pouring into a cone shaped tank at a rate of 8 cubic feet per minute. The height of the tank of the tank is 12 feet and the radius at the top is 4 feet. How fast is the water level rising when the water is 6 feet deep? **Leave answer in terms of π.**

$V = \frac{1}{3}\pi r^2 h$

$V = \frac{1}{3}\pi\left(\frac{1}{3}h\right)^2 h$

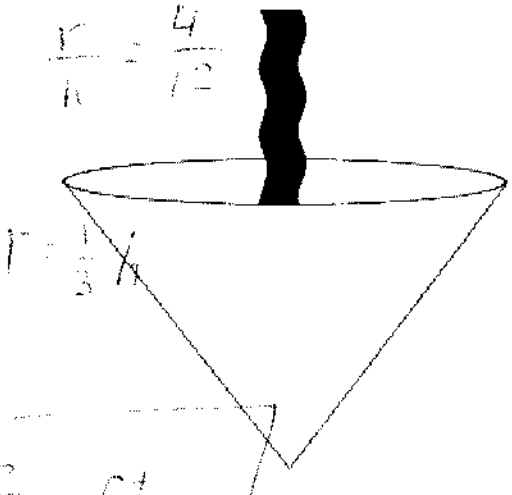
$V = \frac{1}{3}\pi\frac{1}{9}h^3$

$V = \frac{1}{27}\pi h^3$

$\frac{dV}{dt} = \frac{3}{27}\pi h^2\frac{dh}{dt}$

$8 = \frac{1}{9}\pi(6)^2\frac{dh}{dt}$

$\frac{dh}{dt} = \frac{72}{36\pi}$



$\frac{dh}{dt} = \frac{2}{\pi}$ ft/min

$\approx .637$ ft/min

Extra Credit Questions:

1) Find the following limit. $\lim_{x \to -\infty} \dfrac{x^2 - 5x + 3}{3x + 2}$

$-\infty$ DNE

2) Find the derivative of : $x^2 y + 5y^2 = y - 8$

$2xy + x^2 \dfrac{dy}{dx} + 10y \dfrac{dy}{dx} - \dfrac{dy}{dx}$

$x^2 \dfrac{dy}{dx} + 10y \dfrac{dy}{dx} - \dfrac{dy}{dx} = -2xy$

$\dfrac{dy}{dx} = \dfrac{-2xy}{x^2 + 10y - 1}$ or $\dfrac{2xy}{1 - x^2 - 10y}$

Formulas:

Volume of a cone: $v = \dfrac{1}{3}\pi r^2 h$

Surface area of a sphere: $A = 4\pi r^2$

Volume of a sphere: $V = \dfrac{4}{3}\pi r^3$

Area of a circle: $A = \pi r^2$

Volume of a cube: $V = x^3$

Surface area of a cube: $A = 6x^2$