Name________________________________ Related Rates Quiz 2013

Solve the following related rates problems. You must show work to get credit.

1) The length of a rectangle is **increasing** at a rate of 3 cm/sec while the width is **decreasing** at a rate 4 cm/sec. When the length is 10 cm and the width is 7 cm find the rates of change of the area.

Length: X $=10$

Width: Y 7

$\frac{dx}{dt} = 3$

$\frac{dy}{dt} = -4$

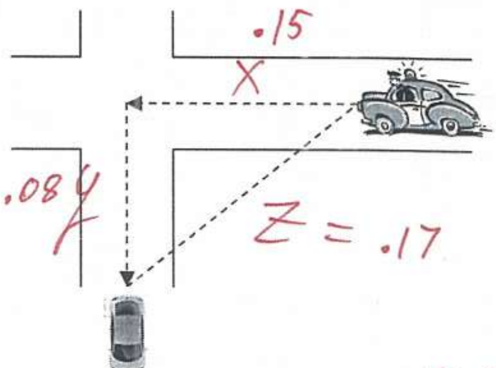
$A = xy$

$$\frac{dA}{dt} = \frac{dx}{dt}y + x\frac{dy}{dt}$$

$$\frac{dA}{dt} = (3)(7) + (10)(-4)$$

$$\boxed{\frac{dA}{dt} = -19 \text{ cm}^2/\text{sec}}$$

2) A police cruiser is traveling west towards an intersection at the rate of 55 mph, while another car is speeding south away from the intersection. When the cruiser is 0.15 miles from the intersection and the other car is 0.08 miles from the intersection, the police determine with radar that the distance between the two vehicles is decreasing at 5 mph. Find the speed of the south bound car in mph.



$$\frac{dx}{dt} = -55$$

$$\frac{dz}{dt} = -5$$

$$x^2 + y^2 = z^2$$

$$2x\frac{dx}{dt} + 2y\frac{dy}{dt} = 2z\frac{dz}{dt}$$

$$2(.15)(-55) + 2(.08)\left(\frac{dy}{dt}\right) = 2(.17)(-5)$$

$$\frac{dy}{dt} = \frac{14.8}{.16}$$

$$\boxed{\frac{dy}{dt} = 92.5 \text{ mph}}$$

3) The surface area of a sphere is increasing at the rate of 5 cm^2/sec. At what rate is its volume increasing when the radius is 3 cm?

$$A = 4\pi r^2$$

$$\frac{dA}{dt} = 8\pi r \frac{dr}{dt}$$

$$5 = 8\pi 3 \frac{dr}{dt}$$

$$\frac{dr}{dt} = \frac{5}{24\pi}$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$$

$$\frac{dV}{dt} = 4\pi 9 \left(\frac{5}{24\pi}\right)$$

$$\frac{dV}{dt} = \frac{180\pi}{24\pi} = \boxed{7.5}$$

OR $\frac{15}{2}$ cm^3/min

4) Gravel is being dumped from a conveyor belt at a rate of 300 ft^3 per min. The gravel forms a conical pile with base radius and height being equal. How fast is the pile increasing when the pile is 10 ft. high? **Leave answer in terms of pi.**

$$V = \frac{1}{3}\pi r^2 h \quad V = \frac{1}{3}\pi h^3 \quad \boxed{r = h}$$

$$\frac{dV}{dt} = \pi h^2 \frac{dh}{dt}$$

$$300 = \pi\, 100 \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{300}{100\pi}$$

$$\boxed{\frac{3}{\pi} \text{ ft/min}}$$

5) A rock drops into a still lake, creating a circular ripple that expands so that the radius grows at a rate of 1.5 feet per second. At the moment when the area is 16π square feet, find the rate of change of the following: **Leave answer in terms of π.**

$16\pi = \pi r^2$

$16 = r^2$

$r = 4$

a) the area

$A = \pi r^2$

$\frac{dA}{dt} = 2\pi r \frac{dr}{dt}$

$\frac{dA}{dt} = 2\pi 4 \cdot 1.5$

$\boxed{\frac{dA}{dt} = 12\pi}$

b) the circumference

$C = 2\pi r$

$\frac{dc}{dt} = 2\pi \frac{dr}{dt}$

$\frac{dc}{dt} = 2\pi 1.5$

$\boxed{\frac{dc}{dt} = 3\pi}$

Bonus Questions:

1) Find the following limit. $\lim_{x \to 2} \frac{x^2 - 4}{x - 2}$ $\frac{(x+2)\cancel{(x-2)}}{\cancel{(x-2)}} = (x+2)$

$2+2$

$\boxed{4}$

2) Find the derivative of : $1 - x^2 y^3 = y - 2x$

$-2xy^3 \; -x^2 3y^2 \frac{dy}{dx} = \frac{dy}{dx} - 2$

$2 - 2xy^3 = \frac{dy}{dx} + 3x^2y^2 \frac{dy}{dx}$

$2 - 2xy^3 = (1 + 3x^2y^2) \frac{dy}{dx}$

$\boxed{\frac{dy}{dx} = \frac{2 - 2xy^3}{1 + 3x^2y^2}}$

Formulas:

Volume of a cone: $v = \frac{1}{3}\pi r^2 h$

Surface area of a sphere: $A = 4\pi r^2$

Volume of a sphere: $V = \frac{4}{3}\pi r^3$

Area of a circle: $A = \pi r^2$

Circumference of a circle: $C = 2\pi r$

Volume of a cube: $V = x^3$