Rational Expressions

Simplifying Rational Expressions

Rational Expressions

Rational expressions can be written in the form $\frac{P}{Q}$ where P and Q are both polynomials and $Q \neq 0$.

Examples of Rational Expressions

$$\frac{3x^2+2x-4}{4x-5} \qquad \frac{4x+3y}{2x^2-3xy+4y^2} \qquad \frac{3x^2}{4}$$

Evaluating Rational Expressions

To evaluate a rational expression for a particular value(s), substitute the replacement value(s) into the rational expression and simplify the result.

Example

Evaluate the following expression for y = –2.

$$\frac{y-2}{-5+y} = \frac{-2-2}{-5+(-2)} = \frac{-4}{-7} = \frac{4}{7}$$

Evaluating Rational Expressions

In the previous example, what would happen if we tried to evaluate the rational expression for y = 5?

$$\frac{y-2}{-5+y} = \frac{5-2}{-5+5} = \frac{3}{0}$$

This expression is undefined!

Undefined Rational Expressions

We have to be able to determine when a rational expression is undefined.

A rational expression is undefined when the denominator is equal to zero.

The numerator being equal to zero is okay (the rational expression simply equals zero).

Undefined Rational Expressions

Find any real numbers that make the following rational expression undefined.

Example

$$\frac{9x^3 + 4x}{15x + 45}$$

The expression is undefined when $15x + 45 = 0$.

So the expression is undefined when $x = -3$.

Simplifying Rational Expressions

Simplifying a rational expression means writing it in lowest terms or simplest form.

To do this, we need to use the

Fundamental Principle of Rational Expressions

If P, Q, and R are polynomials, and Q and R are not 0,

$$\frac{PR}{QR} = \frac{P}{Q}$$

Simplifying Rational Expressions

Simplifying a Rational Expression

1) Completely factor the numerator and denominator.

2) Apply the Fundamental Principle of Rational Expressions to eliminate common factors in the numerator and denominator.

Warning!

Only common FACTORS can be eliminated from the numerator and denominator. Make sure any expression you eliminate is a factor.

Simplifying Rational Expressions

Example

Simplify the following expression.

$$\frac{7x+35}{x^2+5x} = \frac{7(x+5)}{x(x+5)} = \frac{7}{x}$$

Simplifying Rational Expressions

Example

Simplify the following expression.

$$\frac{x^2+3x-4}{x^2-x-20} = \frac{(x+4)(x-1)}{(x-5)(x+4)} = \frac{x-1}{x-5}$$

Simplifying Rational Expressions

Example

Simplify the following expression.

$$\frac{7-y}{y-7}=\frac{-1(y-7)}{y-7}=-1$$

13) $\dfrac{27}{27x+18}$

14) $\dfrac{v^2-7v-30}{v^2-5v-24}$

15) $\dfrac{x^2+8x+12}{x^2+3x-18}$

16) $\dfrac{x^2-11x+18}{x^2+2x-8}$

17) $\dfrac{b^2+3b-28}{b^2-49}$

18) $\dfrac{v^2-3v-40}{v^2-11v+24}$

19) $\dfrac{4n-4}{6n-20}$

20) $\dfrac{v^2-5v-14}{v^2+4v+4}$

Multiplying and Dividing Rational Expressions

Multiplying Rational Expressions

Multiplying rational expressions when P, Q, R, and S are polynomials with $Q \neq 0$ and $S \neq 0$.

$$\frac{P}{Q} \cdot \frac{R}{S} = \frac{PR}{QS}$$

Multiplying Rational Expressions

Note that after multiplying such expressions, our result may not be in simplified form, so we use the following techniques.

Multiplying rational expressions

1) Factor the numerators and denominators.

2) Multiply the numerators and multiply the denominators.

3) Simplify or write the product in lowest terms by applying the fundamental principle to all common factors.

Multiplying Rational Expressions

Example

Multiply the following rational expressions.

$$\frac{6x^2}{10x^3}\cdot\frac{5x}{12}=\frac{\cancel{2}\cdot\cancel{3}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{5}\cdot\cancel{x}}{\cancel{2}\cdot\cancel{5}\cdot\cancel{x}\cdot\cancel{x}\cdot\cancel{x}\cdot 2\cdot 2\cdot\cancel{3}}=\frac{1}{4}$$

Multiplying Rational Expressions

Example

Multiply the following rational expressions.

$$\frac{(m-n)^2}{m+n} \cdot \frac{m}{m^2 - mn} = \frac{\cancel{(m-n)}(m-n)\cdot \cancel{m}}{(m+n)\cdot \cancel{m}\cancel{(m-n)}} =$$

$$\frac{m-n}{m+n}$$

Dividing Rational Expressions

Dividing rational expressions when P, Q, R, and S are polynomials with Q ≠ 0, S ≠ 0 and R ≠ 0.

$$\frac{P}{Q} \div \frac{R}{S} = \frac{P}{Q} \cdot \frac{S}{R} = \frac{PS}{QR}$$

Dividing Rational Expressions

When dividing rational expressions, first change the division into a multiplication problem, where you use the reciprocal of the divisor as the second factor.

Then treat it as a multiplication problem (factor, multiply, simplify).

Dividing Rational Expressions

Example

Divide the following rational expression.

$$\frac{(x+3)^2}{5} \div \frac{5x+15}{25} = \frac{(x+3)^2}{5} \cdot \frac{25}{5x+15} =$$

$$\frac{(x+3)(x+3)\cdot 5\cdot 5}{5\cdot 5(x+3)} = x+3$$

11) $\dfrac{x^2+10x+16}{x^2+6x+8} \div \dfrac{1}{x+4}$

12) $\dfrac{49x+21}{6x} \div \dfrac{42x+18}{6}$

13) $\dfrac{7}{8r-40} \div \dfrac{1}{8r-40}$

14) $\dfrac{1}{2a} \div \dfrac{8a}{2a^2+16a}$

15) $\dfrac{8}{4n^2-16n} \div \dfrac{1}{n-4}$

16) $\dfrac{a-4}{a^2-2a-8} \div \dfrac{1}{a-5}$

17) $\dfrac{b^2-2b-15}{8b+20} \div \dfrac{2}{4b+10}$

18) $\dfrac{10b^2+42b+36}{6b^2-2b-60} \div \dfrac{40b+48}{3b^2-13b+10}$

5) $\dfrac{96}{38n} \cdot \dfrac{25}{45}$

6) $\dfrac{84}{3} \cdot \dfrac{48x}{95}$

7) $\dfrac{6(r+2)}{20} \cdot \dfrac{4r}{6(r+2)}$

8) $\dfrac{7n^2(n+4)}{(n-3)(n+4)} \cdot \dfrac{n-3}{(n+8)(n+6)}$

9) $\dfrac{2(p+6)}{4} \cdot \dfrac{p-3}{2(p-3)}$

10) $\dfrac{9(r+4)}{r+4} \cdot \dfrac{9r}{9(r-5)}$

11) $\dfrac{8(m+1)}{7m} \cdot \dfrac{9}{8(m+1)}$

12) $\dfrac{(p+6)(p-4)}{p-4} \cdot \dfrac{1}{(p-4)(p-2)}$

7) $\dfrac{4n}{n-6} \div \dfrac{4n}{8n-48}$

8) $\dfrac{3}{28b} \div \dfrac{3}{b+1}$

9) $\dfrac{7a^2}{7a^3+56a^2} \div \dfrac{2}{a^2+7a-8}$

10) $\dfrac{6}{28x+4} \div \dfrac{6}{35x+5}$

19) $\dfrac{16x-56}{8} \div \dfrac{8x-28}{4}$

20) $\dfrac{10x^2-28x+16}{2x-4} \div \dfrac{25x^2-25x+4}{5x^2-41x+8}$

21) $\dfrac{6p+27}{18p^2+36p} \div \dfrac{16p+72}{2p+4}$

22) $\dfrac{3x^2-25x-18}{27x+18} \div \dfrac{5x-3}{5x^2-33x+18}$

Units of Measure

Converting Between Units of Measure

Use ***unit fractions*** (equivalent to 1), but with different measurements in the numerator and denominator.

Multiply the unit fractions like rational expressions, canceling common units in the numerators and denominators.

Units of Measure

Example

Convert 1008 square inches into square feet.

$$(1008 \text{ sq in}) \cdot \left(\frac{1 \text{ ft}}{12 \text{ in}}\right) \cdot \left(\frac{1 \text{ ft}}{12 \text{ in}}\right) =$$

$$(\not{2} \cdot \not{2} \cdot \not{2} \cdot \not{2} \cdot \not{3} \cdot \not{3} \cdot 7 \not{\text{in}} \cdot \not{\text{in}}) \cdot \left(\frac{1\, ft}{\not{2} \cdot \not{2} \cdot \not{3}\ \not{in}}\right) \cdot \left(\frac{1\, ft}{\not{2} \cdot \not{2} \cdot \not{3}\ \not{in}}\right) =$$

7 sq. ft.

Pop Quiz

- Convert 190 minutes to A) Hours B)Seconds
- $$\frac{7n^2(n+4)}{(n-3)(n+4)} \cdot \frac{n-3}{(n+8)(n+6)}$$
- $$\frac{b^2-2b-15}{8b+20} \div \frac{2}{4b+10}$$

Adding and Subtracting Rational Expressions with the Same Denominator and Least Common Denominators

Rational Expressions

If P, Q and R are polynomials and Q $\neq$ 0,

$$\frac{P}{R}+\frac{Q}{R}=\frac{P+Q}{R}$$

$$\frac{P}{R}-\frac{Q}{R}=\frac{P-Q}{R}$$

Adding Rational Expressions

Example

Add the following rational expressions.

$$\frac{4p-3}{2p+7}+\frac{3p+8}{2p+7}=\frac{4p-3+3p+8}{2p+7}=\frac{7p+5}{2p+7}$$

Subtracting Rational Expressions

Example

Subtract the following rational expressions.

$$\frac{8y}{y-2}-\frac{16}{y-2}=\frac{8y-16}{y-2}=\frac{8\cancel{(y-2)}}{\cancel{y-2}}=8$$

Subtracting Rational Expressions

Example

Subtract the following rational expressions.

$$\frac{3y}{y^2+3y-10}-\frac{6}{y^2+3y-10}=\frac{3y-6}{y^2+3y-10}=$$

$$\frac{3\cancel{(y-2)}}{(y+5)\cancel{(y-2)}}=\frac{3}{y+5}$$

Least Common Denominators

To add or subtract rational expressions with unlike denominators, you have to change them to equivalent forms that have the same denominator (a ***common denominator***).

This involves finding the ***least common denominator*** of the two original rational expressions.

Least Common Denominators

To find a ***Least Common Denominator***:

1) Factor the given denominators.
2) Take the product of all the *unique* factors.

Each factor should be raised to a power equal to the greatest number of times that factor appears in any one of the factored denominators.

Least Common Denominators

Example

Find the LCD of the following rational expressions.

$$\frac{1}{6y}, \frac{3x}{4y+12}$$

$$6y = 2 \cdot 3y$$

$$4y + 12 = 4(y+3) = 2^2(y+3)$$

So the LCD is $2^2 \cdot 3y(y+3) = 12y(y+3)$

Least Common Denominators

Example

Find the LCD of the following rational expressions.

$$\frac{4}{x^2+4x+3}, \frac{4x-2}{x^2+10x+21}$$

$$x^2+4x+3=(x+3)(x+1)$$

$$x^2+10x+21=(x+3)(x+7)$$

So the LCD is $(x+3)(x+1)(x+7)$

Least Common Denominators

Example

Find the LCD of the following rational expressions.

$$\frac{3x}{5x^2 - 5}, \frac{4x^2}{x^2 - 2x + 1}$$

$$5x^2 - 5 = 5(x^2 - 1) = 5(x+1)(x-1)$$

$$x^2 - 2x + 1 = (x-1)^2$$

So the LCD is $5(x+1)(x-1)^2$

Least Common Denominators

Example

Find the LCD of the following rational expressions.

$$\frac{1}{x-3}, \frac{2}{3-x}$$

Both of the denominators are already factored. Since each is the opposite of the other, you can use either x – 3 or 3 – x as the LCD.

Multiplying by 1

To change rational expressions into equivalent forms, we use the principal that multiplying by 1 (or any form of 1), will give you an equivalent expression.

$$\frac{P}{Q} = \frac{P}{Q} \cdot 1 = \frac{P}{Q} \cdot \frac{R}{R} = \frac{P \cdot R}{Q \cdot R}$$

Equivalent Expressions

Example

Rewrite the rational expression as an equivalent rational expression with the given denominator.

$$\frac{3}{9y^5} = \frac{}{72y^9}$$

$$\frac{3}{9y^5} = \frac{3}{9y^5} \cdot \frac{8y^4}{8y^4} = \frac{24y^4}{72y^9}$$

Adding and Subtracting Rational Expressions with Different Denominators

Unlike Denominators

As stated in the previous section, to add or subtract rational expressions with different denominators, we have to change them to equivalent forms first.

Unlike Denominators

Adding or Subtracting Rational Expressions with Unlike Denominators

1) Find the LCD of all the rational expressions.
2) Rewrite each rational expression as an equivalent one with the LCD as the denominator.
3) Add or subtract numerators and write result over the LCD.
4) Simplify rational expression, if possible.

Adding with Unlike Denominators

Example

Add the following rational expressions.

$$\frac{15}{7a}, \frac{8}{6a}$$

$$\frac{15}{7a} + \frac{8}{6a} = \frac{6 \cdot 15}{6 \cdot 7a} + \frac{7 \cdot 8}{7 \cdot 6a} =$$

$$\frac{90}{42a} + \frac{56}{42a} = \frac{146}{42a} = \frac{73}{21a}$$

Subtracting with Unlike Denominators

Example

Subtract the following rational expressions.

$$\frac{5}{2x-6}, \frac{3}{6-2x}$$

$$\frac{5}{2x-6} - \frac{3}{6-2x} = \frac{5}{2x-6} + \frac{3}{2x-6} =$$

$$\frac{8}{2x-6} = \frac{2 \cdot 2 \cdot 2}{2(x-3)} = \frac{4}{x-3}$$

Subtracting with Unlike Denominators

Example

Subtract the following rational expressions.

$$\frac{7}{2x-3} \text{ and } 3$$

$$\frac{7}{2x-3} - 3 = \frac{7}{2x-3} - \frac{3(2x-3)}{2x-3} =$$

$$\frac{7}{2x-3} - \frac{6x-9}{2x-3} = \frac{7-6x+9}{2x-3} = \frac{16-6x}{2x-3}$$

Adding with Unlike Denominators

Example

Add the following rational expressions.

$$\frac{4}{x^2 - x - 6}, \frac{x}{x^2 + 5x + 6}$$

$$\frac{4}{x^2 - x - 6} + \frac{x}{x^2 + 5x + 6} = \frac{4}{(x-3)(x+2)} + \frac{x}{(x+3)(x+2)} =$$

$$\frac{4(x+3)}{(x-3)(x+2)(x+3)} + \frac{x(x-3)}{(x+3)(x+2)(x-3)} =$$

$$\frac{4x + 12 + x^2 - 3x}{(x+2)(x-3)(x+3)} = \frac{x^2 + x + 12}{(x+2)(x-3)(x+3)}$$

Solving Equations Containing Rational Expressions

Solving Equations

First note that an equation contains an equal sign and an expression does not.

To solve EQUATIONS containing rational expressions, clear the fractions by multiplying both sides of the equation by the LCD of all the fractions.

Then solve as in previous sections.

Note: this works for equations only, not simplifying expressions.

Solving Equations

Example

Solve the following rational equation.

$$\frac{5}{3x}+1=\frac{7}{6}$$

$$6x\left(\frac{5}{3x}+1\right)=\left(\frac{7}{6}\right)6x$$

$$10+6x=7x$$

$$10=x$$

Check in the original equation.

$$\frac{5}{3\cdot 10}+1=\frac{7}{6}$$

$$\frac{5}{30}+1=\frac{7}{6}$$

$$\frac{1}{6}+1=\frac{7}{6}$$

true

Solving Equations

Example

Solve the following rational equation.

$$\frac{1}{2x} - \frac{1}{x+1} = \frac{1}{3x^2 + 3x}$$

$$6x(x+1)\left(\frac{1}{2x} - \frac{1}{x+1}\right) = \left(\frac{1}{3x(x+1)}\right)6x(x+1)$$

$$3(x+1) - 6x = 2$$

$$3x + 3 - 6x = 2$$

$$3 - 3x = 2$$

$$-3x = -1$$

$$x = 1/3$$

Continued.

Solving Equations

Example Continued

Substitute the value for x into the original equation, to check the solution.

$$\frac{1}{2\left(\frac{1}{3}\right)} - \frac{1}{\left(\frac{1}{3}\right)+1} = \frac{1}{3\left(\frac{1}{3}\right)^2 + 3\left(\frac{1}{3}\right)}$$

$$\frac{3}{2} - \frac{3}{4} = \frac{1}{\frac{1}{3}+1}$$

$$\frac{6}{4} - \frac{3}{4} = \frac{3}{4} \quad \textit{true}$$

So the solution is $x = \frac{1}{3}$

Solving Equations

Example

Solve the following rational equation.

$$\frac{x+2}{x^2+7x+10}=\frac{1}{3x+6}-\frac{1}{x+5}$$

$$3(x+2)(x+5)\left(\frac{x+2}{x^2+7x+10}\right)=\left(\frac{1}{3x+6}-\frac{1}{x+5}\right)3(x+2)(x+5)$$

$$3(x+2)=(x+5)-3(x+2)$$

$$3x+6=x+5-3x-6$$

$$3x-x+3x=5-6-6$$

$$5x=-7$$

$$x=-7/5$$

Continued.

Solving Equations

Example Continued

Substitute the value for x into the original equation, to check the solution.

$$\frac{-\frac{7}{5}+2}{\left(-\frac{7}{5}\right)^2+7\left(-\frac{7}{5}\right)+10}=\frac{1}{3\left(-\frac{7}{5}\right)+6}-\frac{1}{\left(-\frac{7}{5}\right)+5}$$

$$\frac{\frac{3}{5}}{\frac{49}{25}-\frac{49}{5}+10}=\frac{1}{-\frac{21}{5}+6}-\frac{1}{\frac{18}{5}}$$

$$\frac{5}{18}=\frac{5}{9}-\frac{5}{18} \qquad \textit{true}$$

So the solution is $x=-\frac{7}{5}$

Solving Equations

Example

Solve the following rational equation.

$$\frac{1}{x-1} = \frac{2}{x+1}$$

$$(x-1)(x+1)\left(\frac{1}{x-1}\right) = \left(\frac{2}{x+1}\right)(x-1)(x+1)$$

$$x+1 = 2(x-1)$$

$$x+1 = 2x-2$$

$$3 = x$$

Continued.

Solving Equations

Example Continued

Substitute the value for x into the original equation, to check the solution.

$$\frac{1}{3-1} = \frac{2}{3+1}$$

$$\frac{1}{2} = \frac{2}{4} \qquad \textit{true}$$

So the solution is $x = 3$.

Solving Equations

Example

Solve the following rational equation.

$$\frac{12}{9-a^2}+\frac{3}{3+a}=\frac{2}{3-a}$$

$$(3-a)(3+a)\left(\frac{12}{9-a^2}+\frac{3}{3+a}\right)=\left(\frac{2}{3-a}\right)(3-a)(3+a)$$

$$12+3(3-a)=2(3+a)$$

$$12+9-3a=6+2a$$

$$21-3a=6+2a$$

$$15=5a$$

$$3=a$$

Continued.

Solving Equations

Example Continued

Substitute the value for x into the original equation, to check the solution.

$$\frac{12}{9-3^2}+\frac{3}{3+3}=\frac{2}{3-3}$$

$$\frac{12}{0}+\frac{3}{5}=\frac{2}{0}$$

Since substituting the suggested value of a into the equation produced undefined expressions, the solution is $\varnothing$.

Solving Equations with Multiple Variables

Solving an Equation With Multiple Variables for One of the Variables

1) Multiply to clear fractions.
2) Use distributive property to remove grouping symbols.
3) Combine like terms to simplify each side.
4) Get all terms containing the specified variable on the same side of the equation, other terms on the opposite side.
5) Isolate the specified variable.

Solving Equations with Multiple Variables

Example

Solve the following equation for R_1

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$RR_1R_2\left(\frac{1}{R}\right) = \left(\frac{1}{R_1} + \frac{1}{R_2}\right)RR_1R_2$$

$$R_1R_2 = RR_2 + RR_1$$

$$R_1R_2 - RR_1 = RR_2$$

$$R_1\left(R_2 - R\right) = RR_2$$

$$R_1 = \frac{RR_2}{R_2 - R}$$

Problem Solving with Rational Equations

Ratios and Rates

Ratio is the quotient of two numbers or two quantities.

The ratio of the numbers a and b can also be written as a:b, or $\frac{a}{b}$.

The units associated with the ratio are important. The units should match.

If the units do not match, it is called a ***rate***, rather than a ratio.

Proportions

Proportion is two ratios (or rates) that are equal to each other.

$$\frac{a}{b} = \frac{c}{d}$$

We can rewrite the proportion by multiplying by the LCD, *bd*.

This simplifies the proportion to $ad = bc$.

This is commonly referred to as the ***cross product***.

Solving Proportions

Example

Solve the proportion for x.

$$\frac{x+1}{x+2}=\frac{5}{3}$$

$$3(x+1)=5(x+2)$$

$$3x+3=5x+10$$

$$-2x=7$$

$$x=-7/2$$

Continued.

Solving Proportions

Example Continued

Substitute the value for x into the original equation, to check the solution.

$$\frac{-\frac{7}{2}+1}{-\frac{7}{2}+2}=\frac{5}{3}$$

$$\frac{-\frac{5}{2}}{-\frac{3}{2}}=\frac{5}{3}$$ *true*

So the solution is $x=-\frac{7}{2}$

Solving Proportions

Example

If a 170-pound person weighs approximately 65 pounds on Mars, how much does a 9000-pound satellite weigh?

$$\frac{170\text{-pound person on Earth}}{9000\text{-pound satellite on Earth}} = \frac{65\text{-pound person on Mars}}{x\text{-pound satellite on Mars}}$$

$$170x = 9000 \cdot 65 = 585{,}000$$

$$x = 585000/170 \approx 3441 \text{ pounds}$$

Solving Proportions

Example

Given the following prices charged for various sizes of picante sauce, find the best buy.

- 10 ounces for $0.99
- 16 ounces for $1.69
- 30 ounces for $3.29

Continued.

Solving Proportions

Example Continued

Size	Price	Unit Price
10 ounces	\$0.99	\$0.99/10 = \$0.099
16 ounces	\$1.69	\$1.69/16 = \$0.105625
30 ounces	\$3.29	\$3.29/30 ≈ \$0.10967

The 10 ounce size has the lower unit price, so it is the best buy.

Similar Triangles

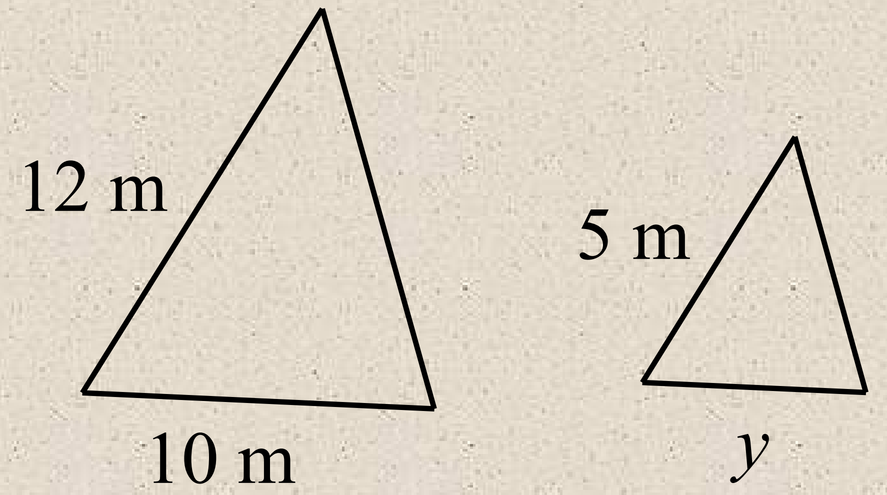
In ***similar triangles***, the measures of corresponding angles are equal, and corresponding sides are in proportion.

Given information about two similar triangles, you can often set up a proportion that will allow you to solve for the missing lengths of sides.

Similar Triangles

Example

Given the following triangles, find the unknown length *y*.



Continued

Similar Triangles

Example

1.) *Understand*

Read and reread the problem. We look for the corresponding sides in the 2 triangles. Then set up a proportion that relates the unknown side, as well.

2.) *Translate*

By setting up a proportion relating lengths of corresponding sides of the two triangles, we get

$$\frac{12}{5} = \frac{10}{y}$$

Continued

Similar Triangles

Example continued

3.) ***Solve***

$$\frac{12}{5} = \frac{10}{y}$$

$$12y = 5 \cdot 10 = 50$$

$$y = {}^{50}\!/_{12} = {}^{25}\!/_{6} \text{ meters}$$

Continued

Similar Triangles

Example continued

4.) ***Interpret***

Check: We substitute the value we found from the proportion calculation back into the problem.

$$\frac{12}{5} = \frac{10}{25/6} = \frac{60}{25} \qquad \textit{true}$$

State: The missing length of the triangle is $25/6$ meters

Finding an Unknown Number

Example

The quotient of a number and 9 times its reciprocal is 1. Find the number.

1.) ***Understand***

Read and reread the problem. If we let

n = the number, then

$\frac{1}{n}$ = the reciprocal of the number

Continued

Finding an Unknown Number

Example continued

2.) ***Translate***

The quotient of a number and 9 times its reciprocal is 1

The quotient of	a number	and 9 times its reciprocal	is	1
$\div$	n	$9\left(\frac{1}{n}\right)$	$=$	1

$$n \div 9\left(\frac{1}{n}\right) = 1$$

Continued

Finding an Unknown Number

Example continued

3.) ***Solve***

$$n \div \left(9 \cdot \frac{1}{n}\right) = 1$$

$$n \div \left(\frac{9}{n}\right) = 1$$

$$n \cdot \frac{n}{9} = 1$$

$$n^2 = 9$$

$$n = 3, -3$$

Continued

Finding an Unknown Number

Example continued

4.) ***Interpret***

Check: We substitute the values we found from the equation back into the problem. Note that nothing in the problem indicates that we are restricted to positive values.

$$3 \div \left(9 \cdot \frac{1}{3}\right) = 1 \qquad -3 \div \left(9 \cdot \frac{1}{-3}\right) = 1$$

$3 \div 3 = 1$ *true* $\qquad -3 \div -3 = 1$ *true*

State: The missing number is 3 or –3.

Solving a Work Problem

Example

An experienced roofer can roof a house in 26 hours. A beginner needs 39 hours to do the same job. How long will it take if the two roofers work together?

1.) ***Understand***

Read and reread the problem. By using the times for each roofer to complete the job alone, we can figure out their corresponding work rates in portion of the job done per hour.

	Time in hrs	Portion job/hr
Experienced roofer	26	1/26
Beginner roofer	39	/39
Together	t	1/t

Continued

Solving a Work Problem

Example continued

2.) ***Translate***

Since the rate of the two roofers working together would be equal to the sum of the rates of the two roofers working independently,

$$\frac{1}{26}+\frac{1}{39}=\frac{1}{t}$$

Continued

Solving a Work Problem

Example continued

3.) ***Solve***

$$\frac{1}{26}+\frac{1}{39}=\frac{1}{t}$$

$$78t\left(\frac{1}{26}+\frac{1}{39}\right)=\left(\frac{1}{t}\right)78t$$

$$3t+2t=78$$

$$5t=78$$

$$t=78/5 \text{ or } 15.6 \text{ hours}$$

Continued

Solving a Work Problem

Example continued

4.) ***Interpret***

Check: We substitute the value we found from the proportion calculation back into the problem.

$$\frac{1}{26}+\frac{1}{39}=\frac{1}{78/5}$$

$$\frac{3}{78}+\frac{2}{78}=\frac{5}{78} \quad \textit{true}$$

State: The roofers would take 15.6 hours working together to finish the job.

Solving a Rate Problem

Example

The speed of Lazy River's current is 5 mph. A boat travels 20 miles downstream in the same time as traveling 10 miles upstream. Find the speed of the boat in still water.

1.) *Understand*

Read and reread the problem. By using the formula $d=rt$, we can rewrite the formula to find that $t = d/r$.

We note that the rate of the boat downstream would be the rate in still water + the water current and the rate of the boat upstream would be the rate in still water – the water current.

	Distance	rate	time = d/r
Down	20	$r + 5$	$20/(r + 5)$
Up	10	$r - 5$	$10/(r - 5)$

Continued

Solving a Rate Problem

Example continued

2.) ***Translate***

Since the problem states that the time to travel downstairs was the same as the time to travel upstairs, we get the equation

$$\frac{20}{r+5} = \frac{10}{r-5}$$

Continued

Solving a Rate Problem

Example continued

3.) ***Solve***

$$\frac{20}{r+5}=\frac{10}{r-5}$$

$$(r+5)(r-5)\left(\frac{20}{r+5}\right)=\left(\frac{10}{r-5}\right)(r+5)(r-5)$$

$$20(r-5)=10(r+5)$$

$$20r-100=10r+50$$

$$10r=150$$

$$r=15\text{ mph}$$

Continued

Solving a Rate Problem

Example continued

4.) ***Interpret***

Check: We substitute the value we found from the proportion calculation back into the problem.

$$\frac{20}{15+5} = \frac{10}{15-5}$$

$$\frac{20}{20} = \frac{10}{10} \quad \textit{true}$$

State: The speed of the boat in still water is 15 mph.

Simplifying Complex Fractions

Complex Rational Fractions

Complex rational expressions (***complex fraction***) are rational expressions whose numerator, denominator, or both contain one or more rational expressions.

There are two methods that can be used when simplifying complex fractions.

Simplifying Complex Fractions

Simplifying a Complex Fraction (Method 1)

1) Simplify the numerator and denominator of the complex fraction so that each is a single fraction.
2) Multiply the numerator of the complex fraction by the reciprocal of the denominator of the complex fraction.
3) Simplify, if possible.

Simplifying Complex Fractions

Example

$$\frac{\frac{x}{2}+2}{\frac{x}{2}-2} = \frac{\frac{x}{2}+\frac{4}{2}}{\frac{x}{2}-\frac{4}{2}} = \frac{\frac{x+4}{2}}{\frac{x-4}{2}} = \frac{x+4}{\cancel{2}} \cdot \frac{\cancel{2}}{x-4} = \frac{x+4}{x-4}$$

Simplifying Complex Fractions

Method 2 for ***simplifying a complex fraction***

1) Find the LCD of all the fractions in both the numerator and the denominator.
2) Multiply both the numerator and the denominator by the LCD.
3) Simplify, if possible.

Simplifying Complex Fractions

Example

$$\frac{\dfrac{1}{y^2}+\dfrac{2}{3}}{\dfrac{1}{y}-\dfrac{5}{6}}\cdot\frac{6y^2}{6y^2}=\frac{6+4y^2}{6y-5y^2}$$