Solving Equations Containing

Rational Expressions

First, we will look at solving these problems algebraically.

Here is an example that we will do together using two different methods.

$$\frac{7}{x+2} = \frac{6}{x-5}$$

The best way to solve a rational equation:

Eliminate the fractions

This can be done by multiplying each side of the equation by the LCD.

What is the LCD? (x+2)(x-5)

$$[(x+2)(x-5)]\frac{7}{x+2} = \frac{6}{x-5}[(x+2)(x-5)]$$

$$7(x-5) = 6(x+2)$$

$$7x - 35 = 6x + 12$$

$$-6x \qquad -6x$$

$$x - 35 = 12$$

$$+35 \quad +35$$

$$x = 47$$

It is VERY important that you check your answers!!!!!!!!!!!!!!!!!

Check:

$$\frac{7}{x+2} = \frac{6}{x-5}$$

$$\frac{7}{47+2} = \frac{6}{47-5}$$

$$\frac{7}{49} = \frac{6}{42}$$

$$\frac{1}{7} = \frac{1}{7}$$

The other method of solving rational equations is cross-multiplication.

$$\frac{7}{x+2} = \frac{6}{x-5}$$

$$7 \cdot (x-5) = 6 \cdot (x+2)$$

$$7x - 35 = 6x + 12$$

$$-6x \qquad -6x$$

$$x - 35 = 12$$

$$+35 \quad +35$$

$$x = 47$$

Here is another example that we will do together:

$$\frac{x+1}{3x+6} = \frac{5x}{6} + \frac{1}{x-2}$$

Step 1: Find the LCD

Hint: Factor the denominator

$$\frac{x+1}{3x-6}$$

This denominator can be factored into 3(x-2)

Therefore….

$$\text{LCD} = 6(x-2)$$

Step 2: Multiply both sides of equation by LCD. This eliminates the fraction.

$$6(x-2)\left[\frac{x+1}{3x-6}=\frac{5x}{6}+\frac{1}{x-2}\right]$$

Step 3: Solve for x

$$2(x+1) = 5x(x-2) + 1 \cdot 6$$

$$2x + 2 = 5x^2 - 10x + 6$$

$$-2x - 2 \qquad\qquad -2x - 2$$

$$0 = 5x^2 - 12x + 4$$

$$0 = (5x - 2)(x - 2)$$

$$x = \frac{2}{5} \qquad x = 2$$

Since there are two answers, there needs to be two checks.

Let $x = \frac{2}{5}$

$$\frac{x+1}{3x-6} = \frac{5x}{6} + \frac{1}{x-2}$$

$$\frac{\frac{2}{5}+1}{3(\frac{2}{5}-6)} = \frac{5(\frac{2}{5})}{6} + \frac{1}{\frac{2}{5}-2}$$

$$\frac{\frac{7}{5}}{3(-\frac{8}{5})} = \frac{2}{6} + \frac{1}{-\frac{8}{5}}$$

$$\frac{\frac{7}{5}}{-\frac{24}{5}} = \frac{2}{6} + -\frac{5}{8}$$

$$-\frac{7}{24} = -\frac{7}{24}$$

Check #2:

Let x = 2

$$\frac{x+1}{3x-6} = \frac{5x}{6} + \frac{1}{x-2}$$

$$\frac{2+1}{3(2)-6} = \frac{5(2)}{6} + \frac{1}{2-2}$$

$$\frac{3}{0} = \frac{10}{6} + \frac{1}{0}$$

When you check the number 2, you get a zero in the denominator. This means that 2 can not be a solution.

Now, you do these on your own.

1) $\dfrac{4}{x-1} = \dfrac{x+1}{12}$ $x = \pm 7$

2) $\dfrac{4t-3}{5} - \dfrac{4-2t}{3} = 1$ $t = 2$

3) $\dfrac{10}{m^2-1} + \dfrac{2m-5}{m-1} = \dfrac{2m+5}{m+1}$ $m = \dfrac{5}{3}$

Example #3:

A car travels 500 miles in the same time that a train travels 300 miles. The speed of the car is 30 miles per hour faster than the speed of the train. Find the speed of the car and the train.

Remember the formula d=rt where:

r = rate of speed

d = distance

t = time

Since both vehicles travel the same amount of time, solve the formula for t.

Identify the variables that you are going to use.

Let r = speed of the train

How do you represent the speed of the car?

Let r+30 = speed of the car

Car's time = Train's time

$$t = \frac{d}{r} \qquad t = \frac{d}{r}$$

$$t = \frac{500}{r+30} \qquad t = \frac{300}{r}$$

$$\frac{500}{r+30} = \frac{300}{r}$$

How would you solve this equation?

$$\frac{500}{r+30} = \frac{300}{r}$$

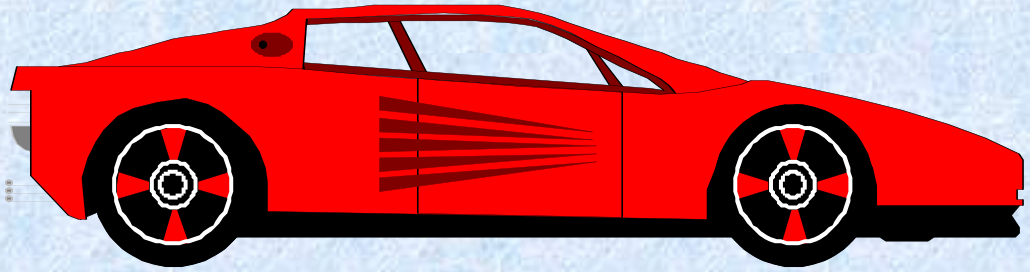
$500 \cdot r = 300(r+30)$ ⟶ Cross-multiply

$$500r = 300r + 9000$$
$$-300r \quad -300r$$
$$200r = 9000$$

$$r = 45$$

Make sure that you answer the question.

ANSWER:



The car travels at a speed of 75mph

The train travels at a speed of 45 mph