

24 pts

Use implicit differentiation to find the derivative of each equation below. Simplify answers as much as possible and leave no negative exponents.

1) $y^5 + 4x^2y^2 + 5x^4 = 12$

$$5y^4 \frac{dy}{dx} + 8xy^2 + 4x^2 \cdot 2y \frac{dy}{dx} + 20x^3 = 0 \quad (1)$$

$$5y^4 \frac{dy}{dx} + 8x^2y \frac{dy}{dx} = -20x^3 - 8xy^2 \quad (2)$$

$$\frac{dy}{dx} = \frac{-20x^3 - 8xy^2}{5y^4 + 8x^2y}$$

2) $2\sqrt{y+x} = 3x^2$

$$2 \cdot \frac{1}{2} (y+x)^{-1/2} \left(\frac{dy}{dx} + 1 \right) = 6x$$

$$\frac{1}{(y+x)^{1/2}} \left(\frac{dy}{dx} + 1 \right) = 6x$$

$$\frac{dy}{dx} = 6x(y+x)^{1/2} - 1$$

$$3) (4y^2 - 1)^3 = (3x^2 - x)^4$$

$$3(4y^2 - 1)^2 (8y \frac{dy}{dx}) = 4(3x^2 - x)^3 (6x - 1)$$

$$\frac{dy}{dx} = \frac{4(3x^2 - x)^3 (6x - 1)}{3(4y^2 - 1)^2 (8y)}$$

$$\boxed{\frac{dy}{dx} = \frac{(3x^2 - x)^3 (6x - 1)}{6y(4y^2 - 1)^2}}$$

4) Find the equation of the tangent line to the implicit function

$$x^3 - 8xy + y^3 = 0 \text{ at the point } (4, 4).$$

$$3x^2 - 8y - 8x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 0$$

$$-8x \frac{dy}{dx} + 3y^2 \frac{dy}{dx} = 8y - 3x^2$$

$$\frac{dy}{dx} = \frac{8y - 3x^2}{-8x + 3y^2}$$

$$\frac{dy}{dx} = \frac{8(4) - 3(4)^2}{-8(4) + 3(4)^2}$$

$$\frac{dy}{dx} = \frac{32 - 48}{-32 + 48} = -1 \quad \text{slope}$$

$$y = mx + b$$

$$4 = (-1)(4) + b$$

$$b = 8$$

$$\boxed{y = -x + 8}$$

