Name________________________ Quiz - Exponential Equations and Logs version 1 2015.doc

1) Write an exponential equation for each situation described. (For the problems below use the non-continuous form unless continuous growth or decay is explicitly stated.)

a) A population is 4,000 and is increasing at a continuous rate of 2.5%.

y = $4000 e^{.025x}$

b) A population 56,000 and is decreasing at a continuous a rate of 6%.

y = $56000 e^{-.06x}$

c) A population is 8,000 and is increasing at a rate of 12% annually.

y = $8000 (1.12)^{x}$

d) A population 1,000 and is decreasing at a rate of 0.09% ~~every 10 years.~~ 10 times ~~per~~ year

y = $1000 (1 - .0009)^{10x}$ or $y = 1000(.9991)^{10x}$

e) A population 2,500 and is doubling every 35 years.

y = $2500 (2)^{x/35}$

f) A bank account is started with an $8,000 deposit and the interest rate is 3% compounded daily.

y = $8000 \left(1 + \frac{.03}{365}\right)^{365x}$

g) A bank account is started with an $8,000 deposit and the interest rate is 3% compounded continuously.

y = $8000 e^{.03x}$

h) A 100 mg sample of an isotope has a half-life of 5700 years.

y = $100 \left(\frac{1}{2}\right)^{x/5700}$

2) Write an exponential equation for each situation described. (For the problems below use the non-continuous form unless continuous growth or decay is explicitly stated.)

a) The population of Ulster County was 177,750 in 2000 and was 181,305 in 2009. Write an exponential equation for population, since 2000, as a function of time in years. (Let 2000 equal time 0)

$\frac{181{,}305}{177{,}750} = 1.02$

$2000 - 2009 = 9$ years

p = $177{,}750\,(1.02)^{x/9}$

b) For every 1000m increase of elevation, the atmospheric pressure is reduced by 11.5%. Write an exponential equation for pressure as a function of elevation in meters.
(Let P_o stand for the atmospheric pressure at 0 meters.)

p = $P_o\,(1 - .115)^{x/1000}$ or $P_o\,(.885)^{x/1000}$

3) The data in the table below can be described by an exponential function. Determine the value of a and y_o for the set of data.

x	y
0	2500
1	250
3	2.5
5	.025

$y = y_o(a)^{x/2}$ $a =$ $.01$ $y_o =$ 2500

$\frac{2.5}{250} = .01$

4) At the beginning of an experiment, two Petri dishes were filled with bacteria. Petri dish A's population can be expressed by $A(x) = 100(0.7)^x$ where x is the number of days. Petri dish B's population can be expressed by $B(x) = 20(1.2)^x$. Answer each question below.

a) Approximately how many days will it take for Petri dish A's population to consist of no more than 1 bacteria?

$$1 = 100(.7)^x$$

$$.01 = .7^x$$

$$\log .01 = x \log .7$$

$$\frac{\log(.01)}{\log(.7)} = x$$

$$x = 12.911$$

$$\boxed{13 \text{ days}}$$

b) After how many days do the two Petri dishes have the same number of bacteria? (Round answer to the nearest thousanth of a day.)

$$100(.7)^x = 20(1.2)^x$$

$$5 = \left(\frac{1.2}{.7}\right)^x$$

$$5 = (1.71428\ldots)^x$$

$$x = 2.985989\ldots$$

$$\boxed{2.986 \text{ days}}$$

5) The population of Appleton is increasing at a rate represented by the equation: $p = p_0(1.025)^x$. At this rate, how will it take for the population to double? Round to the nearest hundredth.

$2 = 1.025^x$

$\log 2 = x \log 1.025$

$x = 28.071$

$\boxed{\text{About 28 days}}$

or

$\log_{1.025}(2) = x$

$x = 28.071$

6) Use **log** or **ln** to solve for x in the following equation. Round answer to the nearest thousandth (3 decimal places). Show your work!!!

$$30e^{0.10x} = 5e^{0.12x}$$

$6 = \dfrac{e^{.12x}}{e^{.10x}}$

$6 = e^{.02x}$

$\ln 6 = .02x$

$89.5879... = x$

$\boxed{x = 89.588}$

or

$6e^{.10x} = e^{.12x}$

$\ln 6 + .10x \ln e = .12x \ln e$

$\ln 6 = .12x - .10x$

$\ln 6 = .02x$

$89.588 \quad x$

Bonus: Find two positive numbers such that the sum of the first and twice the second is 100 and their product is a maximum.

$x + 2y = 100$

$x = 100 - 2y$

$P = xy$

$P = (100 - 2y)y$

$P = 100y - 2y^2$

$P' = 100 - 4y$

$4y = 100$

$\boxed{y = 25 \quad x = 50}$