Developing Spatial Sense and Communication Skills

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Do you find that your students sometimes struggle when communicating their mathematical ideas? Do you believe that developing good spatial sense is important in the mathematics classroom? To jump-start a mathematical conversation among you and your students, complete this exercise: Glance at **figure 1a** for no longer than three seconds, then draw what you saw. When you are done, look at the image and compare it with your drawing. What did you see? How did you draw it? Ask a colleague to do the same. Did he or she draw the diagram differently? How did you describe the image, and how did your colleague?

The NCTM encourages educators to link standards rather than focus on one standard at a time. The Content Standards and the Process Standards work hand in hand. For example, the Number and Operations Standard describes the expectations of developing number sense, making reasonable estimates, and using number knowledge in solving problems (NCTM 2000). The Geometry Standard asks that students use visualization, spatial reasoning, and geometric knowledge to solve problems. The Measurement Standard calls for students to under-

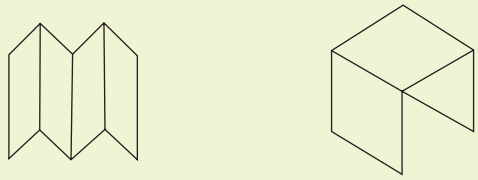
stand measurable attributes and apply appropriate techniques and tools to determine measurement (NCTM 2000). All these goals are greatly enhanced by the use of spatial sense. Communication, a Process Standard, explores the importance of students' being able to explain the process in mathematical terms (NCTM 2000).

In this article, we discuss how we incorporated a spatial activity into our preservice teacher education program. Our approach was to work with preservice teachers to enhance their spatial reasoning both in their education classes and as they worked with a middle-grades student on similar tasks. Our goal in this article is not only to encourage the incorporation of more spatial activities into the middle-grades curriculum but also to encourage spatial instruction with preservice teachers. It is important for them to experience spatial tasks for themselves and then practice implementing the tasks with middle-grades students. However, this article discusses the need for *all* teachers and students to develop spatial sense and communication skills.

QUICK DRAW ACTIVITY: DEVELOPING SPATIAL SENSE AND COMMUNICATION SKILLS

One goal in both elementary and middle-grades mathematics methods classes is to improve preservice teachers' abilities to communicate mathematically. To achieve this goal, we use Quick Draw (Wheatley 2007), a spatial-visualization activity. Participants are shown a geometric figure, displayed using an overhead projector or document camera, for three seconds. They are then asked to draw on unlined paper what they saw. After the drawings are complete, the image is then displayed and discussed. "Quick Draw is designed to develop powerful mental imagery that will come into play in both numerical and geometric settings to encourage students to explore alternative ways to solve a problem" (Wheatley 2007, p. 1). Our preservice teachers are also asked to present Quick Draw in a series of five interactions with students during the semester. **Figure 1** shows examples of Quick Draw illustrations (Wheatley 2007).

Fig. 1 These images begin a mathematical conversation.

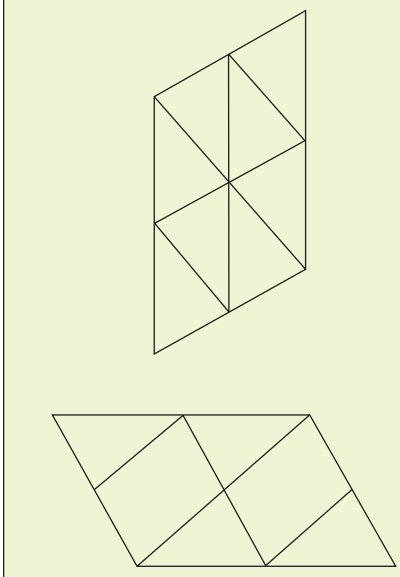


Source: Wheatley 2007

Our mathematics methods classes meet once a week and begin each class by presenting Quick Draw images. Each image is part of a series of two, with similarities between each. The images become more complex as the semester progresses and as the preservice teachers' spatial sense improves. **Figure 2** is an example of a series. After our Quick Draw activity, we review preservice teachers' field experiences (they spend ten hours per week in the schools), discuss assignments from the previous week (articles read, lessons taught), and work on problem-solving activities that relate to the middle-grades curriculum. We find that encouraging our preservice teachers to exercise their spatial abilities at the beginning of each class is an excellent way to then continue to build on a topic through mathematical conversations.

The Process Standard of Communication contains unique qualities for the middle grades. "For example," as noted by NCTM (2000), "mathematics under discussion in grades 6–8 is generally more complex and perhaps more abstract than the mathematics in lower grades" (p. 268). According to *Principles and Standards for School Mathematics,* middle-grades students must explain their thinking in multifaceted ways, and teachers need to establish social norms in the classroom that enable students to communicate openly and freely. We believe that linking communication and spatial abilities in methods classes will help preservice teachers do the same in their future work with students.

Fig. 2 Ths is an example of a series that is shown to students.



Source: Wheatley 2007

During the Quick Draw activity, our preservice teachers vary both in how they draw and interpret the figures. This diversity creates a classroom environment in which communicating mathematically becomes a regular

occurrence. As the semester progresses and more complex figures are used, communication occurs at deeper levels. Vocabulary, visual metaphors, and visual capture strategies are shared. Although the preservice teachers are surprised at the short amount of time they are allowed to view the shape, they soon adjust to the short exposure that is essential in building mental imagery. The conversation about **figure 1a** that follows occurred early in the semester in a middle-grades methods class. Note the simple descriptions given by each preservice teacher. (All names are pseudonyms throughout the article.)

Instructor: How did you draw the figure?
Sal: I just sort of guessed at it and drew upside down Vs and connected them.
Pat: I saw a fan and drew the top of the fan then the bottom of the fan and connected them. I noticed the connecting lines were all parallel, which helped me get the lines in the right place.
Sal: Oh, I didn't realize those lines were parallel. I need to look for that next time.

Later in the semester, the preservice teachers tended to use more geometric vocabulary and elaborate on the explanations, as noted in the following conversation with the same two students who are discussing **figure 1b**:

Sal: At first I saw a three-dimensional cube, but after I really concentrated on the picture in my mind, I realized the outer shape was a hexagon with one missing side. So I erased the missing side and drew the Y inside the shape and drew that other diagonal from the center.
Pat: Wait, I don't see the hexagon.
Sal: See, forget about the inside; look at the outside only.
Pat: Oh! I see! How did you see that? I never would've gotten it that way.
Instructor: How did *you* see it?
Pat: I actually saw it as a cube with a missing bottom, but I drew it much differently than I saw it.
Instructor: What do you mean?
Pat: I drew the rhombus up top first so I would have a starting point. Then I simply connected the lines down from that, and it actually came out looking like the original cube. I like her way though [motioning to the student who initially spoke].

Beginning each class with Quick Draw helps activate our preservice teachers' spatial reasoning and communication skills. They develop their mathematics vocabulary by recognizing that *rhombus* is the mathematics term for what is loosely called a diamond. Our middle-grades preservice teachers more consistently use terms such as *acute angle* and *obtuse angle* rather than *line* and other less precise geometric terms, as especially noted with Sal. There is clear evidence in Sal's activity that she was developing her spatial sense. Students who have well-developed spatial sense tend to solve tasks in more meaningful ways, not just as rote learners (Reynolds and Wheatley 1997), which is why spatial instruction is important. It could be argued that Sal is progressing in her thinking about spatial tasks. It is our hope as mathematics educators that she will provide the same experiences for students in her own classroom. Interaction with students is one way for our preservice teachers to look at what they have experienced using spatial tasks and focus on students' mathematical activity.

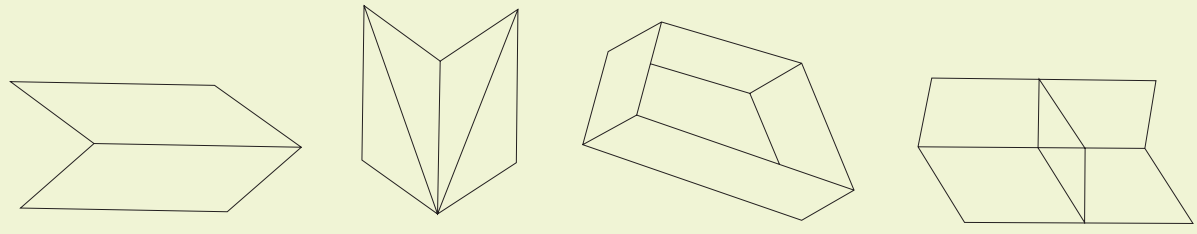
STUDENT ASSIGNMENT AND QUICK DRAW

To help our preservice teachers focus on students' thinking, their assignments include conducting five interactions with middle schoolers using Quick Draw. One purpose of the interview is to learn how a student communicates and thinks mathematically on a spatial-visualization task. During the interviews, the preservice teacher begins the session with Quick Draw, then moves on to a problem-solving task. A written report of one such interview is due every other week, which purposefully coincides with articles assigned concerning spatial sense. After all are completed, a final report summarizes the five interviews.

For the spatial activity portion of the student interaction, we emphasize that our preservice teachers pay close attention to the way that the questions are worded in the Quick Draw instructions. For example, Wheatley (2007) emphasizes that the preservice teachers should avoid either making judgments or encouraging the student to make judgments about the correctness of their drawing. He also offers specific questioning techniques and suggestions regarding administration of the task. Since the goal is to build on mental imagery in particular, the text emphasizes presenting the images for three seconds so the middle-grades students are not just copying the image.

When setting up the student interactions, we instruct our preservice teachers to consult with their cooperating teacher at the beginning of the semester as to which student they choose to interview. In some

Fig. 3 Student and preservice teacher interactions often begin using these images.



Source: Wheatley 2007

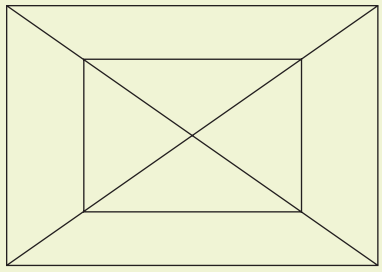
instances, the preservice teacher is allowed to pick whomever they wish; in other instances, the cooperating teacher may recommend a specific student. The interviews are often scheduled one on one in a quiet place, such as the school library. At times we find that our preservice teachers are asked to implement Quick Draw with the entire class they are placed in for their field experience (not their student teaching). When this happens, our preservice teachers experience what Wheatley intended, which is to have rich mathematical discussions between and among students and teachers.

However, our preservice teachers are not always placed in classrooms where implementing Quick Draw with an entire class of middle-grades students (or even small groups) can be done consistently. Keep in mind, the structure of the field experiences in our teacher education program requires that our preservice teachers first *observe* middle-grades classes, then work with small groups of students, and ultimately finish with student teaching. Our preservice teachers typically fall somewhere at the beginning or middle of this scenario when they are taking mathematics methods courses. Therefore, we highlight their Quick Draw experiences with middle-grades students from a one-on-one perspective.

Our preservice teachers often begin their first few student interactions using Quick Draw images such as those shown in **figure 3a** and **3b**. However, they often use their own discretion and choose more advanced images (as shown in **fig. 3c** and **3d**,) based on the reaction and performance of the middle-grades student. For example, if a student quickly draws the image presented to him or her with little effort, that may be a sign to try more difficult images. Our preservice teachers are often surprised at how much quicker middle-grades students catch on to the activity compared with the preservice teachers in methods class. They frequently remark that the middle schoolers ask for more images to review, because they enjoy the activity so much. Their reflections also indicate that as the third, fourth, and fifth interviews are conducted, the middle-grades student tends to explore more and more avenues during the problem-solving portion of the interview, which consists of an open-ended problem-solving task. This is important because our preservice teachers experience how a spatial task can enhance a problem-solving task for a middle-grades student. The following is an example of one task:

> A rectangular box holds 48 cubes. When you open the top, how many might you see? For each possibility, list how many you would see from an end. (Wheatley and Abshire 2002)

Fig. 4 Students describe an illustration differently when they must draw it.



Source: Wheatley 2007

In terms of the mathematical language that comes out of the Quick Draw portion of the student interaction, our preservice teachers often describe rich discussions they had about an image with their middle-grades student. They notice that their middle-grades students saw or described the image differently than how they *drew* the image. For example, one preservice teacher made these comments about the diagram in **figure 4**:

> Prior to revealing the image I had just shown, I asked him how he saw the image. He said he first *saw* four trapezoids but then noticed a hallway. But then when I asked how he *drew* the image, he said he drew a large rectangle, a small one inside of it, and then placed an X across it. I was surprised at the complex way he viewed the image in only three seconds but then drew it using a simple technique.

Every other week, our preservice teachers bring in work from the student interactions, such as Quick Draw images made by the student and any notes or observations about their time together. The purpose is to monitor the preservice teachers' progress with their middle-grades student. In class, we discuss the interactions and encourage questions and conversations about various findings. During these conversations, preservice teachers may mention difficulties in getting their middle-grades student to express ideas. They also use the time to share positive experiences they are having. They often explain how much progress their middle-grades student has made as a result of the tools from methods class discussions. Better communication of ideas and an improvement of the Quick Draw images are the two most noted comments made by our preservice teachers about their middle-grades student.

Once all five interactions are completed, our preservice teachers compile all their artifacts and notes into one report, which is approximately four pages long. The excerpts that follow are from the reports that we found to be the most noteworthy.

This excerpt is from the report of preservice teacher 1, referring to a seventh grader:

> I also believe beginning with Quick Draw made visualization more of a viable option for her. This translated for me into the knowledge that spatial sense and visualization can give children the power to think about tasks in a more concrete way.

Preservice teacher 2 discusses an eighth grader:

> As the interactions continued, Brad's mathematical thinking became more developed. For instance, when initially explaining his Quick Draws, Brad explained his drawings by relating them to common objects, like accordions and tents and baseballs or mirrors. . . . [In a later interaction, instead] of relating the pictures to common objects, Brad used terms like "cube, trapezoid, and triangle." In explaining his interactions, Brad would really think before speaking. . . . Brad throughout his interactions would use geometric terminology whenever he felt certain he was using the correct word. However, if he wasn't sure of the shape he'd drawn, he would relate his picture back to a common object.

The preservice teachers noticed that the students started using terms and speaking more "mathematically" because they were making connections with their prior knowledge

In addition to learning about students' thinking and communication, preservice teachers learned the importance of fostering communication as they teach mathematics. They learned that communication occurred more and more as they enabled students to think and talk about a particular image. They often noted that when they displayed the image after their student had drawn it (the accurate way of implementing Quick Draw), even more communication occurred. It was often indicated that the communication aspect of the spatial activity is what made the student's thinking come alive, as found in the excerpts below.

This excerpt is from the report of preservice teacher 3, referring to an eighth grader:

> My reaction at the end of the first interaction was that I really had to consider exactly what I asked and how I asked questions. In order to learn about Emily's thinking and learning, I didn't want to lead Emily but also wanted her to understand what I needed her to do. A good example came from our first Quick Draw activity. Emily was fairly successful in drawing the shapes . . . but I wanted her to explain how she saw the shape. I asked her, "How did you draw the shape?" Her answer was, "I just drew it, I drew this line, then this one, etc.!" I realized that I needed to rephrase the question so she would understand that I wanted to learn how she looked at the original picture in order to remember how to lay it out on her paper. I then asked her, "When you looked at the shape, what did you do to remember it so you could draw it when I covered it up?" This question led to a much more complete answer. Teaching involves carefully planning what you want to say.

Preservice teacher 4's report contains this information:

> This child interaction has given me power in teaching mathematics. . . . Through this experience, I have learned how valuable it is not to automatically give affirmation and answers. Before, I would automatically tell a child what was right or wrong. I would tell a child with the right answer "good job" without asking how they got that answer. . . . But in the end, we both enjoyed the exchange of mathematical knowledge

that occurred when we had conversations over what would make sense rather than merely checking a right or wrong answer. Quick Draw also gave a great opportunity for math dialogue. Overall, I learned the importance of a two-way, student dominated conversation about math.

Preservice teacher 5 discusses a seventh grader:

When we discussed the images, he related them to things in his life. . . . After learning some geometric vocabulary, he started using it to describe other images. I think that the Quick Draw activities also ran over into his other work during our interactions. He was normally a little distracted during our interactions. He would talk to other students or randomly draw something, not really paying attention; however, after working with Quick Draw, he began to concentrate on the problems I gave him. I also tried Quick Draw with the Algebra class that I taught. . . . It's a great icebreaker at the beginning of class. I think that it helped them focus on the task at hand for the day. I had worked with that class before and it was hard to get them settled down, but with Quick Draw they were instantly engaged and class started off a lot smoother than normal. Quick Draw is nothing but a complete advantage, and I plan on using it in my future math classes.

This excerpt is from the report of preservice teacher 6, referring to an eighth grader:

Her vocabulary was very mathematical. She used words such as triangles, squares, and parallelograms. . . . From the beginning, she used mathematical terms to describe the pictures. As we progressed through the interactions, I increased the level of difficulty of the drawings. She was challenged by the drawings that used dashed lines. . . . I asked her at our last session if she learned anything from doing Quick Draw. She said that this activity has made her more observant of details in her classes. By doing the Quick Draw activity, I learned a couple of things. For someone who is relatively quiet, she [the middle-grades student] was eager to talk about her drawing and how she did it. It appears to be an activity that inspires children to talk . . . she was very consistent in use of geometric terms and became more thorough in her descriptions of her drawings.

Another point we found interesting was how Quick Draw seemed to draw out not only communication about spatial tasks but also mathematical language. The preservice teachers noticed that the students started using terms and speaking more "mathematically" because they were making connections with their prior knowledge.

One preservice teacher, who regularly implemented Quick Draw with an entire class of middle-grades students, noted that although the students were exposed to mathematical vocabulary on a continuing basis in the classroom, they seemed to use it more readily during the Quick Draw activity. She compared the experience with learning a list of vocabulary words, then being asked to use those words in a sentence. Students also began to discuss terminology that they heard their classmates using. For example, once two or three students began using a word, others would pick it up as well. Overall, we thoroughly enjoyed and continue to enjoy implementing Quick Draw with our preservice teachers and find that they feel the same as they implement the activity with their own students.

REFLECT AND DISCUSS: Developing Spatial Sense and Communication Skills

Reflective teaching is a process of self-observation and self-evaluation. It means looking at your classroom practice, thinking about what you do and why you do it, then evaluating whether it works. By collecting information about what goes on in our classrooms and then analyzing and evaluating these data, we identify and explore our own practices and underlying beliefs.

The following questions related to "Developing Spatial Sense and Communication Skills" are suggested prompts to help you reflect on the article and on how the authors' ideas might benefit your own classroom practice. You are encouraged to reflect on the article independently as well as discuss it with your colleagues.

- In what ways is communication about mathematics encouraged in your classroom or classrooms you have observed?
- What type of classroom environment is needed for students to express their mathematical thinking?
- How do you think students and teachers can benefit from spatial instruction? How do these benefits align with both state and national mathematics standards?
- How can the integration of spatial instruction with communication enhance problem solving and number sense in your classroom?

You are invited to tell us how you used "Reflect and Discuss" as part of your professional development. The Editorial Panel appreciates the interest and values the views of those who take the time to send us their comments. Letters may be submitted to *Mathematics Teaching in the Middle School* at **mtms@nctm.org**. Please include "Readers Write" in the subject line. Because of space limitations, letters and rejoinders from authors beyond the 250-word limit may be subject to abridgement. Letters are also edited for style and content.

FINAL THOUGHTS

In this article, we have described how we incorporated meaningful spatial-reasoning tasks into our mathematics teacher education program. A key component in our activities is giving preservice teachers time to communicate. The majority of teachers have had minimal experiences with spatial tasks as part of their own K–12 mathematics curriculum; thus, their spatial ability is underdeveloped. The importance of thinking spatially is being increasingly recognized (National Research Council 2006). Not only do we believe it is important for our preservice teachers to develop their own spatial abilities, we also think it is equally important for them to present the in-class activities to the middle-grades or elementary students. In the future, we plan to implement "The Wheatley Test of Spatial Ability" with our preservice teachers and with middle-grades students to collect empirical data on the progress of their spatial abilities. Spatial tasks are an important component of our methods curriculum, because they develop the thinking of our preservice teachers.

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