P-V Diagrams Have More to Offer

Textbooks that cover thermodynamics at an introductory level routinely follow an introduction to the First Law with a number of applications involving ideal gas processes. These processes are usually illustrated by the use of appropriate *P-V* diagrams.

However, use of these diagrams is commonly restricted to a few of the following four simple cases: reversible or constant pressure expansions under isothermal conditions and reversible or constant pressure expansions under adiabatic conditions. Several other applications of *P-V* diagrams are suggested here.

Various Thermodynamic Processes

The individual, verbal definitions of isobaric *(dP* = 0), isothermal *(dT* = 0), adiabatic *(Q* = 0), and isochoric *(dV*= 0) processes are easily understood by students. Figure 1 is a useful way to present a visual distinction between these four processes. It also emphasizes that an ideal gas subjected to these processes may, and in most instances will, end up at a different temperature in each case. This is an important fact that is not apparent from the verbal definitions.

There is a pedagogic value of superposing all four processes on a single *P-V* diagram because it not only identifies the process, but simultaneously shows how it differs from the other processes.

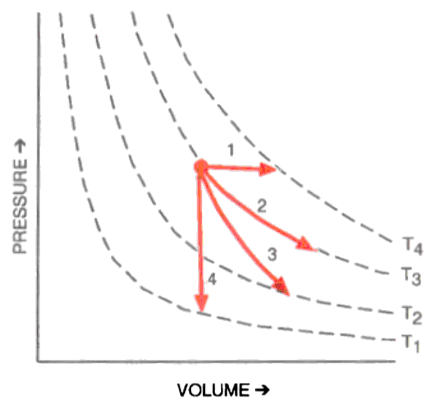

Fig. 1. Various thermodynamic processes. 1. Isobaric (Constant pressure; Charles Law); 2. Isothermal (Constant temperature; Boyle's Law); 3. Adiabatic (No heat exchange with the surroundings); 4. Isochoric (Constant volume; Amonton's Law).

Why C_P *May Be Used for Constant Volume Processes*

Students readily accept that $\Delta U = C_v \Delta T$ when an ideal gas is heated *at constant volume,* but find it difficult to understand why the C_P is *not* substituted for C_v when the process occurs *at constant pressure.* Figure 2 provides a simple visual explanation: If the gas undergoes an isochoric (constant volume) process, it will follow arrow 1, for which $\Delta U = C_v \Delta T$. On the other hand, if the gas undergoes an isobaric (constant pressure) process, it will follow arrow 2. To show that *AU* is *still* equal to $C_v \Delta T$ *for* this process, it is only necessary to remember that ΔU is a state function and note that:

$$
\begin{aligned}
\Delta U_2 &= \Delta U_1 + \Delta U_3 \\
&= \Delta U_V + \Delta U_T \\
&= C_V \Delta T + 0 \\
&= C_V \Delta T
\end{aligned}
$$

Thus $\Delta U = C_v \Delta T$ for *both* constant volume *and* constant pressure processes. Similarly, $\Delta H = C_P \Delta T$ for *both* constant pressure *and* constant volume processes.

Processes in Which* P, V, *and*T *AU Change

The P-V diagrams used in textbooks are invariably confined to processes in which a particular variable is held constant—as in Figs. 1 and 2. This is a sound way to introduce the basic concepts to students, but it is a particularly useful exercise to link these various processes by considering cases in which none of the basic variables is held constant (see Fig. 3). Such an example also provides the opportunity to emphasize the fact that changes in state functions depend only on the initial and final states of the system and are independent of the pathway between these states.

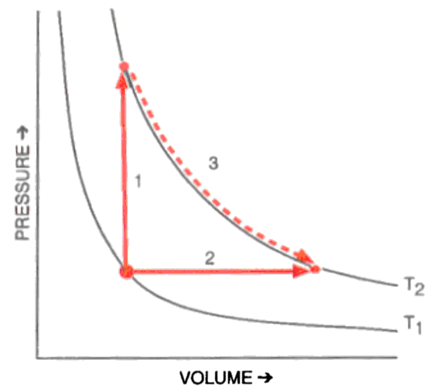

Fig. 2. ΔU = C_V ΔT for *both* isochoric (1) and for isobaric (2) processes.

In Fig. 3, arrow 1 is the process of interest, in which *P, V,* and *T* all undergo a change of state from $P_1V_1T_1$ *to* $P_2V_2T_2$. For such a case we have:

$$\begin{aligned}
\Delta U_2 &= \Delta U_1 + \Delta U_3 \\
&= \Delta U_V + \Delta U_T \\
&= C_V \Delta T + 0 \\
&= C_V \Delta T
\end{aligned}$$

or

$$\begin{aligned}
\Delta U_1 &= \Delta U_4 + \Delta U_5 \\
&= \Delta U_P + \Delta U_T \\
&= C_V \Delta T + 0 \\
&= C_V \Delta T
\end{aligned}$$

Similarly $$\Delta H_1 = \Delta H_2 + \Delta H_3 + \Delta H_4 + \Delta H_5 = C_P \Delta T$$

In Fig. 3. the second step (3 or 5) is isothermal. This provides the easiest and most convenient solution. However. the independence of pathway may be further illustrated by electing to make the second step adiabatic. Despite the change of pathway, the same end result is still obtained:

$$\begin{aligned}\Delta U_1 &= \left(\Delta U_2 + \Delta U_6\right) + \Delta U_7 \\ &= \Delta U_V + \Delta U_{(rev,\ adiabatic)} \\ &= C_V\left(T_3 - T_1\right) + C_V\left(T_2 - T_3\right) \\ &= C_V\left(T_2 - T_1\right) \\ &= C_V \Delta T\end{aligned}$$

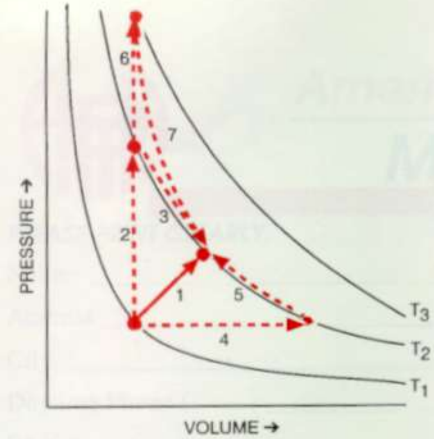

Fig. 3. A process in which *P*, *V*, and *T* all change.

Pressure against Which a Gas Expands Not Necessarily Final Pressure of the Gas

Textbooks and teachers generally take care to explain that the work done by an expanding system depends, not on the pressure of the system, but on the *external* pressure against which the system expands. Having done that, most of the supporting examples, and virtually all of the relevant *P-V* diagrams are of such a nature that the system expands against a pressure that is exactly equal to the final pressure of the system (see Fig. 4a). This is not wrong, but allows students to obtain correct results by incorrectly using *P* (final) instead of *P* (external). This is best avoided by making use of examples, illustrated by a *P-V* diagram such as Fig. 4b, in which *P* (external) < *P* (final).

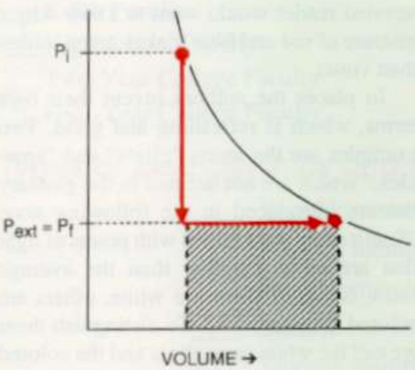

Fig. 4a. Isothermal expansion against an external pressure that is equal to the final pressure of the system.

What about Compressions?

The vast majority of *P-V* diagrams illustrate expansion processes only. It is well worth mentioning to students that compression processes are equally important. The use of *P-V* diagrams such as Fig. 5 (in conjunction with Fig. 4) is one of the many ways in which *P-V* diagrams may be employed to illustrate this point. By comparing Figs. 4 and 5, it is also apparent that the work done (shaded area) by an expanding system is always less than (or, for reversible processes, equal to) the work required to restore the system to its original state— goodbye to perpetual motion and such.

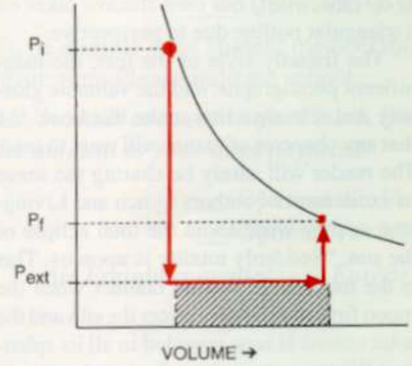

Fig. 4b. Isothermal expansion against an external pressure that is less than the final pressure of the system.

Conclusion

It is a basic pedagogic principle (i.e., common sense!) that improved comprehension results from the reinforcement of an idea by approaching it from several different angles. Thus many scientific concepts may be defined verbally, reinforced by an equation, further reinforced by an appropriate diagram and finally, illustrated by a suitable calculation. Diagrams are especially appreciated by students, for whom the old adage, "A picture is worth a thousand words" seems to apply. As a result, *P-V* diagrams are a particularly useful, but under-exploited, teaching aid; this note provides some suggestions for extending their usefulness.

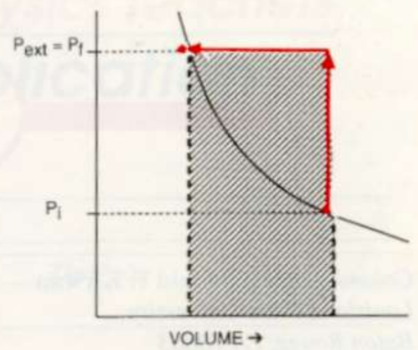

Fig. 5a. Isothermal compression by an external pressure that is equal to the final pressure of the system.

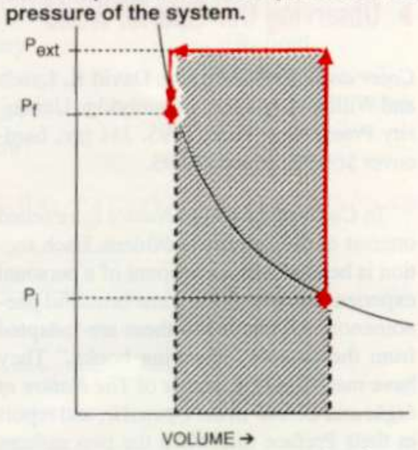

Fig. 5b. Isothermal compression by an external pressure that is greater than the final pressure of the system.