

# Honors Physics

## Projectile Motion

# Solving a projectile kinematics problem

A projectile problem is actually two kinematics problems.

The key is that when you are identifying your information, you must decide which problem each item goes in.

Horizontal distances and velocities go in the x-direction problem.

Vertical distances, velocities, and accelerations go in the y-direction problem.

# Solving a projectile kinematics problem

In the horizontal (x-direction) it has constant speed. We will always use the equation  $x=vt$  to solve the x-direction problem.

$$a = \frac{v_f - v_i}{t}$$

$$x = \left( \frac{v_i + v_f}{2} \right) t$$

In the vertical (y-direction) it is accelerating, so we will use one of the kinematics equations we learned earlier.

$$x = v_i t + \frac{1}{2} a t^2$$

The most important part of this is that x-direction values cannot be used in the y-direction problem, and y-direction values cannot be used in the x-direction problem. Time does not have a direction so it can be used in both problems.

$$v_f^2 = v_i^2 + 2ax$$

# Horizontal projectiles

Example problem:

A ball is rolling across a table. It rolls horizontally across the table at  $2 \text{ m/s}$  and falls to a vertical distance of  $1.5 \text{ meters}$  to the floor. How far from the table does it land?

# Horizontal projectiles

Example problem:

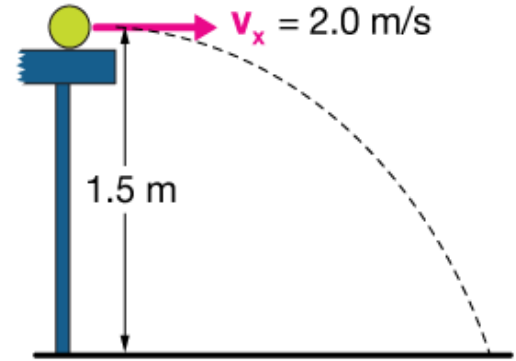
A ball is rolling across a table. It rolls horizontally across the table at 2 m/s and falls to a vertical distance of 1.5 meters to the floor. How far from the table does it land?

*Step 1 - what information are we given?*

*Horizontal (x-direction):  $v = 2 \text{ m/s}$*

*Vertical (y-direction):  $x = -1.5 \text{ m}$  (negative since it is down),  $a = -9.81 \text{ m/s}^2$  (negative since it is down),  $v_i = 0$*

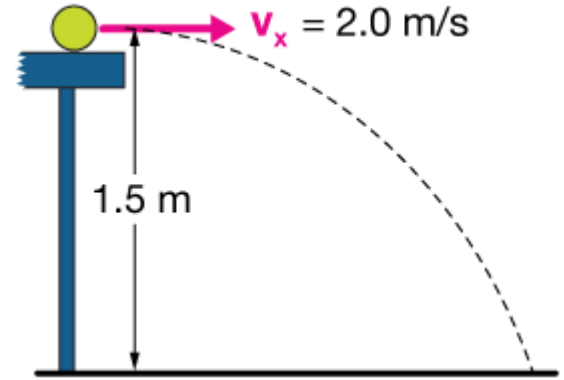
*(The tricky one to see is that  $v_i = 0$  in the y-direction.)*



# Horizontal projectiles

Example problem:

A ball is rolling across a table at 2 m/s. It rolls horizontally across the table and falls to a vertical distance of 1.5 meters to the floor. How far from the table does it land?



*Step 2 - what are we solving for?*

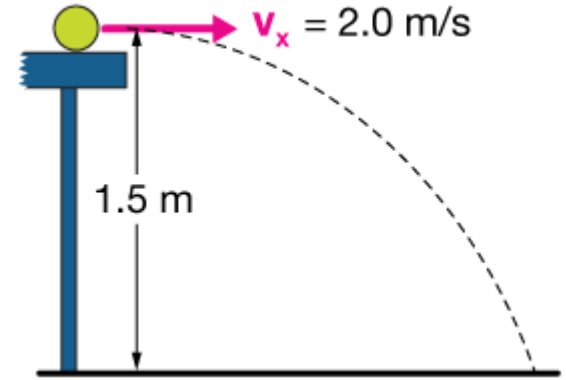
*We are trying to find a distance (x) in the horizontal direction (x-direction)*

*In the x-direction problem we always use  $x=vt$ . We have  $v$ , but we need to know  $t$ . To find  $t$  we solve a y-direction problem.*

# Horizontal projectiles

Example problem:

A ball is rolling across a table at 3 m/s. It rolls horizontally across the table and falls to a vertical distance of 1.5 meters to the floor.



*Step 3: Set up separate x-direction and y-direction problems. Choose appropriate equation in the y-direction.*

X-direction

*We always use  $x=vt$*

Y-direction

*We will use the equation  $x=v_i t + \frac{1}{2}at^2$  because we know  $x$ ,  $v_i$  and  $a$  and need to get  $t$*

# Horizontal projectiles

Example problem:

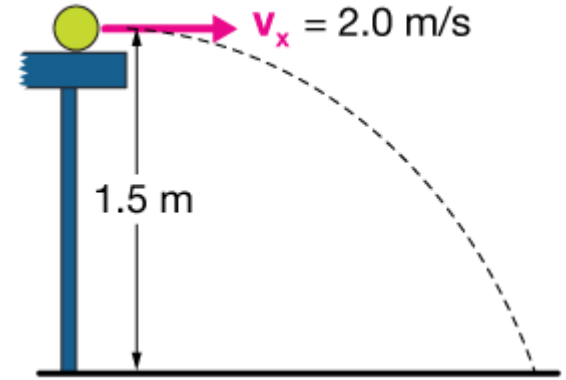
A ball is rolling across a table at 3 m/s. It rolls horizontally across the table and falls to a vertical distance of 1.5 meters to the floor. How far from the table does it land?

X-direction

$$x = vt$$

$$x = (3)(0.55)$$

$$\underline{x = 1.65 \text{ m}}$$



Y-direction

$$x = v_i t + \frac{1}{2} a t^2$$

$$-1.5 = 0t + \frac{1}{2}(-9.81)t^2$$

$$-1.5 = -4.905 t^2$$

$$t = 0.55 \text{ sec}$$



# Horizontal projectile practice

Try solving these practice problems.

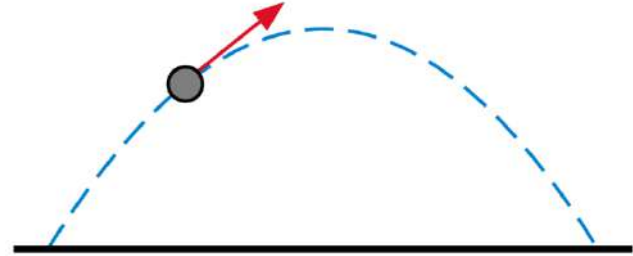
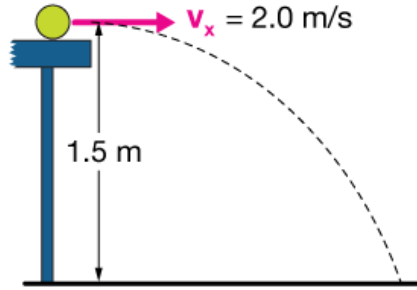
An arrow is shot from a bow. It is initially moving directly horizontally at 25 m/s. The arrow begins its motion at a height of 1.2 meters above the ground. How far from the point where it is fired will it hit the ground?

Follow up question: if the arrow was shot at 30 m/s instead, does that change the time it takes to hit the ground?

(this next one is slightly different)

A toy car is rolling horizontally across a student's desk. It is moving at 2 meters per second when it rolls off the edge of the desk. It lands a horizontal distance of 1 meter from the desk. How tall is the desk?

# Projectiles at angles



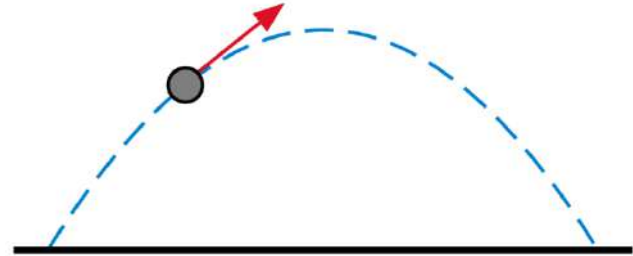
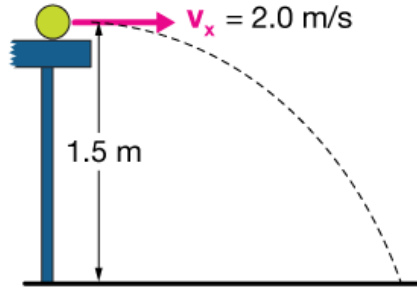
What is different between these two projectiles?

Do they both accelerate in the y-direction?

Do they both have constant speed in the x-direction?

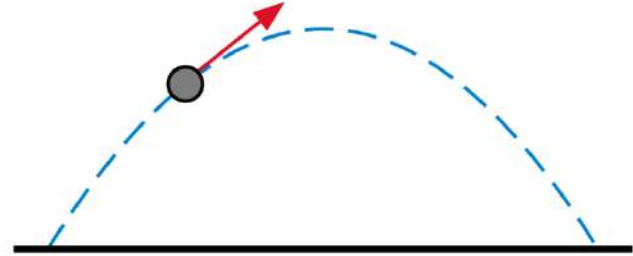
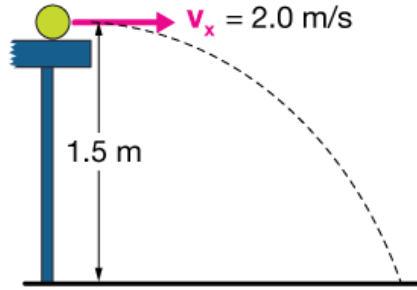
Do they both have a  $v_i$  of zero in the y-direction?

# Projectiles at angles



- Do they both accelerate in the y-direction?
  - *Yes! They both accelerate downward due to gravity with  $a = -9.81 \text{ m/s}^2$*
- Do they both have constant speed in the x-direction?
  - *Yes! Gravity only acts downward. There are no forces pushing left or right so they don't accelerate in the x-direction.*
- Do they both have a  $v_i$  of zero in the y-direction?
  - *No! The projectile on the left is initially only moving to the right, so it starts out with no vertical speed. The projectile on the right is moving upward from the very start of the motion, so it has a non-zero  $v_i$*

# Projectiles at angles



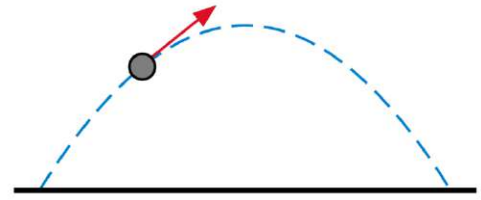
1. *What information are we given? Identify what values you have. Separate them into x-direction and y-direction values*
2. *What are you solving for?*
3. *Set up an x-direction problem with  $x=vt$  and a y-direction problem where we choose a kinematics equation based on what we are given*
4. *Solve one problem for  $t$ . Use it in the other problem to get what we are looking for.*

# Projectiles at angles

Example problem:

A soccer ball is kicked at an angle, as shown in the picture above. It is in the air for 4 seconds and lands a horizontal distance of 25 meters from where it was kicked. Calculate the speed that the ball was kicked.

# Projectiles at angles



Example problem:

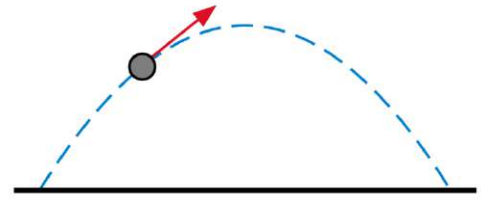
A soccer ball is kicked at an angle, as shown in the picture above. It is in the air for 4 seconds and lands a horizontal distance of 25 meters from where it was kicked. Calculate the speed that the ball was kicked.

*Step 1: what information are we given?*

*We have a horizontal distance of 25 m, so that goes in the x-direction problem. We also have a time of 4 seconds. That can go in either problem.*

*We also know that the ball ends at its original height. That means the y-direction displacement is zero! We also know that there is an acceleration of  $-9.81 \text{ m/s}^2$  in the y-direction.*

# Projectiles at angles



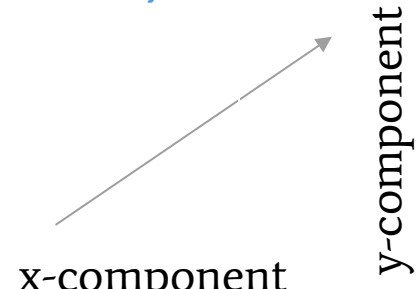
Example problem:

A soccer ball is kicked at an angle, as shown in the picture above. It is in the air for 4 seconds and lands a horizontal distance of 25 meters from where it was kicked. Calculate the speed that the ball was kicked.

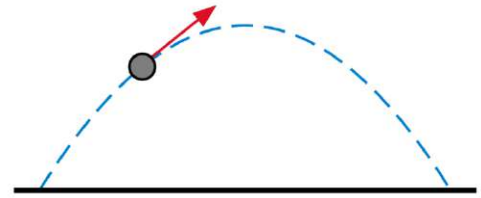
*Step 2: What are we trying to find?*

*We want to find the initial speed of the ball. The way we can do that is by finding the speed in the x-direction and the initial speed in the y-direction. Then those are the components of the velocity vector.*

*We will use trigonometry to find the velocity.*



# Projectiles at angles



Example problem:

A soccer ball is kicked at an angle, as shown in the picture above. It is in the air for 4 seconds and lands a horizontal distance of 25 meters from where it was kicked. Calculate the speed that the ball was kicked.

*Step 3: Set up an x-direction problem with  $x=vt$  and a y-direction problem where we choose a kinematics equation based on what we are given*

*X-direction*

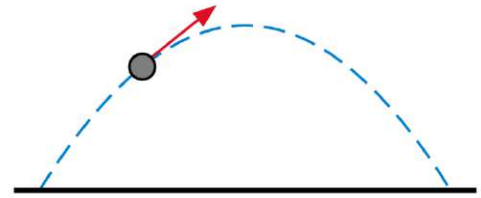
*We always use  $x=vt$*

*Y-direction*

*We will use the equation  $x=v_i t + \frac{1}{2}at^2$  because we know  $x$ ,  $t$ , and  $a$  and need to get  $v_i$*



# Projectiles at angles



Example problem:

A soccer ball is kicked at an angle, as shown in the picture above. It is in the air for 4 seconds and lands a horizontal distance of 25 meters from where it was kicked. Calculate the speed that the ball was kicked.

*Step 4: Solve one problem for  $t$ . Use it in the other problem to get what we are looking for.*

$X$ -direction

$$x = vt$$

$$25 = v(4)$$

$$v = 6.25 \text{ m/s}$$

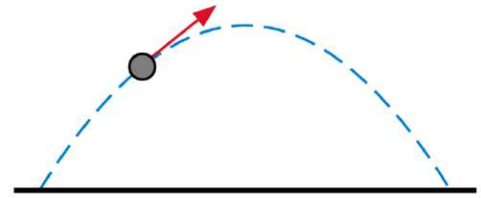
$Y$ -direction

$$x = v_i t + \frac{1}{2}at^2$$

$$0 = v_i(4) + \frac{1}{2}(-9.81)(4)^2$$

$$v_i = 19.62 \text{ m/s}$$

# Projectiles at angles



Example problem:

A soccer ball is kicked at an angle, as shown in the picture above. It is in the air for 4 seconds and lands a horizontal distance of 25 meters from where it was kicked. Calculate the speed that the ball was kicked.

*Now use trigonometry to get the final problem.*

$$a^2 + b^2 = c^2$$

$$6.25^2 + 19.62^2 = c^2$$

$$c = 20.6 \text{ m/s}$$

*the kick*

Ball's  
velocity

6.25 m/s

19.62 m/s

*You can also find the angle of*

# Projectiles at angles - Practice

Try one on your own:

A person kicks a football in a parabolic arc. The ball is in the air for 3.2 seconds. It lands a horizontal distance of 45 meters from the kicker. Calculate the velocity of the ball when it was kicked.