

Complements and At Least One

Complement of an event A consists of all outcomes in which A does NOT occur
 $P(A')$, $P(A^c)$ and $P(\bar{A})$ all denote the probability of A not occurring

Complementary Rules

$$P(A) + P(\bar{A}) = 1$$

$$P(\bar{A}) = 1 - P(A)$$

$$P(A) = 1 - P(\bar{A})$$

At least One

At least one is equivalent to 1 or more

The complement of at least one is "None"

$$P(\text{at least one}) + P(\text{none}) = 1$$

$$P(\text{at least one}) = 1 - P(\text{none})$$

Example

If a couple plans to have three children, what is the probability that they will have at least one girl?

The complement of "at least one" is "none:

$$P(\text{at least one girl}) = 1 - P(\text{no girls}) = 1 - P(\text{all boys})$$

$$P(\text{all boys}) = (1/2)^3 = 0.125$$

$$P(\text{at least one girl}) = 1 - 0.125 = 0.875$$

If a couple plans to have five children, what is the probability that they will have at least one girl?

$$P(\text{at least one girl}) = 1 - P(\text{no girls})$$

$$P(\text{no girls}) = (1/2)^5 = 0.03125$$

$$P(\text{at least one girl}) = 1 - 0.03125 = 0.96875$$

Example

If you roll a die five times, what is the probability of getting at least one 6?

$$P(\text{at least one six}) = 1 - P(\text{no six})$$

$$P(\text{no six}) = P(\text{all not six}) = (5/6)^5 = 0.833^5 = 0.401878$$

$$P(\text{at least one six}) = 1 - 0.401878 = 0.598122$$

Example

A student experiences difficulties with malfunctioning alarm clocks. Instead of using one alarm clock, he decides to use three. What is the probability that at least one of his alarm clocks works correctly if each individual alarm clock has a 99% chance of working correctly?

$$P(\text{at least one works}) = 1 - P(\text{none work}) = 1 - P(\text{all Fail}) = 1 - .01^3 = 1 - 0.000001 = 0.999999$$

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Example

In a lab there are eight technicians. Three are male and five are female. If three technicians are selected, find the probability that at least one is female.

$$P(\text{at least one female}) = 1 - P(\text{no female}) = 1 - P(\text{all male})$$

$$P(\text{all male}) = \frac{3}{8} \cdot \frac{2}{7} \cdot \frac{1}{6} = \frac{1}{56}$$

$$P(\text{at least one female}) = 1 - \frac{1}{56} = \frac{55}{56} = 0.9821$$

Example

There are three chemistry instructors and six physics instructors at a college. If a committee of four instructors is selected, find the probability that of at least one of them being a physics instructor.

$$P(\text{at least one physics}) = 1 - P(\text{no physics}) = 1 - P(\text{all chemistry})$$

$$P(\text{all chemistry}) = \frac{3}{9} \cdot \frac{2}{8} \cdot \frac{1}{7} \cdot \frac{0}{6} = 0$$

$$P(\text{at least one physics}) = 1 - 0 = 1. \text{ It is certain! Think about the situation.}$$

Example

On a surprise quiz consisting of five true-false questions, an unprepared student guesses each answer. Find the probability that he gets at least one correct.

$$P(\text{at least one correct}) = 1 - P(\text{no correct}) = 1 - P(\text{all wrong})$$

$$P(\text{all wrong}) = (1/2)^5 = .03125$$

$$P(\text{at least one correct}) = 1 - .03125 = 0.9687$$

Example

A carpool contains three kindergartners and five first-graders. If two children are ill, find the probability that at least one of them is a kindergartner.

$$P(\text{at least one kindergartner}) = 1 - P(\text{no kindergartner}) = 1 - P(\text{all first-graders})$$

$$P(\text{all first graders}) = \frac{5}{8} \cdot \frac{4}{7} = \frac{5}{14}$$

$$P(\text{at least one kindergartner}) = 1 - \frac{5}{14} = \frac{9}{14}$$