Probability and Statistics

Unit Title: Analyzing & Collecting Data
Stage 1: Desired Results

Standards & Indicators:

S-ID Interpreting Categorical and Quantitative Data:

Summarize, represent, and interpret data on a single count or measurement variable:

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
4. Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.

Summarize, represent, and interpret data on two categorical and quantitative variables:

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.

   a. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.*

   b. Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology.

   c. Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models:

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

Career Readiness, Life Literacies and Key Skills		
Standard	**Performance Expectations**	**Core Ideas**
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.1.1.12prof.CR3a).	With a growth mindset, failure is an important part of success.
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3)	Collaboration with individuals with diverse experiences can aid in the problem-solving process, particularly for global issues where diverse solutions are needed.

Probability and Statistics

9.4.12.GCA.1	Collaborate with individuals to analyze a variety of potential solutions to climate change effects and determine why some solutions (e.g., political. economic, cultural) may work better than others (e.g., SL.11-12.1., HS-ETS1-1, HS-ETS1-2, HS-ETS1-4, 6.3.12.GeoGI.1, 7.1.IH.IPERS.6, 7.1.IL.IPERS.7, 8.2.12.ETW.3).	Solutions to the problems faced by a global society require the contribution of individuals with different points of view and experiences.

Central Idea/Enduring Understanding:	**Essential/Guiding Question:**
Chapter 1: ● Classifying data ● Displaying Categorical Data ● Displaying Quantitative Data ● Measuring Center ● Measuring Variability ● Summarizing Quantitative Data ● Describing Location in a Distribution **Chapter 2:** ● Relationships Between 2 Categorical Variables ● Relationships Between 2 Quantitative Variables ● Correlations ● Making Predictions ● Least-Squares Regression Lines ● Residual Plots **Chapter 3:** ● Population vs Samples ● Observational Studies vs Experiments ● Sampling Techniques ● Simple Random Samples ● Margin of Error ● Undercoverage, Nonresponse and other Bias ● Random Assignment ● Inference for Experiments	**Chapter 1:** At the end of this chapter, students should be able to answer the Essential Question: *Why do we study Statistics?* Answer: We live in a world of data. A solid understanding of Statistics will help one become a more informed consumer, make better decisions based on data and be able to read, analyze and interpret data. **Chapter 2:** At the end of this chapter, students should be able to answer the Essential Question: *Does correlation imply causation? Why or why not?* Answer: Correlation does not imply causation. Two variables might have a strong correlation, but changes in one variable are very unlikely to cause changes in the other variable. **Chapter 3:** At the end of this chapter, students should be able to answer the Essential Question: *What are the various ways one can collect a random sample?* Answer: Using slips of paper, a random number generator, flipping a coin, etc.
Content: **Chapter 1:** 1.1 The Science and Art of Data	**Skills(Objectives):** 1.1 Classify data. Summarize data using a frequency table.

Probability and Statistics

1.2 Displaying Categorical Data
1.3 Dotplots
1.4 Stemplots
1.5 Histograms
1.6 Measuring Center
1.7 Measuring Variability
1.8 Boxplots and Outliers
1.9 Describing Location

Chapter 2:
2.1 Relationships between 2 Categorical Variables
2.2 Relationships between 2 Quantitative Variables
2.3 Correlation
2.4 Calculating the Correlation
2.5 Regression Lines
2.6 The Least-Squares Regression Line
2.7 Assessing a Regression Model

Chapter 3:
3.1 Data Collection
3.2 Bias
3.3 Simple Random Sample
3.4 Margin of Error
3.5 Sampling and Surveys
3.6 Observational Studies and Experiments
3.7 Random Assignment
3.8 Inference for Experiments
3.9 Using Studies Wisely

1.2 Create and interpret charts and graphs.

1.3 Make and interpret dotplots. Describe the shape of a distribution.

1.4 Make and interpret stemplots.

1.5 Make and interpret histograms.

1.6 Calculate and interpret the mean and median of a distribution of quantitative data. Compare the mean and median.

1.7 Calculate and interpret the range, IQR and standard deviation of quantitative data.

1.8 Identify boxplots. Make and interpret boxplots.

1.9 Find and interpret a percentile in a distribution of quantitative data. Find and interpret a standardized score (z-score) in a distribution of quantitative data.

Chapter 2:
2.1 Distinguish between explanatory and response variables for categorical data. Determine if there is an association between 2 categorical variables and describe the association if it exists.

2.2 Make a scatterplot to display the relationship between two quantitative variables. Describe the direction, form, and strength of a relationship displayed in a scatterplot and recognize outliers in a scatterplot.

2.3 Interpret the correlation. Understand the basic properties of correlation. Distinguish correlation from causation.

2.4 Calculate the correlation between 2 quantitative variables. Apply the properties of the correlation. Describe how outliers influence the correlation.

2.5 Interpret the slope and y intercept of a least-squares regression line. Use the least-squares regression line to predict y for a given x. Explain the dangers of extrapolation. Calculate and interpret residuals.

Probability and Statistics

	2.6 Determine the equation of a least-squares regression line using technology and summary statistics. Describe how outliers affect the least-squares regression line. 2.7 Construct and interpret residual plots to assess if a linear model is appropriate. Interpret the standard deviation of the residuals and coefficient of determination. **Chapter 3:** 3.1 Identify the population and sample in a statistical study. Distinguish between an observational study and an experiment. 3.2 Describe how types of sampling can lead to bias. 3.3 Describe how to obtain a simple random sample. Explain the concept of sampling variability and the effect of increasing sample size. Use simulation to test a claim about a population proportion. 3.4 Use simulation to approximate the margin of error for a sample proportion and interpret the margin of error. Use simulation to approximate the margin of error for a sample mean and interpret the margin of error. 3.5 Explain how undercoverage, nonresponse, question wording, and other aspects of a sample survey can lead to bias. 3.6 Explain the concept of confounding and how it limits the ability to make cause-and-effect conclusions. 3.7 Explain the purpose of comparison, random assignment, control, and replication in an experiment. 3.8 Outline an experiment that uses a completely randomized design. Explain the concept of statistical significance in the context of an experiment. 3.9 Evaluate whether a statistical study has been carried out in an ethical manner.

Interdisciplinary Connections:

Interdisciplinary connections are integrated in each unit with connections to the mathematical practices.
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics

Probability and Statistics

5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Stage 2: Assessment Evidence

Performance Task(s):

Chapter 1:
Lesson App 1.1 - What are my classmates like?
Lesson App 1.2 - Which phone speaks to you?
Lesson App 1.3 - How can we check the health of a stream?
Lesson App 1.4 - How many shoes are too many shoes?
Lesson App 1.5 - How old are US Presidents?
Lesson App 1.6 - Is the pace slower in smaller cities?
Lesson App 1.7 - Have we found the beef?
Lesson App 1.8 - What is best at reducing stress?
Lesson App 1.9 - Which states are rich?

Chapter 2:
Lesson App 2.1- Which finger is longer?
Lesson App 2.2- More sugar, more calories?
Lesson App 2.3- If I eat more chocolate, will I win a Nobel Prize?
Lesson App 2.4- Flying dinosaur or early bird?
Lesson App 2.5- Do cut flowers benefit from sugar in the water?
Lesson App 2.6- Did the Broncos buck the trend?
Lesson App 2.7- Do higher priced tablets have better battery life?

Chapter 3:
Lesson App 3.1- Do you have dinner plans?
Lesson App 3.2- Still on the phone?
Lesson App 3.3- Do you tweet?
Lesson App 3.4- Can you roll your tongue?
Lesson App 3.5- Who did you say was calling?
Lesson App 3.6- What happens when physicians study themselves?

Other Evidence:

Classwork
Written & Online Homework
Section Quizzes
Chapter Tests
Online Student Assessments
End of Unit Assessment
Graphing Calculators
Applets
Statistical Software

Probability and Statistics

Lesson App 3.7- Multitasking? Or making distractions? Lesson App 3.8- Does fish oil affect blood pressure? Lesson App 3.9- Is foster care better for children than an orphanage?	

Stage 3: Learning Plan

Learning Opportunities/Strategies:	**Resources:**
- Data Collection - Simulation - Rolling Dice - Graphing with a Graphing Calculator - Graphing with Statistical Software - Think-Pair-Share - Collaboration - Matching Graphs with their Data - Random Number Generators	eBook-Statistics and Probability with Applications (High School) Daren S. Starnes (The Lawrenceville School) , Josh Tabor (Canyon del Oro (AZ) High School) 2016 LaunchPad Google Doc of Extra Examples Google Slide Lecture Presentations Prepared Tests and Test Bank Prepared Quizzes Prepared Worksheets PD Videos Applets Statistical Software IXL LGBT and Disabilities Resources: LGBTQ-Inclusive Lesson & Resources by Garden State Equality and Make it Better for Youth LGBTQ+ Books DEI Resources: Learning for Justice GLSEN Educator Resources Supporting LGBTQIA Youth Resource List Respect Ability: Fighting Stigmas, Advancing Opportunities NJDOE Diversity, Equity & Inclusion Educational Resources Diversity Calendar

Differentiation

*Please note: Teachers who have students with 504 plans that require curricular accommodations are to refer to Struggling and/or Special Needs Section for differentiation

High-Achieving Students	On Grade Level Students	Struggling Students	Special Needs/ELL
Projects Problems with higher degree difficulty Higher order thinking challenges Use of Statistical Software	Guided Notes Chapter Summaries Formula Sheets Graphing Calculators Think-Pair-Share Group Work Collaboration Hands-On activities	Guided Notes Allow use of formula sheets on tests Hands-On Activities Real-life applications Require use of calculators Think-Pair-Share	Any student requiring further accommodations and/or modifications will have them individually listed in their 504 Plan or IEP. These might include, but are not limited to: breaking assignments into smaller tasks, giving directions through several channels (auditory, visual,

Probability and Statistics

	Real-life application of statistics Chapter Reviews prior to tests	Allow students to work in groups to complete class assignments. Study Sheets Practice Tests Online review of content Pair Student with a High-Achieving student One-on-One Tutoring	kinesthetic, model), and/or small group instruction for reading/writing ELL supports should include, but are not limited to, the following:: Extended time Provide visual aids Repeated directions Differentiate based on proficiency Provide word banks Allow for translators, dictionaries

Unit Title: Exploring Random Variables & Probability

Stage 1: Desired Results

Standards & Indicators:

S-CP Conditional Probability and the Rules of Probability:
A. Understand independence and conditional probability and use them to interpret data:

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of A given B as $P(A$ and $B)/P(B)$, and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

B. Use the rules of probability to compute probabilities of compound events in a uniform probability model:
6. Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
7. Apply the Addition Rule, $P(A$ or $B) = P(A) + P(B) – P(A$ and $B)$, and interpret the answer in terms of the model.
8. Apply the general Multiplication Rule in a uniform probability model, $P(A$ and $B) = P(A)P(B|A) = P(B)P(A|B)$, and interpret the answer in terms of the model.

Probability and Statistics

9. Use permutations and combinations to compute probabilities of compound events and solve problems.

S-MD Using Probability to Make Decisions:
A. Calculate expected values and use them to solve problems
1. Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.
2. Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
3. Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
4. Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*

B. Use probability to evaluate outcomes of decisions :
5. Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.
   a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast food restaurant.*
   b. Evaluate and compare strategies on the basis of expected values. *For example, compare high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*
6. Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
7. Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).

Career Readiness, Life Literacies and Key Skills		
Standard	**Performance Expectations**	**Core Ideas**
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9.4.12.GCA.1	Collaborate with individuals to analyze a variety of potential solutions to climate change effects and determine why some solutions (e.g., political. economic, cultural) may work better than others (e.g., SL.11-12.1.,	Solutions to the problems faced by a global society require the contribution of individuals with different points of view and experiences.

Probability and Statistics

	HS-ETS1-1, HS-ETS1-2, HS-ETS1-4, 6.3.12.GeoGI.1, 7.1.IH.IPERS.6, 7.1.IL.IPERS.7, 8.2.12.ETW.3).	

Central Idea/Enduring Understanding:

Chapter 4:
- Randomness
- Simulation
- Probability Rules
- Two-Way Tables
- Venn Diagrams
- Addition Rule
- Conditional Probability
- Independence
- Multiplication Rule
- Tree Diagrams
- Independent Events
- Counting Principle
- Permutations
- Combinations

Chapter 5:
- Continuous Random Variables
- Discrete Random Variables
- Binomial Random Variables
- Mean of a Binomial Distribution
- Standard Deviation of a Binomial Distribution
- Continuous Random Variables
- 68-95-99.7 Rule
- Normal Distribution Calculations

Chapter 6:
- Parameters
- Statistics
- Sampling Distribution
- Unbiased Estimators
- Center
- Variability
- Large Counts Condition
- Sample Proportion
- Sample Mean
- Central Limit Theorem

Essential/Guiding Question:

Chapter 4:
At the end of this chapter, students should be able to answer the Essential Question:

Why do we use simulations?

Answer:
Simulation is a way to model random events, such that simulated outcomes closely match real-world outcomes. By observing simulated outcomes, researchers gain insight on the real world. Some situations do not lend themselves to precise mathematical treatment. Others may be difficult, time-consuming, or expensive to analyze. In these situations, simulation may approximate real-world results; yet, require less time, effort, and/or money than other approaches.

Chapter 5:
At the end of this chapter, students should be able to answer the Essential Question:

How can we find the median of a discrete random variable?

Answer:
The median of a discrete random variable is the 50th percentile of its probability distribution. We can find the median by adding a cumulative probability row to the probability distribution table, and then locating the smallest value for which the cumulative probability equals or exceeds 0.50.

Chapter 6:
At the end of this chapter, students should be able to answer the Essential Question:

How can we check for bias in a survey?

Answer:
One way of checking the effect of undercoverage, nonresponse, and other sources of bias in a sample

Probability and Statistics

	survey is to compare the sample with known facts about the population.
Content: **Chapter 4:** 4.1 Randomness 4.2 Probability Rules 4.3 Addition Rule 4.4 Conditional Probability & Independence 4.5 Multiplication Rule & Tree Diagrams 4.6 Independent Events 4.7 Counting Principle and Permutations 4.8 Combinations & Probability **Chapter 5:** 5.1 Random Variables 5.2 Discrete Random Variables 5.3 Binomial Random Variables 5.4 Analyzing Binomial Random Variables 5.5 Continuous Random Variables 5.6 Standard Normal Distribution 5.7 Normal Distribution Calculations **Chapter 6:** 6.1 Parameter vs. Statistic 6.2 Center & Variability 6.3 Sampling Distribution 6.4 Sample Proportion 6.5 Sample Mean 6.6 Central Limit Theorem	**Skills(Objectives):** **Chapter 4:** 4.1 Define and Interpret Probability. Use simulation to model chance behavior. 4.2 List the sample space of an event. Apply the complement rule. Apply the addition rule. 4.3 Use a two-way table and a Venn Diagram to find probabilities. 4.4 Find and interpret conditional probabilities using two-way tables. Use the conditional probability formula to calculate probabilities. Determine whether two events are independent. 4.5 Apply the multiplication rule. Use a tree diagram to model a chance process and to calculate conditional probabilities. 4.6 Apply the multiplication rule for independent events. 4.7 Apply the counting rule. Use factorials to count the number of permutations of a group of individuals. 4.8 Use combinations to calculate probabilities. **Chapter 5:** 5.1 Verify the validity of a discrete random variable. Determine if a random variable is discrete or continuous. Calculate probabilities involving a discrete random variable. 5.2 Create a histogram to display the probability distribution of a discrete random variable. Calculate and interpret the expected value of a discrete random variable. Calculate and interpret the standard deviation of a discrete random variable. 5.3 Determine if a given setting is binomial. Calculate probabilities involving a single value of a binomial random variable. Make a histogram to display a binomial distribution and describe its shape. 5.4 Calculate and interpret the mean and standard deviation of a binomial distribution. Find probabilities

involving several values of a binomial random variable. Use technology to calculate cumulative binomial probabilities.

5.5 Show that the probability distribution of a continuous random variable is valid and use the distribution to calculate probabilities. Determine the relative locations of the mean and median of a continuous random variable form the shape of its probability distribution. Draw a normal probability distribution with a given mean and standard deviation.

5.6 Use the Empirical Rule to approximate probabilities. You a table to find a z-score and area under the curve.

5.7 Calculate the probability that a value falls within a given interval in a normal distribution. Find the value corresponding to a given probability in a normal distribution.

Chapter 6:
6.1 Distinguish between a parameter and a statistic. Understand the definition of a sampling distribution. Distinguish between population distribution, sampling distribution, and the distribution of sample data.

6.2 Determine whether a statistic is an unbiased estimator of a population parameter. Understand the relationship between sample size and the variability of an estimator.

6.3 Calculate the mean and the standard deviation of the sampling distribution of a sample count and interpret the standard deviation. Determine if the sampling distribution of a sample count is approximately normal. If appropriate, use the normal approximation to the binomial distribution to calculate probabilities involving a sample count.

6.4 Find the mean and standard deviation of the sampling distribution of a sample proportion for an SRS of size n from a population having proportion p of successes. Interpret the standard deviation. Determine if the sampling distribution of p is approximately normal. If appropriate, use a normal distribution to calculate probabilities involving p.

Probability and Statistics

	6.5 Find the mean and standard deviation of the sampling distribution of a sample mean from an SRS of size *n*. Interpret the standard deviation. Calculate probabilities involving a sample mean when the population distribution is Normal. 6.6 Explain how the shape of the sampling distribution of is related to the shape of the population distribution. Use the central limit theorem to help find probabilities involving a sample mean .

Interdisciplinary Connections:

Interdisciplinary connections are integrated in each unit with connections to the mathematical practices.
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Stage 2: Assessment Evidence

Performance Task(s):	**Other Evidence:**
Chapter 4: Lesson App 4.1- Will the train arrive on time? Lesson App 4.2- How prevalent is high cholesterol? Lesson App 4.3- Who owns a home? Lesson App 4.4- Who earns A's in college? Lesson App 4.5- Not milk? Lesson App 4.6- How should we interpret genetic screening? Lesson App 4.7- Do you scream for ice cream? Lesson App 4.8- How many songs can you set up an iPod playlist? **Chapter 5:** Lesson App 5.1- Making the grade? Lesson App 5.2- How much do colleges grades vary? Lesson App 5.3- Is the train binomial? Lesson App 5.4- Free lunch? Lesson App 5.5- Still waiting for the server? Lesson App 5.6- What's a good batting average?	Classwork Written & Online Homework Section Quizzes Chapter Tests Online Student Assessments End of Unit Assessment Graphing Calculators Applets Statistical Software

Probability and Statistics

Lesson App 5.7- What cholesterol levels are unhealthy for teen boys?

Chapter 6:
Lesson App 6.1- How cold is it inside the cabin?
Lesson App 6.2- How many tanks does that emeny have?
Lesson App 6.3- How can we check for bias in a survey?
Lesson App 6.4- What's that spot on my potato chip?
Lesson App 6.5- Are college women taller?
Lesson App 6.6- Keeping things cooler with statistics?

Stage 3: Learning Plan

Learning Opportunities/Strategies:
- Dice Games
- Simulation
- Probability Games
- Random Number Generator
- Data Collection
- Applets
- Card Games
- Venn Diagrams
- Two-Way Tables
- Graphing Calculators
- Statistical Software
- Think-Pair Share
- Small Group Discussion

Resources:

eBook-Statistics and Probability with Applications (High School) Daren S. Starnes (The Lawrenceville School) , Josh Tabor (Canyon del Oro (AZ) High School) 2016 LaunchPad
Google Doc of Extra Examples
Google Slide Lecture Presentations
Prepared Tests and Test Bank
Prepared Quizzes
Prepared Worksheets
PD Videos
Applets
Statistical Software
IXL

LGBT and Disabilities Resources:
- LGBTQ-Inclusive Lesson & Resources by Garden State Equality and Make it Better for Youth
- LGBTQ+ Books

DEI Resources:
- Learning for Justice
- GLSEN Educator Resources
- Supporting LGBTQIA Youth Resource List
- Respect Ability: Fighting Stigmas, Advancing Opportunities
- NJDOE Diversity, Equity & Inclusion Educational Resources
- Diversity Calendar

Probability and Statistics

Unit Title: Estimating a Parameter & Testing a Claim

Stage 1: Desired Results

Standards & Indicators:

S-IC Making Inferences and Justifying Conclusions:

A. Understand and evaluate random processes underlying statistical experiments
1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies
3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

Probability and Statistics

<table>
<tr><td colspan="3" align="center">Career Readiness, Life Literacies and Key Skills</td></tr>
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<u>**Central Idea/Enduring Understanding:**</u>

<u>**Chapter 7:**</u>
- Confidence Levels
- Confidence Interval Interpretation
- Margin of Error
- Critical Values
- Reading Tables
- Area under the Curve
- 4-Step Process
- Finding t-Values
- Standard Error
- Conditions

<u>**Chapter 8:**</u>
- Significance Test
- Stating Hypothesis
- Interpreting P-values
- Making Conclusions
- Significance Level
- Type I & Type II Errors
- Conditions of a Significance Test
- 4-Step Process
- Two-Sided Tests
- Significance Tests for Proportion
- Testing a Claim about a Mean
- Significance Tests for a Mean

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<u>**Essential/Guiding Question:**</u>

<u>**Chapter 7:**</u>
At the end of this chapter, students should be able to answer the Essential Question:

What does a confidence level tell us?

<u>Answer:</u>
The confidence level does not tell us the probability that a particular confidence interval captures the population parameter. The confidence level reveals how likely it is that the method we are using will produce an interval that captures the population parameter if we use it many times.

<u>**Chapter 8:**</u>
At the end of this chapter, students should be able to answer the Essential Question:

How do you make a conclusion in a significance test?

<u>Answer:</u>
If the P-value is low we reject the Ho. If the P-value is high we fail to reject the Ho.

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Probability and Statistics

Content:	Skills(Objectives):
Chapter 7: 7.1 Confidence Intervals 7.2 Interpreting Confidence Intervals 7.3 Estimating a Proportion 7.4 Confidence Intervals for a Proportion 7.5 Estimating a Mean 7.6 Confidence Intervals for a Mean **Chapter 8:** 8.1 Interpreting P-values 8.2 Type I & Type II Errors 8.3 Conditions of a Significance Test 8.4 Significance Tests for Proportion 8.5 Testing a Claim about a Mean 8.6 Significance Tests for a Mean	**Chapter 7:** 7.1 Interpret a confidence interval in context. Determine the point estimate and margin of error from a confidence interval. Use confidence intervals to make decisions. 7.2 Interpret a confidence level in context. Describe how the confidence level and sample size affect the margin of error. Explain how practical issues like nonresponse, undercoverage, and response bias can affect the interpretation of a confidence interval. 7.3 Carry out the steps in constructing a confidence interval for a population proportion: define the parameter, check conditions, perform calculations and interpret results in context. 7.4 Use the 4-Step Process to construct and interpret a confidence interval for a population proportion. Determine the sample size required to obtain a level **C** confidence interval for a population proportion with a specified margin of error. 7.5 Construct and interpret a confidence interval for a population mean. Carry out the steps in constructing a confidence interval for a population mean: define the parameter, check conditions, perform calculations and interpret results in context. 7.6 Use sample data to check the Normal/Large Sample Condition. Use the 4-Step Process to construct and interpret a confidence interval for a population mean. **Chapter 8:** 8.1 State appropriate hypothesis for a significance test about a population parameter. Interpret P-value in context. Make an appropriate conclusion for a significance test based on P-value. 8.2 Determine if the results of a study are statistically significant and make an appropriate conclusion using a significance level. Interpret a Type I and Type II error in context. Give a consequence of a Type I and Type II error in a given setting. 8.3 Check the Random and Large Counts conditions for performing a significance test about a population

Probability and Statistics

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proportion. Calculate the standardized test statistic for a significance test about a population proportion. Find the P-value for a one-sided significance test about a population proportion using a table or technology.

8.4 Use the 4-Step process to perform a one-sided test about a population proportion. Calculate the P-value for a 2-sided significance test about a population proportion using a table or technology. Use the 4-Step process to perform a 2-sided significance test about a population proportion.

8.5 Check the Random and Large Counts conditions for performing a significance test about a population mean. Calculate the standardized test statistic for a significance test about a population mean. Find the P-value for a significance test about a population mean using a table.

8.6 Use the 4-Step process to perform a significance test about a population mean. Use a confidence interval to draw a conclusion about a 2-sided test for a population mean.

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Interdisciplinary Connections:

Interdisciplinary connections are integrated in each unit with connections to the mathematical practices.
1. Make sense of problems and persevere in solving them
2. Reason abstractly and quantitatively
3. Construct viable arguments and critique the reasoning of others
4. Model with mathematics
5. Use appropriate tools strategically
6. Attend to precision
7. Look for and make use of structure
8. Look for and express regularity in repeated reasoning

Stage 2: Assessment Evidence

Performance Task(s):	**Other Evidence:**
Chapter 7: Lesson App 7.1- Do you approve of the president's job performance? Lesson App 7.2- Do you like my picture? Lesson App 7.3-Do you know your government? Lesson App 7.4- TV in bed? Lesson App 7.5- What does an Oreo weigh? Lesson App 7.6- How tense are the video screens? **Chapter 8:**	Classwork Written & Online Homework Section Quizzes Chapter Tests Online Student Assessments End of Unit Assessment Graphing Calculators Applets Statistical Software

Probability and Statistics

Lesson App 8.1- Do you kiss people the "right" way? Lesson App 8.2- Are these potatoes keepers? Lesson App 8.3- Is it better to be last? Lesson App 8.4- Who feels job stress? Lesson App 8.5- Who needs an aspirin? Lesson App 8.6- Do our employees have high blood pressure?	

Stage 3: Learning Plan

Learning Opportunities/Strategies:	**Resources:**
● Mystery Mean Activity ● Simulation ● Data Collection ● Collaboration ● Survey ● Polling ● Applets ● Statistical Software ● Graphing Calculators ● Think-Pair Share ● Reading Tables and Charts ● Small Group Discussion	eBook-Statistics and Probability with Applications (High School) Daren S. Starnes (The Lawrenceville School) , Josh Tabor (Canyon del Oro (AZ) High School) 2016 LaunchPad Google Doc of Extra Examples Google Slide Lecture Presentations Prepared Tests and Test Bank Prepared Quizzes Prepared Worksheets PD Videos Applets Statistical Software IXL LGBT and Disabilities Resources: ● LGBTQ-Inclusive Lesson & Resources by Garden State Equality and Make it Better for Youth ● LGBTQ+ Books DEI Resources: ● Learning for Justice ● GLSEN Educator Resources ● Supporting LGBTQIA Youth Resource List ● Respect Ability: Fighting Stigmas, Advancing Opportunities ● NJDOE Diversity, Equity & Inclusion Educational Resources ● Diversity Calendar

Differentiation
*Please note: Teachers who have students with 504 plans that require curricular accommodations are to refer to Struggling and/or Special Needs Section for differentiation

High-Achieving Students	**On Grade Level Students**	**Struggling Students**	**Special Needs/ELL**

Probability and Statistics

Projects Problems with higher degree difficulty Higher order thinking challenges Use of Statistical Software	Guided Notes Chapter Summaries Formula Sheets Graphing Calculators Think-Pair-Share Group Work Collaboration Hands-On activities Real-life application of statistics Chapter Reviews prior to tests	Guided Notes Allow use of formula sheets on tests Hands-On Activities Real-life applications Require use of calculators Think-Pair-Share Allow students to work in groups to complete class assignments. Study Sheets Practice Tests Online review of content Pair Student with a High-Achieving student One-on-One Tutoring	Any student requiring further accommodations and/or modifications will have them individually listed in their 504 Plan or IEP. These might include, but are not limited to: breaking assignments into smaller tasks, giving directions through several channels (auditory, visual, kinesthetic, model), and/or small group instruction for reading/writing ELL supports should include, but are not limited to, the following:: Extended time Provide visual aids Repeated directions Differentiate based on proficiency Provide word banks Allow for translators, dictionaries

Unit Title: **Comparing Populations & Inference**	
Stage 1: Desired Results	

Standards & Indicators:
 S-IC Making Inferences and Justifying Conclusions:
 	
B. Make inferences and justify conclusions from sample surveys, experiments, and observational studies
    1. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
    2. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.
    3. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
    4. Evaluate reports based on data.	

Career Readiness, Life Literacies and Key Skills		
Standard	**Performance Expectations**	**Core Ideas**
9.4.12.CI.1	Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.1.1.12prof.CR3a).	With a growth mindset, failure is an important part of success.
9.4.12.CT.1	Identify problem-solving strategies used in the development of an innovative product or practice (e.g., 1.1.12acc.C1b, 2.2.12.PF.3)	Collaboration with individuals with diverse experiences can aid in the problem-solving process, particularly for global issues where diverse solutions are needed.

Probability and Statistics

9.4.12.GCA.1	Collaborate with individuals to analyze a variety of potential solutions to climate change effects and determine why some solutions (e.g., political. economic, cultural) may work better than others (e.g., SL.11-12.1., HS-ETS1-1, HS-ETS1-2, HS-ETS1-4, 6.3.12.GeoGI.1, 7.1.IH.IPERS.6, 7.1.IL.IPERS.7, 8.2.12.ETW.3).	Solutions to the problems faced by a global society require the contribution of individuals with different points of view and experiences.

Central Idea/Enduring Understanding:	**Essential/Guiding Question:**
Chapter 9: ● Describing Sampling Distributions ● Checking Conditions ● Calculating Difference Between 2 Proportions ● Testing a Claim about a Difference Between 2 Proportions ● Estimating Difference Between 2 Means ● Testing a Claim about a Difference Between 2 Means ● Analyzing Paired Data ● Testing a Claim about Mean Difference ● Reading Graphs, Charts & Tables **Chapter 10:** ● Stating Hypothesis ● Calculating Expected Counts ● Chi-Square Tests ● Testing the Distribution of a Categorical Variable ● Goodness of Fit ● Testing a Relationship Between 2 Categorical Variables ● Chi-Square Test for Association	**Chapter 9:** At the end of this chapter, students should be able to answer the Essential Question: *What's the relationship between the mean difference and the difference of the means?* <u>Answer:</u> The mean difference is equal to the difference of the means. **Chapter 10:** At the end of this chapter, students should be able to answer the Essential Question: *When is a chi-square test for goodness of fit used?* <u>Answer:</u> The Goodness of Fit test is applied when you have one categorical variable from a single population. It is used to determine whether sample data are consistent with a hypothesized distribution.
Content:	**Skills(Objectives):**
Chapter 9: 9.1 Difference Between 2 Proportions 9.2 Testing a Claim about a Difference Between 2 Proportions 9.3 Estimating Difference Between 2 Means 9.4 Testing a Claim about a Difference Between 2 Means 9.5 Analyzing Paired Data 9.6 Testing a Claim about Mean Difference	**Chapter 9:** 9.1 Describe the characteristics of the sampling distribution of the difference between 2 proportions. Calculate probabilities using the sampling distribution of the difference between 2 proportions. Determine whether the conditions for performing inference are met. Construct and interpret a confidence interval to compare two proportions. 9.2 State the hypothesis and check the conditions for performing a significance test about a difference between 2 proportions. Calculate the standardized test statistic

Probability and Statistics

| **Chapter 10:**
10.1 Testing the Distribution of a Categorical Variable
10.2 Goodness of Fit
10.3 Testing a Relationship Between 2 Categorical Variables
10.4 Chi-Square Test for Association | and P-value for a significance test about a difference between 2 proportions. Perform a significance test to compare two proportions. Interpret the results of inference procedures in a randomized experiment.

9.3 Describe the shape, center and variability of the sampling distribution of a difference between 2 sample means. Check the Random and Normal/Large Counts conditions for constructing a confidence interval for a difference between 2 means. Use the 4-Step process to construct and interpret a confidence interval for the difference between 2 means.

9.4 State hypotheses and check conditions for performing a significance test about a difference between 2 means. Calculate the standardized test statistic and P-value for a significance test about a difference between 2 means. Use the 4-Step process to perform a significance test about a difference between 2 means.

9.5 Use a graph to analyze the distribution of differences in a paired data set. Calculate the mean and standard deviation of the differences in a paired data set, and interpret the mean difference in context. Use the 4-Step process to construct and interpret a confidence interval for the true mean difference.

9.6 Use the 4-Step process to construct and interpret a confidence interval for the true mean difference. Determine whether you should use 2-sample *t* procedures for inference in a given setting.

Chapter 10:
10.1 State hypothesis for a test about the distribution of a categorical variable. Calculate expected counts for a test about the distribution of a categorical variable. Calculate expected counts for a test about the distribution of a categorical variable. Calculate test statistic for a test about the distribution of a categorical variable.

10.2 Test conditions for a test about the distribution of a categorical variable. Calculate the P-value for a test about the distribution of a categorical variable. Use the 4-Step process to perform a chi-squared test for goodness of fit. |

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	10.3 State hypotheses for a test about the relationship between 2 categorical variables. Calculate expected counts for a test about the relationship between 2 categorical variables. Calculate the test statistic for a test about the relationship between 2 categorical variables. 10.4 Check conditions for a test about the relationship between 2 categorical variables. Use the 4-Step process to perform a chi-square for association.
Interdisciplinary Connections: Interdisciplinary connections are integrated in each unit with connections to the mathematical practices. 1. Make sense of problems and persevere in solving them 2. Reason abstractly and quantitatively 3. Construct viable arguments and critique the reasoning of others 4. Model with mathematics 5. Use appropriate tools strategically 6. Attend to precision 7. Look for and make use of structure 8. Look for and express regularity in repeated reasoning	

Stage 2: Assessment Evidence

Performance Task(s): **Chapter 9:** Lesson App 9.1- Who likes rap music more? Lesson App 9.2- Does taking aspirin help prevent heart attacks? Lesson App 9.3- Do bigger apartments cost more money? Lesson App 9.4- Is name-brand popcorn better than store-brand? Lesson App 9.5- Is caffeine dependence real? Lesson App 9.6- Does generic ice cream melt faster? **Chapter 10:** Lesson App 10.1- Are fruit flies predictable? Lesson App 10.2 - Is the die fair? Lesson App 10.3- Is there an association between gender and superpower preference? Lesson App 10.4- Should angry people go to the sauna?	**Other Evidence:** Classwork Written & Online Homework Section Quizzes Chapter Tests Online Student Assessments End of Unit Assessment Graphing Calculators Applets Statistical Software

Stage 3: Learning Plan

Probability and Statistics

- Mystery Mean Activity
- Simulation
- Data Collection
- Collaboration
- Survey
- Polling
- Applets
- Statistical Software
- Graphing Calculators
- Think-Pair Share
- Reading Tables and Charts
- Small Group Discussion

Resources:

eBook-Statistics and Probability with Applications (High School) Daren S. Starnes (The Lawrenceville School) , Josh Tabor (Canyon del Oro (AZ) High School) 2016
LaunchPad
Google Doc of Extra Examples
Google Slide Lecture Presentations
Prepared Tests and Test Bank
Prepared Quizzes
Prepared Worksheets
PD Videos
Applets
Statistical Software
IXL

LGBT and Disabilities Resources:
- LGBTQ-Inclusive Lesson & Resources by Garden State Equality and Make it Better for Youth
- LGBTQ+ Books

DEI Resources:
- Learning for Justice
- GLSEN Educator Resources
- Supporting LGBTQIA Youth Resource List
- Respect Ability: Fighting Stigmas, Advancing Opportunities
- NJDOE Diversity, Equity & Inclusion Educational Resources
- Diversity Calendar

Differentiation

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High-Achieving Students	On Grade Level Students	Struggling Students	Special Needs/ELL
Projects Problems with higher degree difficulty Higher order thinking challenges Use of Statistical Software	Guided Notes Chapter Summaries Formula Sheets Graphing Calculators Think-Pair-Share Group Work Collaboration Hands-On activities Real-life application of statistics Chapter Reviews prior to tests	Guided Notes Allow use of formula sheets on tests Hands-On Activities Real-life applications Require use of calculators Think-Pair-Share Allow students to work in groups to complete class assignments. Study Sheets Practice Tests	Any student requiring further accommodations and/or modifications will have them individually listed in their 504 Plan or IEP. These might include, but are not limited to: breaking assignments into smaller tasks, giving directions through several channels (auditory, visual, kinesthetic, model), and/or small group instruction for reading/writing ELL supports should include, but are not limited to, the following:: Extended time Provide visual aids

Probability and Statistics

		Online review of content	Repeated directions
		Pair Student with a High-Achieving student	Differentiate based on proficiency
		One-on-One Tutoring	Provide word banks
			Allow for translators, dictionaries

Pacing Guide

Probability & Statistics	TextBook: Statistics and Probability with Applications	NJSLS-Mathematics
UNIT 1 Analyzing and Collecting Data(21 Days)	**CHAPTERS** 1: (7 Days) 2: (7 Days) 3: (7 Days)	S.ID 1 - 9
MID MP		
UNIT 2 Exploring Random Variables and Probability (21 Days)	**CHAPTERS** 4: (7 Days) 5: (7 Days) 6: (7 Days)	S.CP 1 - 9 S.MD 1- 7
END OF MP		
UNIT 3 Estimating a Parameter & Testing a Claim (21 Days)	**CHAPTERS** 7: (10 Days) 8: (11 Days)	S.IC 1-4
MID MP		
UNIT 4 Comparing Population & Inference (21 Days)	**CHAPTERS** 9: (11 Days) 10: (10 Days)	S.IC 3-5
END OF MP		