Tracker-assisted Modeling: Simultaneous Validation of the Conservation Law of Energy and the Work-Energy Theorem

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Abstract

The video of a free-falling object was analyzed in Tracker in order to extract the position and the time data. On the basis of these data, the velocity, gravitational potential energy, kinetic energy, and the work done by gravity were obtained. These led to a rather simultaneous validation of the conservation law of energy and the work-energy theorem. The superimposed plots of the kinetic energy, gravitational potential energy, and the total energy as respective functions of time and position demonstrate energy conservation quite well. The same results were observed from the plots of the potential energy against the kinetic energy. The total energy values obtained from the said plots are $0.391 \pm 1.12 \times 10^{-17}$ J, $0.391 \pm 1.69 \times 10^{-17}$ J, $0.391 \pm 8.14 \times 10^{-18}$ J, $0.391 \pm 7.86 \times 10^{-18}$ J. On the other hand, the work-energy theorem has emerged from the plot of the total work-done against the change in kinetic energy, the slope of the two linear plots being equal to 1.00 ± 0.03 and $1.00 \pm 6.23 \times 10^{-16}$ Because of the accessibility of the setup, the current work is seen as suitable for a home-based activity, during these times of the pandemic in particular in which online learning has remained to be the format in some countries. With the guidance of a teacher, online or face-to-face, students in their junior or senior high school – as well as for those who are enrolled in basic physics in college– will be able to benefit from this work.

1. Introduction

The conservation law of energy and the work-energy theorem are usually introduced in high schools. up to introductory college, in the manner that these topics are presented in texts, conceptually and theoretically. In this technique, teachers would demonstrate the validity of the concepts by applying them in solving exercise problems which are equally solvable in the framework of Newton's laws of motion or with the equations of kinematics. To the fascination of the students upon seeing total agreement in the results. But there is nothing more demonstrative of the correctness of a physical theory or concept other than with an experimental verification. Indeed, experimental demonstrations, for pedagogical purposes, of the conservation law of energy [1,2] and the work-energy theorem [3,4] are well-documented in the literature using varied systems and techniques.

The current article aims to present yet another way of demonstrating the two concepts using the well-known video-analysis technique applied on a free-falling system. The setup is common but our modified work, to the best of our knowledge – the way the data analysis is relatively extended in particular – appears to be lacking presentation in the literature. Video-based study of free-falling motion is well-known among teachers, but it re-appears in the current work because some data extracted from it were needed in validating energy conservation and the work-energy theorem. This makes the current work a simultaneous validation of three physical concepts.

2. Theoretical background

The theoretical relationship between the position and the time associated with a free-falling object is given by the equation

$$y = y_o + v_o t - \frac{1}{2} g t^2 . \tag{1}$$

where y is the position, t is the time, v_o is the initial velocity, g is the acceleration due to gravity, and y_o is the initial position. The conservation law of energy associated with a free-falling object is expressed as

$$\frac{1}{2} m v^2 + mgy = E_T , \tag{2}$$

where m is the mass of the object, v is its instantaneous speed, and E_T is the conserved total mechanical energy. The first and the second terms (on the left-hand side) are the kinetic and potential energy, respectively. Employing kinematical relations, equation (2) can be written explicitly as a function of time:

$$\frac{1}{2} m \left(v_o - gt\right)^2 + mg\left(y_o + v_o t - \frac{1}{2} g t^2\right) = E_T . \tag{3}$$

Moreover, equation (2) can be written in terms of the position alone:

$$\frac{1}{2}m\left[v_o^2-2g(y-y_o)\right]+mgy=E_T. \quad (4)$$

With KE as the kinetic energy and PE the gravitational potential energy of the object, equations (2), (3), or (4) can be written as

$$PE=E_T-KE. \quad (5)$$

Equation (5) expresses a linear relationship between the gravitational potential energy and the kinetic energy with a slope of negative one (-1). The intercept gives the total mechanical energy. Notice that equations (3) and (4) can be inferred to be clearer than equation (2) – but the emphasis of which is apparently hidden in texts– as expressions for the conservation law of energy. This is because the expressions are in terms of the position or time alone, a technique which would likely invite the students to think that, indeed, the total energy of a system is conserved in whatever point in space and time. This has been employed in a related work [5] but the data is not as prominent. It is in the case of projectile motion of a basketball which is subject to some effects of air friction. For its part, the work-energy theorem theoretically states that

$$W_T=\frac{1}{2}mv_2^2-\frac{1}{2}mv_1^2, \quad (6)$$

where W_T is the total work done by all the forces on a body while the first and the second term, on the right-hand side, are the object's final and initial kinetic energy, respectively. In compact form, we write equation (6) as

$$W_T=\Delta KE, \quad (7)$$

where the right-hand side is identified as the change in kinetic energy. Thus, we see that the total work done and the change in kinetic energy are linearly related to each other with the value of the slope being equal to one.

3. Experimental procedure and data analysis

A spherical object (a small-sized billiard ball) was dropped freely and all the while a slow-motion video of it was recorded with a high-speed smartphone camera. The experimental setup is depicted in figure 1. Later, the video having been transferred to a PC and then analyzed in Tracker [6,7], a popular video-based physics modeling software, the position and the time data were obtained. The origin of the coordinate system was designated about $y_o = 1.20\ m$ below the initial position (any convenient point below the point where the ball was hand-released) of the ball. Such initial position (where the tracking was set to start) was chosen because it was supposed to be more convenient than the very point where the ball was hand-released. This is illustrated in figure 1. The data were subsequently copied from Tracker and then pasted into MS Excel where the data analysis was performed.

At first, we were interested in the value of the initial velocity, so we plotted the position against the time. This plot, together with a polynomial fit, is presented in figure 2. The function fitting in MS Excel was done by right-clicking on the scatter plot of the datapoints and then selecting 'add trendline' in which a polynomial fit was selected together with the selection of the 'equation display' option and of the R^2 value. Now, by comparing the coefficients in equation (1) with that in the quadratic fit, we obtained the initial velocity to be equal to *–1.21 m/s*. Additionally, the same plot demonstrates equation (1) while also revealing (also by comparison of the coefficients in the fit with that in equation (1)) an experimental magnitude of *g* to be equal to 9.90 m/s^2. This value is in error by 1.02% in comparison to the average value of 9.80 m/s^2.

Using the experimental value of *g* (= *9.90) m/s^2*), initial velocity (=*–1.21 m/s)*, mass of the object *(=0.031 kg)*, and the time data, we have computed, the values of the kinetic energy, potential energy, and the total energy using the respective terms in equation (3). That is, the kinetic energy was computed using the first term, potential energy with the second term, and the total energy by summing up the results of the two terms (the same process was used in what follows).

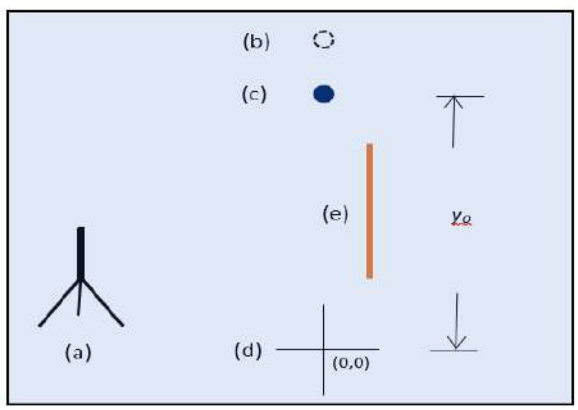

Figure 1. Experimental setup. (a) Smartphone camera running at 120 fps (slow motion); (b) point where the ball was hand-released; (c) position of the ball where the tracking was designated to begin; (d) designated origin of the coordinate system; the meter stick in (e) attached to a wall was used in calibrating the video. The ball was hand-held and released very close to the plane of the wall.

The position, kinetic energy, potential energy, and the total energy were all rounded off to 3 decimal digits to be consistent with the time resolution that Tracker acquired the time data which was equal to 0.017 sec. Next, we have superimposed the plots of the kinetic energy, potential energy, and the total energy against the time. Our output is shown in figure 3. The respective fit functions agreeing well with what are expressed in equation (3) are observed as well. Better yet, the uppermost plot, which is that of the total energy, is a horizontal line, and this validates the constancy of the total energy of the system at any instant in time. The total energy is equal to the intercept of the time-based plot of the total energy which is equal to 0.391 J. By employing the linear regression analysis feature of MS Excel, a relatively parallel process is detailed in another work [8], we obtained the error in the energy (error in the intercept) and so the total energy is $0.391 \pm 1.12 \times 10^{-17}\,J$. The same process in obtaining errors in the total energy was employed in what follows.

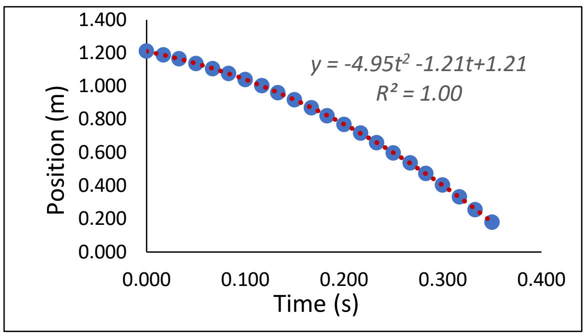

Figure 2. Plot of the position against the time together with a function fit (dashed-red (the same color code is used in what follows).

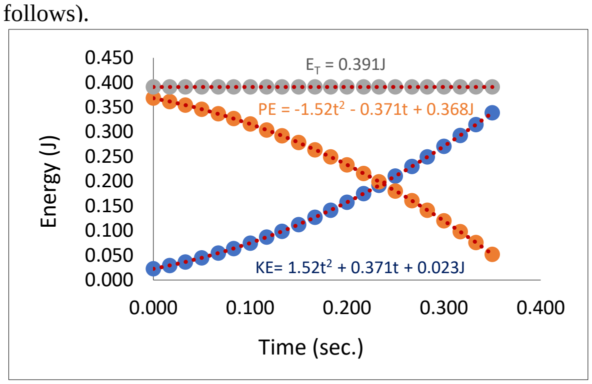

Figure 3. Superimposed time plots of the kinetic energy, potential energy, and the total energy, together with their respective fit functions (when both kinetic and potential energy are expressed as functions of time).

Furthermore, we have plotted the potential energy against the kinetic energy and the generated graph is shown in figure 4. The linear fit appears to be a perfect match to equation (5) as demonstrated by the value of the slope *(= -1)*. Hence, conservation of mechanical energy is alternatively demonstrated by figure 4 with

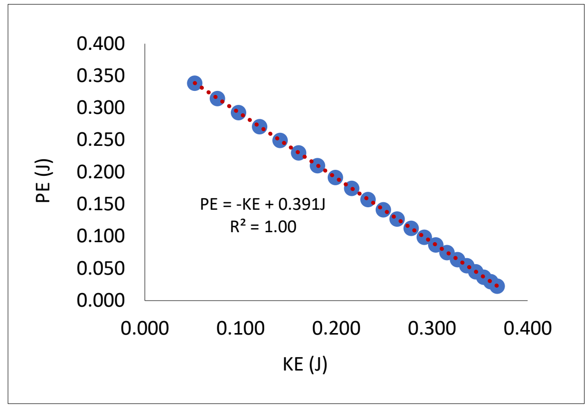

Figure 4. Plot of the potential energy against the kinetic energy (when both are expressed as functions of time).

the total mechanical energy, $0.391 \pm 1.69 \times 10^{-17}\,J$, being equal to the intercept of the linear fit. Subsequently, the kinetic energy, potential energy, and their sum (total energy) were computed by similarly inserting the experimental value of the gravitational acceleration, the obtained initial velocity, the mass of the object, and the position data in equation (4). Figure 5 shows the superimposed plots of the resulting values of kinetic energy, potential energy, and the total energy as functions of the position.

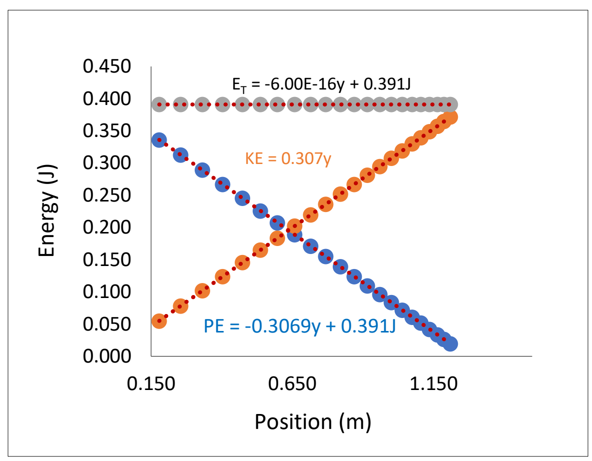

Figure 5. Superimposed position plots of the kinetic energy, potential energy, and total energy, together with their respective fit functions (when both kinetic energy and potential energy are expressed in terms of the position).

Likewise, we could see a consistency between the linear functions fit to the kinetic energy and potential energy and with what are described in equation (4). Most interestingly, though, is the linear fit to the total energy being near-horizontal, validating total energy conservation. The total energy obtained is $0.391 \pm 8.14 \times 10^{-18} J$. Here, we could observe that the plots of the total energy – in both figures 3 and 5 – similarly reveal the conserved total energy of the system being equal to 0.391 *J*, except for a little discrepancy in the error. Then again, as an alternative graphical validation of the same concept, the potential energy was plotted as a function of the kinetic energy and the resulting plot is shown in figure 6. This time, with the slope being also equal to negative one, revealing a quite exact match between the linear function fit and equation (5). Therefore, the demonstration of equation (5) that is the conservation of the total mechanical energy. The total energy together with the error is $0.391 \pm 7.86 \times 10^{-18} J$.

Finally, to validate equation (6) or (7) we needed to compute the work done by gravity, which was the total work done because there was no other force (air friction was neglected) on the object as it moved from

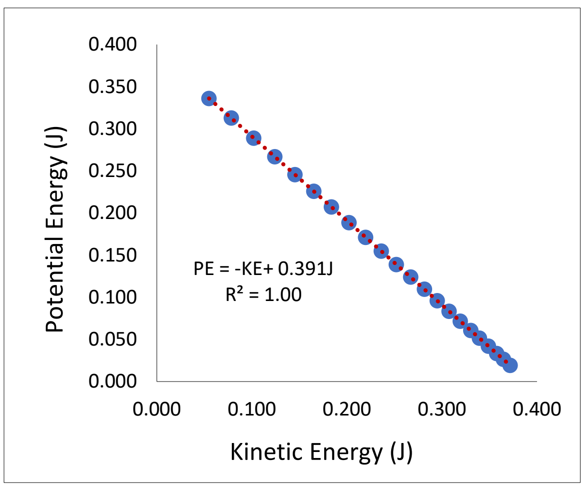

Figure 6. Plot of the potential energy against the kinetic energy (when both are expressed as functions of the position).

one position to the immediate next position. To do this, the displacements were first computed by subtracting the first position datapoint from the immediate second position datapoint and dragging down, we produced all the target displacement datapoints. After multiplying the absolute values of these displacement data to the weight (= *0.30 N*) of the object, we obtained all the datapoints for the total work done. Then by using the same procedure on the kinetic energy datapoints (though the absolute values need not be taken)

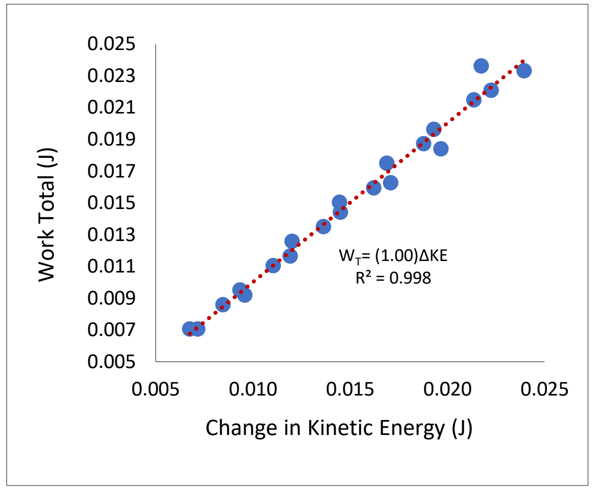

Figure 7. Scatter plot of the total work done against the change in kinetic energy (from equation (3)) along with a linear function fit.

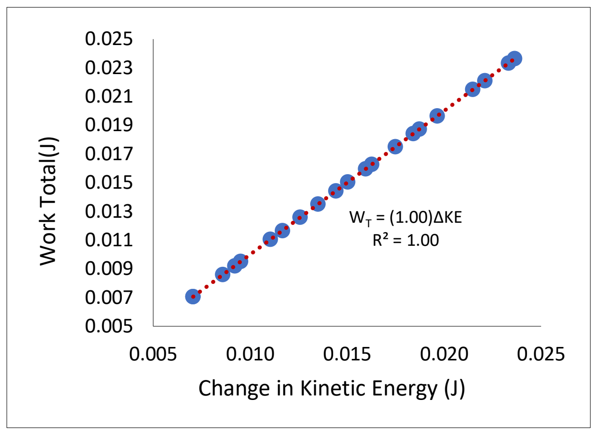

Figure 8. Scatter plot of the total work done against the change in kinetic energy (from equation (4)) along with a linear function fit.

used to obtain the target displacements, the corresponding changes in kinetic energy were obtained. Here the kinetic energies involved were those computed using the first term in the left-hand side of equation (3). The total work done was then plotted against the change in kinetic energy and the resulting plot, along with a linear fit, is presented in figure 7. Indeed, it is easy to see that the linear fit matches equation (7) very well in which the slope, together with the error is equal to 1.00 ± 0.03. Thus, the validation of the work-energy theorem. The same process of linear regression in MS Excel was used in obtaining the error, except that that the target data was the error in the slope instead of the error in the intercept. These two error values simultaneously appear in the tabular output of the linear regression in MS Excel.

In addition, changes in kinetic energy were also computed by taking into account the kinetic energies which were obtained using the second term in the left-hand side of equation (4). Now plotting the same total work done (used in obtaining the plot shown in figure 7) against these changes in kinetic energy, the work-energy theorem had emerged with even greater accuracy as indicated by the slope of the linear fit being equal to $1.00 \pm 6.23 \times 10^{-16}$.This linear plot, which proves to be remarkably consistent with equation (7), is shown in figure 8.

4. Conclusions and recommendations

By using a free-falling system and video analysis, via Tracker, we have experimentally validated the conservation law of energy and the work-energy theorem. The validation of these two important physics concepts –including the kinematics of free-falling motion – was rather simultaneous because the same raw data were needed to prove the three theoretical concepts. The Tracker software is free and the entire setup is accessible and convenient. Certainly, the computer and the smartphone with a high-speed camera are still expensive but these gadgets have long become ubiquitous, making the present work easy to duplicate – both in school and at home.

In some countries, where access to Covid-19 vaccines is economically-constrained, the online mode of instruction has remained. In this format, the delivery of teaching –laboratory subjects in particular – has been fairly a challenge. Good thing for introductory laboratory physics, computer simulations have been extremely helpful, besides the fact that simulated experiments are freely accessible online. But then again, a simulated experiment would fit well as a technique in expounding the theoretical concept in a lecture setting rather than in a laboratory activity. That said, we believe that the present work is a contribution in addressing such a downside in the online teaching of introductory physics labs.

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