1.8 Combinations of Functions: Composite Functions

What you should learn

- Add, subtract, multiply, and divide functions.
- Find the composition of one function with another function.
- Use combinations and compositions of functions to model and solve real-life problems.

Definition of Composition of Two Functions

The **composition** of the function f with the function g is

$$(f \circ g)(x) = f(g(x)).$$

"f of g of x"

The domain of $(f \circ g)$ is the set of all x in the domain of g such that $g(x)$ is in the domain of f. (See Figure 1.90.)

Plug $g(x)$ into $f(x)$

Steps to finding domain

1. Identify domain of inner function (g)
2. Identify domain of composition (f(g))
3. Combine Domains.

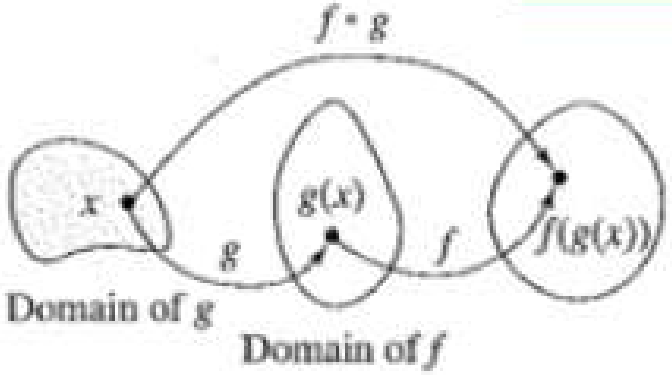


FIGURE 1.90

Example 4 **Composition of Functions**

Given $f(x) = x + 2$ and $g(x) = 4 - x^2$, find the following.

a. $(f \circ g)(x)$ **b.** $(g \circ f)(x)$ **c.** $(g \circ f)(-2)$

a) $f(g(x)) = f(4-x^2) = 4-x^2+2 = \boxed{6-x^2}$

b) $g(f(x)) = g(x+2) = 4-(x+2)^2 =$

$= 4-(x^2+4x+4)$ ← $(x+2)(x+2)$

$= \cancel{4}-x^2-4x-\cancel{4} = \boxed{-x^2-4x}$

c) $g(f(-2)) = -(-2)^2-4(-2) = \boxed{4}$

Example 5 **Finding the Domain of a Composite Function**

Given $f(x) = x^2 - 9$ and $g(x) = \sqrt{9 - x^2}$, find the composition $(f \circ g)(x)$. Then find the domain of $(f \circ g)$.

$f(g(x)) = f(\sqrt{9-x^2}) = (\sqrt{9-x^2})^2 - 9 =$

$= 9 - x^2 - 9 = \boxed{-x^2}$

D of f(g): $[-3, 3]$

Domain

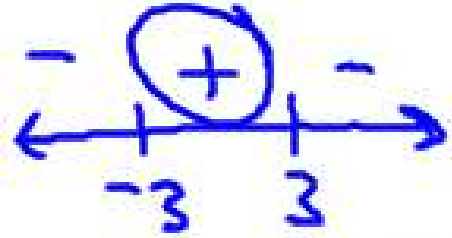
① D of g?

$9 - x^2 \geq 0$

$9 - x^2 = 0$

$\sqrt{9} = \sqrt{x^2}$

$\pm 3 = x$



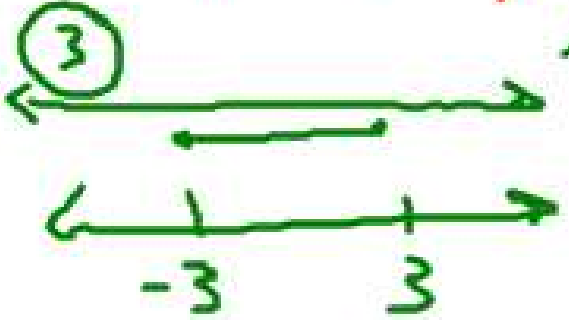
-4	$9-(-4)^2 = -7$
0	$9-(0)^2 = 9$
4	$9-(4)^2 = -7$

D of g: $[-3, 3]$

② D of $-x^2$

$(-\infty, \infty)$

③



Example 6 **Decomposing a Composite Function** (Backwards)

Write the function given by $h(x) = \frac{1}{(x-2)^2}$ as a composition of two functions.

$h(x) = f(g(x)) = \frac{1}{(x-2)^2}$

$f(x) = \frac{1}{x^2}$

$g(x) = x - 2$

$\frac{1}{x^2}$; $x - 2$

(f) (g)

Example 7 **Bacteria Count**

The number N of bacteria in a refrigerated food is given by

$$N(T) = 20T^2 - 80T + 500, \qquad 2 \le T \le 14$$

where T is the temperature of the food in degrees Celsius. When the food is removed from refrigeration, the temperature of the food is given by

$$T(t) = 4t + 2, \qquad 0 \le t \le 3$$

where t is the time in hours. (a) Find the composition $N(T(t))$ and interpret its meaning in context. (b) Find the time when the bacterial count reaches 2000.

$(4t+2)(4t+2)$

a) $N(T(t)) = N(4t+2) = 20(4t+2)^2 - 80(4t+2) + 500$

$= 20(16t^2 + 16t + 4) - 80(4t+2) + 500$

$= 320t^2 + 320t + 80 - 320t - 160 + 500$

$N = 320t^2 + 420$

# of Bacteria after t hours out of the fridge

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$N = 2000$

b) $N = 320t^2 + 420$

$2000 = 320t^2 + 420$

$-420 \qquad -420$

$1580 = 320t^2$

$\frac{1580}{320} = \frac{320t^2}{320}$

$\sqrt{4.9375} = \sqrt{t^2}$

$t = 2.2 \text{ hr}$

Homework: p. 89-92 #31-41(odd), 47-53(odd), 63, 65, 67