Section 1-4: Day 1

Building Functions from Functions

- Students will be able to combine functions algebraically
- Students will be able to find and use composites of functions
- Students will be able to use relations and implicitly defined functions

Combining Functions Algebraically

Let f and g be two functions with intersecting domains. Then for all values of x in the intersection, the algebraic combinations of f and g are defined by the following rules:

Sum: $(f+g)(x) = f(x) + g(x)$

Difference: $(f-g)(x) = f(x) - g(x)$

Product: $(fg)(x) = f(x)g(x)$

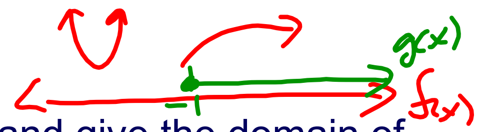
Quotient: $\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$, provided $g(x) \neq 0$

In each case, the domain of the new function consists of all numbers that belong to both the domain of f and the domain of g.

Defining New Functions Algebraically

Let $f(x) = x^2$ and $g(x) = \sqrt{x+1}$

D: $(-\infty, \infty)$ D: $[-1, \infty)$



Find formulas for the functions below and give the domain of each.

f + g $f(x) + g(x) = x^2 + \sqrt{x+1}$ $[-1, \infty)$

f - g $f(x) - g(x) = x^2 - \sqrt{x+1}$ $[-1, \infty)$

fg $f(x)g(x) = x^2\sqrt{x+1}$ $[-1, \infty)$

f/g $\frac{f(x)}{g(x)} = \frac{x^2}{\sqrt{x+1}} \neq 0$ $(-1, \infty)$

gg $g(x)g(x) = (\sqrt{x+1})^2$ *tricky $[-1, \infty)$

$x+1$ $(-\infty, \infty)$

#1

$f(x) = \sqrt{x+5}$ and $g(x) = |x+3|$

D: $[-5, \infty)$ D: $(-\infty, \infty)$

Find formulas for f + g, f - g and fg. Give the domain of each.

$f + g = \sqrt{x+5} + |x+3|$ D: $[-5, \infty)$

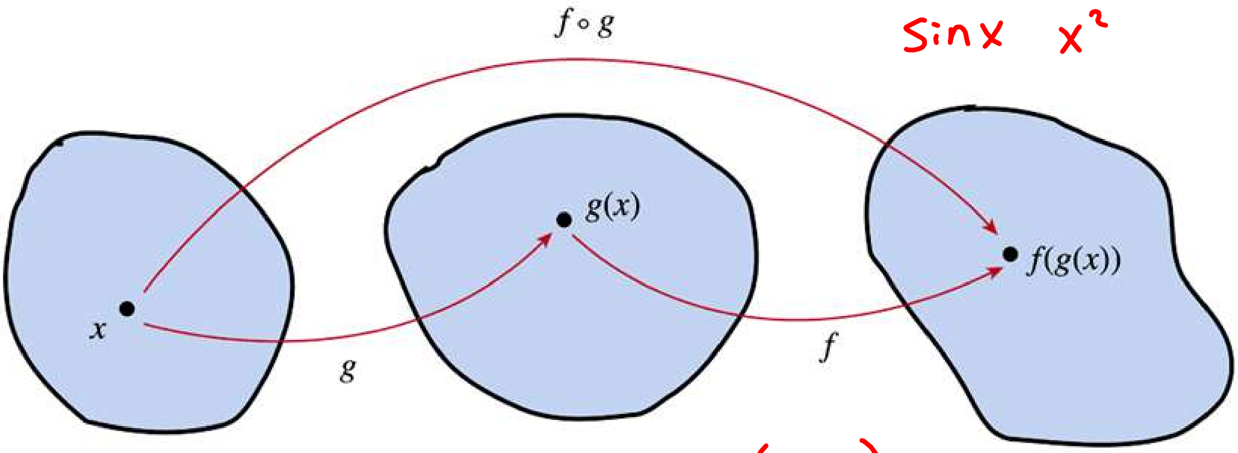
$f - g = \sqrt{x+5} - |x+3|$ D: $[-5, \infty)$

$fg = \sqrt{x+5}\,|x+3|$ D: $[-5, \infty)$

Composition of Functions

$\sin x^2$

$\sin x \quad x^2$



x must be in the domain of g and $g(x)$ must be in the domain of f

$(f \circ g)x = f(g(x))$

right to left   inside out

Composition of Functions

$f(g(3))$ $g(f(-2))$

$f(g(x))$ $g(f(x)) =$

Find $(f \circ g)(3)$ and $(g \circ f)(-2)$ $= 5$ $f(x) = \dfrac{x}{x+1}; g(x) = 9 - x^2$

$g(x) = 9 - x^2$

$g(3) = 9 - 3^2$

$= 9 - 9$

$g(3) = 0$

$f(0) = \dfrac{x}{x+1}$

$= \dfrac{0}{0+1}$

$= \dfrac{0}{1}$

$f(0) = 0$

$f(-2) = \dfrac{-2}{-2+1}$

$= \dfrac{-2}{-1}$

$= 2$

$g(2) = 9 - 2^2$

$= 9 - 4$

$= 5$

$(g \circ f)(-2) = 5$

$(f \circ g)(3)$

$= 0$

Composing Functions

Let $f(x) = e^x$ and $g(x) = \sqrt{x}$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$ and verify numerically that the resulting functions are not the same.

$(f \circ g)x = e^{\sqrt{x}}$

$(g \circ f)x = \sqrt{e^x}$

let $x = 1$

$\boxed{(f \circ g)x \neq (g \circ f)x}$

$e^{\sqrt{1}} = e$ $e \neq \sqrt{e}$

$\sqrt{e^1} = \sqrt{e}$

graphically or domain

Finding the Domain of a Composition

Let $f(x) = x^2 - 1$ and let $g(x) = \sqrt{x}$. Find the domains of the composite functions...

$(-\infty, \infty)$ $[0, \infty)$

$g \circ f$ $g(f(x)) = \sqrt{x^2 - 1}$

$x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

$D: (-\infty, -1] \cup [1, \infty)$

$f \circ g$

$f(g(x)) = \sqrt{x}^2 - 1$

$= (\sqrt{x})^2 - 1$

$x - 1$ tricky *

$D\ [0, \infty)$

HW 3 - 15 by 3's, 16-20 all