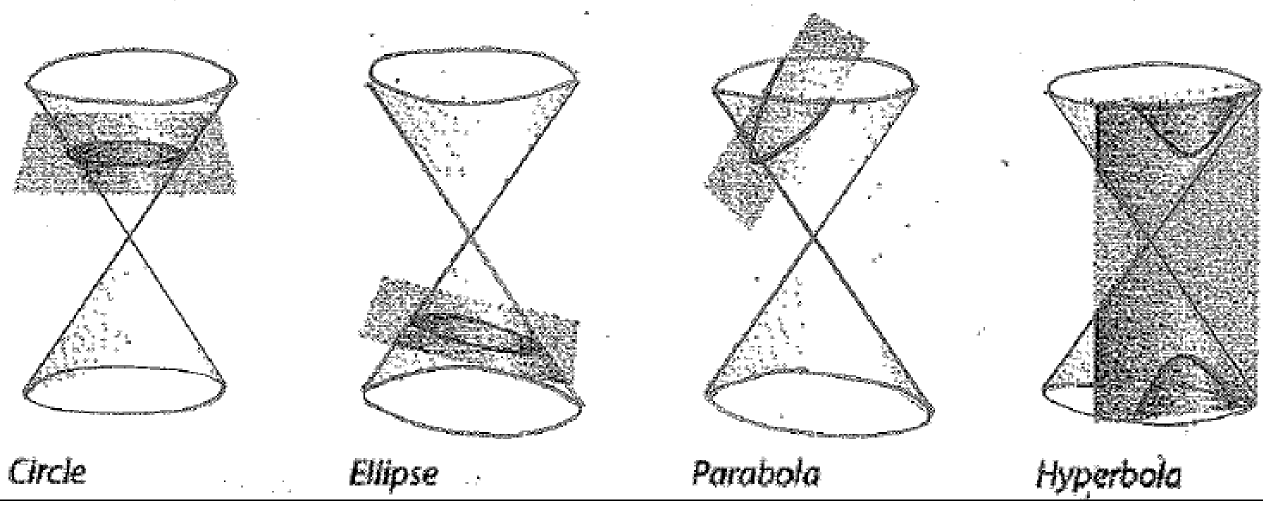
Precalculus Notes **Section 10.2: Introduction to Conics: Parabolas**

What you should learn:
1) Write equations of parabolas in standard form and graph parabolas.

conic section: the intersection of a plane and a double-napped cone

Basic Conics (p. 735)

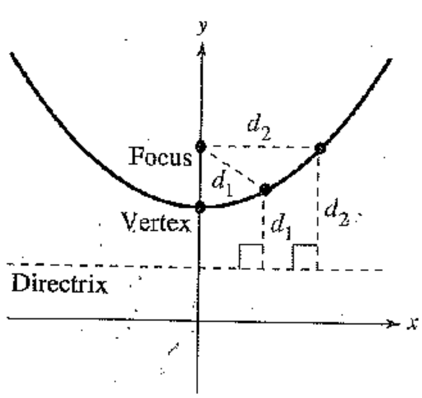


Equation of a Circle [with center (h, k) and radius r]

$$(x-h)^2+(y-k)^2=r^2.$$

Definition of Parabola

A **parabola** is the set of all points (x, y) in a plane that are equidistant from a fixed line **(directrix)** and a fixed point **(focus)** not on the line.



Standard Equation of a Parabola

The **standard form of the equation of a parabola** with vertex at (h, k) is as follows.

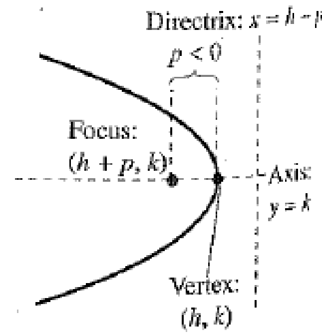
$$(x - h)^2 = 4p(y - k),\ p \neq 0 \qquad \text{Vertical axis, directrix: } y = k - p$$

$$(y - k)^2 = 4p(x - h),\ p \neq 0 \qquad \text{Horizontal axis, directrix: } x = h - p$$

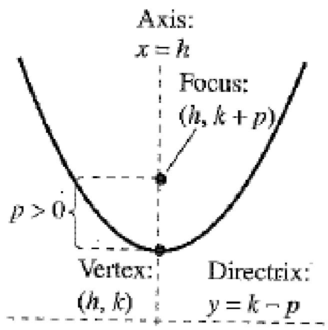
The focus lies on the axis p units (*directed distance*) from the vertex. If the vertex is at the origin $(0, 0)$, the equation takes one of the following forms.

$$x^2 = 4py \qquad \text{Vertical axis}$$

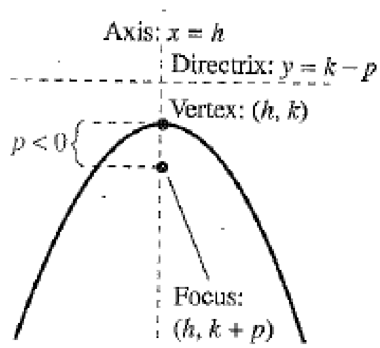
$$y^2 = 4px \qquad \text{Horizontal axis}$$



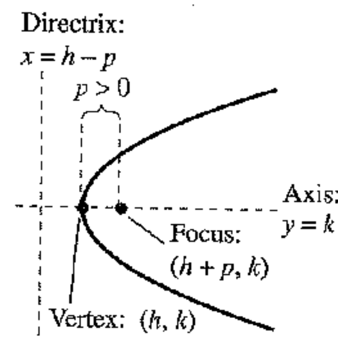
(d) $(y - k)^2 = 4p(x - h)$
Horizontal axis: $p < 0$



(a) $(x - h)^2 = 4p(y - k)$
Vertical axis: $p > 0$



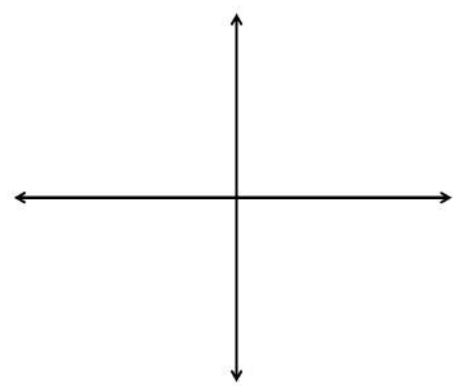
(b) $(x - h)^2 = 4p(y - k)$
Vertical axis: $p < 0$



(c) $(y - k)^2 = 4p(x - h)$
Horizontal axis: $p > 0$

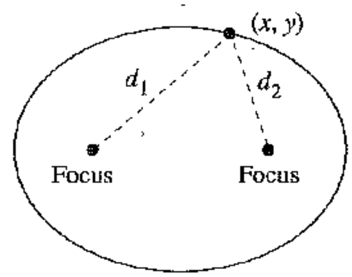
Example 2 **Finding the Focus of a Parabola**

Find the focus of the parabola given by $y = -\frac{1}{2}x^2 - x + \frac{1}{2}$.



HW: p. 740 #23, 25

What you should learn:
1) Write equations of ellipses in standard form and graph ellipses.
2) Find eccentricities of ellipses.



Definition of Ellipse

An **ellipse** is the set of all points (x, y) in a plane, the sum of whose distances from two distinct fixed points **(foci)** is constant. See Figure 10.18.

Standard Equation of an Ellipse

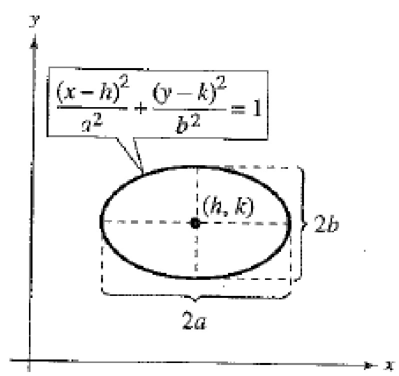
The **standard form of the equation of an ellipse,** with center (h, k) and major and minor axes of lengths $2a$ and $2b$, respectively, where $0 < b < a$, is

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \qquad \text{Major axis is horizontal.}$$

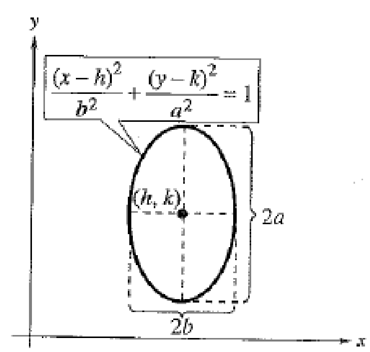
$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1. \qquad \text{Major axis is vertical.}$$

The foci lie on the major axis, c units from the center, with $c^2 = a^2 - b^2$. If the center is at the origin $(0, 0)$, the equation takes one of the following forms.

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1 \quad \text{Major axis is horizontal.} \qquad \frac{x^2}{b^2} + \frac{y^2}{a^2} = 1 \quad \text{Major axis is vertical.}$$



Major axis is horizontal.



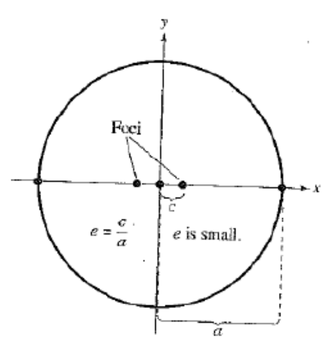
Major axis is vertical.

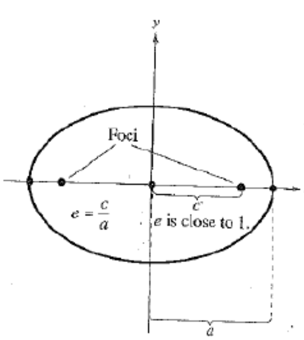
<u>eccentricity</u>: used to measure the ovalness of an ellipse

Definition of Eccentricity

The **eccentricity** e of an ellipse is given by the ratio

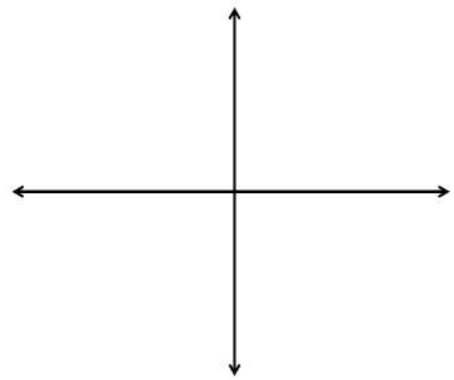
$$e = \frac{c}{a}. \qquad \text{Note that } 0 < e < 1 \text{ for } \textit{every} \text{ ellipse.}$$





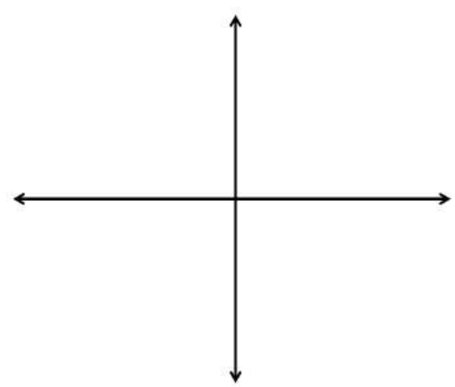
Example 2 Sketching an Ellipse

Sketch the ellipse given by $x^2 + 4y^2 + 6x - 8y + 9 = 0$.



Example 3 Analyzing an Ellipse

Find the center, vertices, and foci of the ellipse $4x^2 + y^2 - 8x + 4y - 8 = 0$.

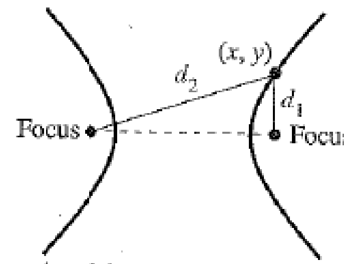


HW: p. 750 #19, 21, 23, 25

What you should learn:

1) Write equations of hyperbolas in standard form.

2) Find asymptotes of and graph hyperbolas.



$d_2 - d_1$ is a positive constant.

Definition of Hyperbola

A **hyperbola** is the set of all points (x, y) in a plane, the difference of whose distances from two distinct fixed points **(foci)** is a positive constant. See Figure 10.29.

Standard Equation of a Hyperbola

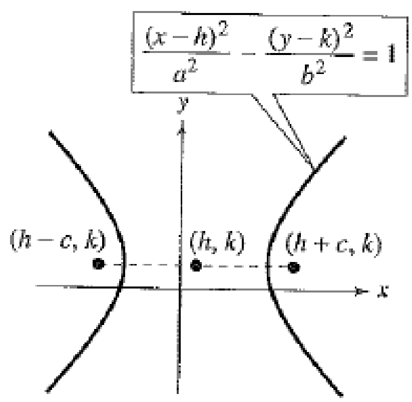
The **standard form of the equation of a hyperbola** with center (h, k) is

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1 \qquad \text{Transverse axis is horizontal.}$$

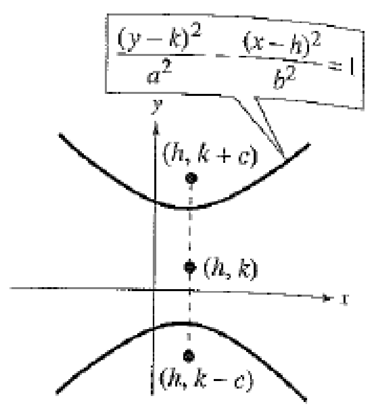
$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1. \qquad \text{Transverse axis is vertical.}$$

The vertices are a units from the center, and the foci are c units from the center. Moreover, $c^2 = a^2 + b^2$. If the center of the hyperbola is at the origin $(0, 0)$, the equation takes one of the following forms.

$$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \quad \text{Transverse axis is horizontal.} \qquad \frac{y^2}{a^2} - \frac{x^2}{b^2} = 1 \quad \text{Transverse axis is vertical.}$$



Transverse axis is horizontal.



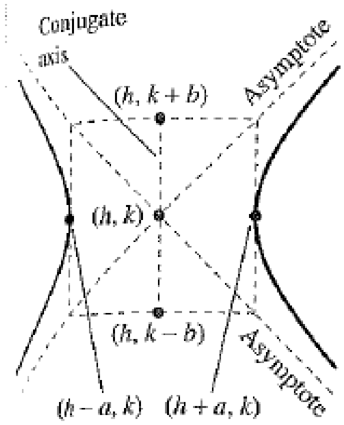
Transverse axis is vertical.

Asymptotes of a Hyperbola

The equations of the asymptotes of a hyperbola are

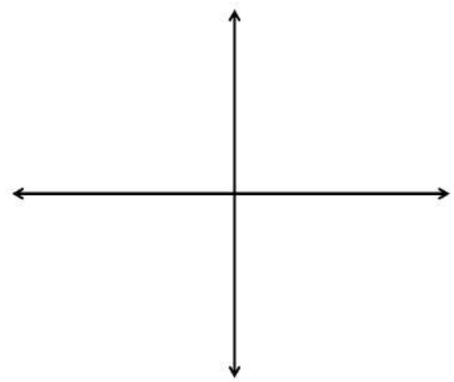
$$y = k \pm \frac{b}{a}(x - h) \qquad \text{Transverse axis is horizontal.}$$

$$y = k \pm \frac{a}{b}(x - h). \qquad \text{Transverse axis is vertical.}$$



Example 3 **Finding the Asymptotes of a Hyperbola**

Sketch the hyperbola given by $4x^2 - 3y^2 + 8x + 16 = 0$ and find the equations of its asymptotes and the foci.



HW: p. 760 #13, 19

Extra Practice: p. 762 #49, 51, 57, 59