Summit Public Schools

Summit, New Jersey

Grade Levels 10th - 11th / Content Area: Mathematics

Length of Course: Full Academic Year

Pre-Calculus Honors

Submitted by:

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Curriculum

Course Description:

Pre-Calculus Honors is a full-year course offered as part of the mathematics curriculum. This course begins with a review of important parent functions, focusing on their properties and applications. An in-depth study of trigonometry will follow. Students will solve problems that arise in a variety of fields using algebraic and graphic properties of trigonometry, often requiring the application of important identities. The course will conclude with the study of parametric and polar functions, conic sections, sequences, series, and an introduction to calculus. Students will be expected to use a variety of technological tools in solving the complex problems that they will encounter in this course. Skills in using the TI-83 graphing calculator and a variety of computer programs will be taught in parallel with the material.

(Corresponding textbook sections are included in parentheses)

Unit 1 – Functions and Their Graphs

Topic	Time Frame
Algebraic Functions (1.1, 1.2)	
Function Transformations (1.3, 1.6)	
Function Operations (1.4, 1.5)	
Functions, Graphically (7.1, 7.2)	
Functions, Numerically (7.3)	
Logarithmic, Exponential and Logistic Functions (7.4, 7.5, 7.6)	
Polynomial Functions (15.1, 15.2, 15.3)	
Review	
Assessments	
Total	

Unit 2 – Introduction to Trigonometry

Periodic Functions (2.1, 2.2, 2.3)	
Trigonometric Function Values (2.4)	
Inverse Trig Functions (2.5)	
Review	
Assessments	
Total	

Unit 3 – Trigonometric Graphs and Identities

Topic	Time Frame
Graphs of the Six Trigonometric Functions (3.1, 3.2, 3.3)	
Radian Angle Measurement (3.4)	
Circular Functions (3.5, 3.6)	
Modeling with Sinusoidal Functions (3.7)	
Pythagorean and Reciprocal Identities (4.1, 4.2, 4.3)	
Inverse Trig Functions and Graphs (4.4, 4.6)	
Parametric Functions (4.5)	
Review	
Assessments	
Total	

Unit 4 – Applications of Trigonometry

Composite Arguments and Linear Combinations (5.2, 5.3)	
Composition of Ordinates and Harmonic Analysis (5.4)	
Sum, Product, Double, and Half Angle Identities (5.5, 5.6)	
Law of Cosines (6.2)	
Law of Sines (6.4, 6.5)	
Area of Triangles (6.3)	
Real-World Triangle Problems (6.7)	
Review	
Assessments	
Total	

Unit 5 – Vectors

Topic	**Time Frame**
Two-Dimensional Vectors (6.6, 10.1, 10.2)	
Vectors in Space (10.3)	
Scalar Products and Vector Projections (10.4)	
Planes in Space (10.5)	
Vector Product of Two Vectors (10.6)	
Direction Angles and Direction Cosines (10.7)	
Review	
Assessments	
TOTAL	

Unit 6 – Conic Sections

Equations and Graphs of the Conic Sections (12.1, 12.2)	
Quadratic Surfaces and Inscribed Figures (12.3)	
Analytic Geometry of the Conic Sections (12.4)	
Parametric Equations of Conic Sections (12.5)	
Applications of Conic Sections (12.6)	
Review	
Assessments	
TOTAL	

Unit 7 – Polar Coordinates and Complex Numbers

Topic	Time Frame
Polar Equations (13.1, 13.2)	
Intersections of Polar Curves (13.3)	
Complex Numbers in Polar Form (13.4)	
Parametric Equations for Motion (13.5) **optional, as time allows	
Review	
Assessments	
TOTAL	

Unit 8 – Discrete Math and an Introduction to Calculus

Arithmetic, Geometric and Other Sequences (14.1, 14.2)	
Series and Partial Sums (14.3)	
Limits and Continuity (15.4)	
Instantaneous Rate of Change (15.5)	
Mathematical Induction (not in text)	
Review	
Assessments	
TOTAL	

Unit 1 - Functions and Their Graphs

Standards: Interpreting Functions (F-IF), Building Functions (F-BF), Arithmetic with Polynomials (A-APR)	
Big Ideas: *Course Objectives / Content Statement(s)* • To review functions and learn to identify, categorize, describe and graph functions. • To express numerical relationships algebraically, graphically, and numerically. • To examine, with great detail, polynomial, rational, logarithmic, exponential, and logistic functions.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
	Students will understand that…
What are the physical and algebraic characteristics of functions?	Translations, dilations, and reflections can be applied to any parent function by changing parameters in the parent function's equation.
How can functions be represented in different ways?	Functions can be represented in algebraic, graphic, numberic, and verbal forms.
What are the important characteristics of, and similarities and differences between, the important families of functions?	Polynomial, rational, exponential, logarithmic, and logistic functions each have defining characteristics such as a domain, range, end behavior, and critical points.

<table>
<tr><th>Areas of Focus: Proficiencies

(Cumulative Progress Indicators)</th><th>Examples, Outcomes, Assessments</th></tr>
<tr><td>Students will:</td><td rowspan="21">Instructional Focus:

1. Functions: Algebraically, Numerically, Graphically, and Verbally (1.1)
2. Kinds of Functions (1.2)
3. Dilation and Translation of Function Graphs (1.3)
4. Composition of Functions (1.4)
5. Inverse of Functions (1.5)
6. Reflections, Absolute Values, and other Transformations (1.6)
7. Shapes of Function Graphs (7.1)
8. Identifying Functions from Graphical Patterns (7.2)
9. Identifying Functions from Numeric Patterns (7.3)
10. Logarithms (7.4)
11. Logarithmic Properties (7.5)
12. Logistic Functions (7.6)
13. Polynomial Functions (15.1)
14. Graphs and Zeros of Polynomial Functions (15.2)
15. Fitting Polynomial Functions to Data (15.3)

Sample Assessments:

1) EX: Let A={2, 3, 4, 5} and B={-3, -2, -1, 0, 1}. Which of the following sets of ordered pairs represent functions from set A to set B?

a. (-2, 2), (3, 0), (4, 1)
b. (3, -2), (5, 0), (2, -3)

2) EX: Let $f(x) = 10 - 3x^2$ and find the following:
a. f(2)
b. f(-4)
c. f(x - 1)

3) EX: Find the domain and range of $g(x) = \sqrt{x-16}$.

4) Explain to your friend how you would classify the following equations. What would each graph look like?

a. $y = 4x + 5$

b. $x^2 - y^2 = 5$</td></tr>
<tr><td>• (A-APR-1) Perform arithmetic operations on polynomials</td></tr>
<tr><td>• (A-APR-2,3) Understand the relationship between zeros and factors of polynomials</td></tr>
<tr><td>• (A-APR-4,5) Use polynomial identities to solve problems</td></tr>
<tr><td>• (F-BF-3) Identify horizontal and vertical translation of functions</td></tr>
<tr><td>▪ (F-BF-3) Reflect the graph of a function in the x- or y-axis</td></tr>
<tr><td>▪ (F-BF-3) Learn how to stretch graphs</td></tr>
<tr><td>▪ (F-BF-3) Perform arithmetic combinations of functions</td></tr>
<tr><td>▪ (F-BF-1c) Compositions of Functions</td></tr>
<tr><td>▪ (F-BF-4) Determining if a function has an inverse</td></tr>
<tr><td>▪ (F-BF-4) Find the inverse of functions</td></tr>
<tr><td>▪ (F-BF-5) Understand inverse relationship between exponents and logarithms</td></tr>
<tr><td>▪ (F-IF-7A) Graph quadratic functions</td></tr>
<tr><td>▪ (F-IF-8A) Identify and write the standard form of a quadratic function</td></tr>
<tr><td>▪ (F-IF-7C) Graph of higher degree polynomials</td></tr>
<tr><td>▪ (F-IF-7C) Use the leading coefficient test to determine the end behavior of polynomial functions</td></tr>
<tr><td>▪ (F-IF-7C) Find zeros of polynomial functions</td></tr>
<tr><td>▪ (F-IF-7C) Write the polynomial function given the zeros of a function</td></tr>
<tr><td>▪ (F-IF-7) Perform long division of polynomials</td></tr>
<tr><td>▪ (F-IF-7) Apply the Remainder Theorem to evaluate a function</td></tr>
<tr><td>▪ (F-IF-7) Apply the rational zeros of a polynomial</td></tr>
</table>

- (F-IF-7) Apply the Fundamental Theorem of Algebra
- (F-IF-7) Determine the bounds for real zeros of polynomial functions
- (F-LE-2, 3, 4, 5) Evaluating exponential expressions
- (F-LE- 3 & F-IF 7e) Graphing exponential functions
- (F-LE-1c, 5) Compound interest problems
- (F-LE-4) Evaluating logarithms
- (F-LE) Solving exponential equations
- (F-LE- 4) Solving logarithmic equations
- (F-LE- 3 & F-IF 7e) Graphs of logarithmic functions
- (F-LE- 4) Rewriting logarithmic expressions
- (F-LE- 4) Condensing logarithmic expressions
- (F-LE-1c, 5) Problems that deal with logarithms and exponents
- (F-IF-7) Perform synthetic division of polynomials

c. $y^2 = 4x + 5$

d. $x^2 + y^2 = 5$

e. $y = |4x + 5|$

f. $y = \sqrt{x} + 4$

g. $y = x^3 + 5$

h. $y = (x + 2)^2$

7) Sketch the graph of:

$$f(x) = \begin{cases} -\frac{1}{2}x - 6, & x \le -4 \\ x + 5, & x > -4 \end{cases}$$

$$f(x) = \begin{cases} -\frac{1}{2}x - 6, & x \le -4 \\ x + 5, & x > -4 \end{cases}$$

8) Determine whether $f(x) = 5 - 3x$ is even, odd, or neither.

9) For $f(x) = x^2 - 2x + 9$, find $\frac{f(4+h) - f(4)}{h}, h \ne 0$.

10) Use the graph to describe the increasing, decreasing behavior of the function: $f(x) = x^3 + 3x^2 - 10$.

11) Graph the following function using shifts:

$f(x) = x^3$

$g(x) = (x - 2)^3$

$h(x) = -x^3$

12) *If* $f(x) = x^2 + 2$, *find* $\frac{f(h+1) - f(1)}{h}$, $x \ne 0$.

13) Perform the given operations and simplify.

a) $\left(3x^2+2x-1\right)+\left(4x-x^2+10\right)$

b) $\left(x^2-5\right)-\left(x^2-5x+4\right)$

c) $4\left(x^2+\frac{1}{2}\right)+5$

d) $\frac{10x^2+30x}{10x}$

13) Given: $f(x)=x^2,\ g(x)=1-x$

Find:

a) (f + g) (x)

b) (f – g)(x)

c) (f g) (x)

d. (f / g) (x)

e. What is the domain of f/g?

14) Given $f(x)=x^2,\ \ g(x)=\sqrt{x-6}, find\ (f\circ g)(-1).$ Determine if $(f\circ g)=(g\circ f).$

15) Given the graph below graph each of the following:

a) y = f(x) + 2
b) y = -f(x)
c) y= f(x-2)
d) y = f(x+3)
e) y = 2f(x)
f) y = f(-x)
g) y = f(½ x)

16. Use the function $f(x)=5^x$ to identify the transformation:

a) $g(x)=f(x)+1$

b) $h(x)=f(x)-3$

	c) $s(x) = f(x+2)$ d) $j(x) = f(x-5)$ e) $c(x) = -2f(x)$ f) $t(x) = f(-x) + 4$ 17. Solve the equation for x. a. $\log_2 x = 4$ b. $\log_x 64 = 2$ c. $\log_a 1 = x$ d. $\log_a a = x$ e. $\log_a a^x = x$ f. $\log_a \left(\frac{1}{a^x}\right) = x$ 18. Solve: a) $16 = 2^{7x-5}$ b) $3^x + 4 = 9$ c) $25^{x-2} = 5^{3x}$ d) $e^x = 60$ e) $\log(x+1) + 3 = 5$ f) 3 + 4ln x = 31 g) $6^{5x} = 3000$ 19. The population P of a city is given by $P = 240{,}360e^{0.015t}$ where t represents 1990. According to this model, when will the population reach 275,000? Sample question: Students will analyze the path of a ball of a Major league player's hit. Students will be able to approximate how high the ball will go before it starts to fall back to the ground. In addition, students will be able to determine the length of time it will take for the ball to return to the ground. Students will be able to analyze the ball's flight graph and, based on changes to the original function, examine the impacts of those changes on the flight of the

	ball. Instructional Strategies: Interdisciplinary Connections Students will study functions that represent the bacteria in foods. Students will be able to find the composition of the temperature in the refrigerator and foods, the number of bacteria in the food after X amount of hours and the time when bacteria count reaches a certain amount. Exponential and logarithmic functions have many real-life applications. Some include radioactive decay, sound intensity and Newton's Law of Cooling. Technology Integration Students will use a graphing calculator to find maximums and minimums of functions. In addition, students will use a graphing calculator to determine when a function has a vertical stretch or shrink. Media Literacy Integration Global Perspectives Students can approximate maximum and minimum temperatures of a particular function representing a particular city. Students will select a well-known, worldwide city and gather the monthly average temperatures. They will develop functions representing those temperatures and use the functions to compare temperatures around the world at given moments during the year. Students will research and examine the impact of earthquakes throughout the world. Each student will review the data for a given earthquake and examine how the logarithmic function can be used to model the strength (Richter scale) of that earthquake. As an extension, students can discuss the impact of those earthquakes on the communities where the activity occurred.

Unit 2 – Introduction to Trigonometry

<table>
<tr><td colspan="2">Standards: Trigonometric Functions (F-TF),
Similarity, Right Triangles, and Trigonometry (G-SRT)</td></tr>
<tr><td colspan="2">Big Ideas: Course Objectives / Content Statement(s)
Students will learn about how angles are measured in the coordinate plane in standard position.
Students will learn and apply the definitions of the six trigonometric functions.
Students will learn about inverse trigonometric functions and their domain restrictions.</td></tr>
<tr><td>Essential Questions
What provocative questions will foster inquiry, understanding, and transfer of learning?</td><td>Enduring Understandings
What will students understand about the big ideas?</td></tr>
<tr><td>What is an angle in standard position? Why might the system have been set up in this way?

How can we quickly evaluate trigonometric functions?

Why must inverse trig functions be restricted?</td><td>Students will understand that…

Angles in standard position are measured from the positive x-axis. Positive angles rotate counter-clockwise and negative angles rotate clockwise. Such a system exploits the sign of coordinates in the Cartesian plane to define the trigonometric functions.

Assuming they've memorized the 30-60-90 and 45-45-90 triangles, students can use reference angles and the sign of the coordinates in the angle's quadrant to quickly evaluate a trig function.

The trig functions are not one-to-one, and therefore are not invertible. Students must restrict the trig functions before inverting them to ensure that the inverted relations are in fact functions.</td></tr>
<tr><td>Areas of Focus: Proficiencies
(Cumulative Progress Indicators)</td><td>Examples, Outcomes, Assessments</td></tr>
<tr><td>Students will:
(F-TF-2) explain how the unit circle enables the extension of trig functions to all real numbers
(F-TF-3) use special triangles to determine geometrically the values of trig functions
(F-TF-4) use the unit circle to explain the symmetry and periodicity of trigonometric functions
(F-TF-6) understand that restricting</td><td>Instructional Focus:
Periodic Functions (2.1)
Measurement of Rotation (2.2)
Sine and Cosine Functions (2.3)
Values of the Six Trigonometric Functions (2.4)
Inverse Trigonometric Functions and Triangle Problems (2.5)</td></tr>
</table>

a trigonometric function to a domain on which it is always increasing or decreasing allows its inverse to be constructed

- (G-SRT-6) understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios
- (G-SRT-7) explain and use the relationship between the sine and cosine of complementary angles
- (G-SRT-8) use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems

Sample Assessments:

1) The initial current charge in an electrical circuit is zero. The current when 100 volts is applied to the circuit is given by

$$I = 5e^{-2t}\sin t$$

where the resistance, inductance, and capacitance are 80 ohms, 20 henrys, and 0.01 farad, respectively. Approximate the current (in amperes) t=0.7 second after the voltage is applied.

2) Evaluate:

a) $\csc\left(\frac{\pi}{6}\right)$ b) $\sin\left(\frac{3\pi}{2}\right)$ c) $\tan\left(\frac{5\pi}{4}\right)$

3) Use $\sec\theta = 5,\ \tan\theta = 2\sqrt{6}$ to find the indicated trig functions.

$a.\cos\theta$ $b.\cot\theta$ $c)\cot(90^\circ - \theta)$ $d)\sin\theta$

4) Find each value of θ in degrees $(0^\circ < \theta < 90^\circ)$ and radians $\left(0 < \theta < \frac{\pi}{2}\right)$ without using a calculator.

A. $\cos\theta = \frac{\sqrt{2}}{2}$ B. $\tan\theta = 1$ C. $\sec\theta = 2$

5) TRUE OF FALSE:

$A.\ \sin 170^\circ = -\sin 350^\circ$

$B.\ \cos\frac{3\pi}{7} = \cos\left(-\frac{10\pi}{7}\right)$

$C.\ -\tan 220^\circ = \tan 140^\circ$

6) A ship leaves port at noon and has a bearing of S 27 W. If the ship sails at 20 knots (nautical miles per hour), how many nautical miles south and how many nautical miles west will the

	ship have traveled by 6:00 P.M.? 7) An observer in a lighthouse 350 feet above sea level observes two ships directly offshore. The angles of depression to the ships are 4 and 6.5. How far apart are the ships? Instructional Strategies: Interdisciplinary Connections Students will discuss how trigonometry is used in the development of astronomy, navigation, and surveying. Students will be given an irregularly shaped plot of land and apply the laws of trigonometry to evaluate the dimensions and area of that land. Technology Integration Through the use of the graphing calculator, students will examine a given function and locate all zeros, real and non-real, of function. Student will describe the location of those zeros in relation to the x- and y-axes. The students will then apply various transformations to the function, view the corresponding graphs simultaneously, and discuss the impact it has on the zeros of those functions. Media Literacy Integration

Unit 3 – Trigonometric Graphs and Identities

<table>
<tr><th colspan="2">Standard: Trigonometric Functions (F-TF),
Interpreting Functions (F-IF)</th></tr>
<tr><td colspan="2">Big Ideas: Course Objectives / Content Statement(s)
Students will create the graphs of the six trigonometric and inverse trigonometric functions.
Students will apply trigonometry to the system of radian angle measurement.
Students will understand and apply basic trigonometric identities, including Pythagorean and reciprocal properties.</td></tr>
<tr><th>Essential Questions
What provocative questions will foster inquiry, understanding, and transfer of learning?</th><th>Enduring Understandings
What will students understand about the big ideas?</th></tr>
<tr><td>How can you identify the amplitude and period of a sinusoidal function from an equation or a graph?</td><td>Students will understand that…

The parameters that determine amplitude and period are the same parameters that affect dilations, as described in the first unit.</td></tr>
<tr><td>What is the benefit of using radians to measure angles, as opposed to degrees?</td><td>Radians are a unit-less measurement, and as such do not create a conflict when trigonometric functions are combined with algebraic or other transcendental functions. Students will also appreciate the logical, geometric system that radian angles establish.</td></tr>
<tr><td>How can trigonometric identities help in simplifying expressions or solving equations?</td><td>Trigonometric identities allow us to exchange trig functions or to change the argument on a trig function, both of which are helpful in creating consistency in an equation or expression.</td></tr>
<tr><th>Areas of Focus: Proficiencies
(Cumulative Progress Indicators)</th><th>Examples, Outcomes, Assessments</th></tr>
<tr><td>Students will:</td><td rowspan="3">Instructional Focus:
Amplitude, period, and Cycles of Sinusoids (3.1)
General Sinusoidal Graphs (3.2)
Graphs of Tangent, Cotangent, Secant, and Cosecant Functions (3.3)</td></tr>
<tr><td>(F-IF-7) graph trigonometric functions, showing period, midline, and amplitude</td></tr>
<tr><td>(F-IF-4) interpret key features of graphs and tables in terms of the</td></tr>
</table>

quantities, and sketch graphs showing key features given a verbal description

- (F-TF-1) understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle
- (F-TF-5) choose trigonometric functions to model periodic phenomena with specified period, amplitude, and midline
- (F-TF-8) prove the trigonometric Pythagorean identities, and use them to evaluate trig functions

4. Radian Measure of Angles (3.4)
5. Circular Functions (3.5)
6. Inverse Circular Functions (3.6)
7. Sinusoidal Functions as Mathematical Models (3.7)
8. Pythagorean, Reciprocal, and Quotient Identities (4.1, 4.2)
9. Identities and Algebraic Transformation of Expressions (4.3)
10. Using Inverse Trig to Solve Equations (4.4)
11. Inverse Trigonometric Relation Graphs (4.6)

Sample Assessments:

1) Use the trig identities to transform one side into the other.

$\csc\theta\tan\theta = \sec\theta$

$(\csc\theta + \cot\theta)(\csc\theta - \cot\theta) = 1$

2) Analyze and sketch the graph:

$$y = 3\cos\left(\frac{\pi x}{2} + \frac{\pi}{2}\right) - 3$$

3) Sketch at least **ONE CYCLE** of the graph of the function: $f(x) = -2\csc 4x + 2$. State two points and the asymptotes in the same cycle.

4) Sketch **TWO CYCLES** of the graph of the function: $f(x) = \frac{1}{3}\cot\left(x - \frac{\pi}{4}\right)$. State two points and the asymptotes in the same cycle.

5) Evaluate:

a) $\tan(\arctan 1)$ b) $\sec\left(\arcsin\frac{4}{5}\right)$

c) $\csc\left(\arctan\left(-\frac{5}{12}\right)\right)$

6) Write an algebraic expression that is equivalent to the expression: $\sec(\arctan 3x)$.

	Instructional Strategies: Interdisciplinary Connections Students will discuss that trig is used in applications involving vibrations, sound waves, light rays, planetary orbits, vibrating strings, pendulums and orbits of atomic particles. Technology Integration Students will use the following website to discover the graph of the sine curve: http://www.intmath.com/trigonometric-graphs/1-graphs-sine-cosine-amplitude.php. Media Literacy Integration Global Perspectives

Unit 4 – Applications of Trigonometry

<table>
<tr><th colspan="2">Standards: Trigonometric Functions (F-TF),
Similarity, Right Triangles, and Trigonometry (G-SRT)</th></tr>
<tr><td colspan="2">Big Ideas: Course Objectives / Content Statement(s)
• Students will examine the graphical, algebraic, and numeric result of combining sinusoids.
• Students will learn and apply the sum, difference, double, and half angle identities.
• Students will solve oblique triangles and find their area.</td></tr>
<tr><th>Essential Questions
What provocative questions will foster inquiry, understanding, and transfer of learning?</th><th>Enduring Understandings
What will students understand about the big ideas?</th></tr>
<tr><td>
How are the periodicity, amplitude, and other defining features of sinusoids affected by adding or multiplying sinusoids?

How can trigonometric functions with complex arguments be simplified?</td><td>Students will understand that…

Adding or multiplying trigonometric functions with different periods create “variable sinusoids”, where the critical points and amplitudes of the waves vary at regular increments.

Sum and difference identities can be used to create double and half angle identities, allowing us to re-write trigonometric expressions with several terms or coefficients.</td></tr>
<tr><th>Areas of Focus: Proficiencies
(Cumulative Progress Indicators)</th><th>Examples, Outcomes, Assessments</th></tr>
<tr><td>▪ (F-TF-9) prove the addition and subtractions formulas for sine, cosine, and tangent and use them to solve problems

▪ (G-SRT-9) derive the formulas for area of oblique triangles

▪ (G-SRT-10) prove the Law of Sines and Cosines and use them to solve problems

▪ (G-SRT-11) understand and apply the Law of Sines and Cosines to find unknown measurements in right and oblique triangles</td><td>Instructional Focus:
1. Composite Argument and Linear Combination Properties (5.2, 5.3)
2. Composition of Ordinates and Harmonic Analysis (5.4)
3. Sum and Product Properties (5.5)
4. Double and Half Angle Properties (5.6)
5. Law of Cosines (6.2)
6. Area of a Triangle (6.3)
7. Law of Sines (6.4)
8. The Ambiguous Case (6.5)
9. Real-World Triangle Problems (6.7)</td></tr>
</table>

	Sample Assessments: Instructional Strategies: Interdisciplinary Connections Technology Integration Media Literacy Integration Global Perspective

Unit 5 - Vectors

<table>
<tr><td colspan="2">Standard: Vector and Matrix Quantities (N-VM)</td></tr>
<tr><td colspan="2">Big Ideas: Course Objectives / Content Statement(s)
Students will learn how vectors can model physical phenomena.
Students will learn how to perform operations and computations, and interpret the results of these computations, on vectors in 2 and 3 dimensions.</td></tr>
<tr><td>Essential Questions
What provocative questions will foster inquiry, understanding, and transfer of learning?</td><td>Enduring Understandings
What will students understand about the big ideas?</td></tr>
<tr><td>What natural phenomenon can be modeled using vectors?

How are vectors and vector operations used to represent quantities in real life?

How can vectors be used to describe planes and other 3-dimensional objects?</td><td>Students will understand that…

The students will learn that a vector is used to represent quantities that involve both magnitude and direction. The students will understand how vectors are used to model real life situations, especially in physics problems involving finding force on an inclined ramp or a wind-adjusted bearing in airplane navigation.

Students will be able to apply vector operations of addition, multiplication, scalar multiplication, and dot product. They will be able to identify which operation to use in real life situations.

The cross-product of two vectors creates a normal vector in either direction. Planes and other geometric objects in space can be generated using this simple fact.</td></tr>
<tr><td>Areas of Focus: Proficiencies
(Cumulative Progress Indicators)</td><td>Examples, Outcomes, Assessments</td></tr>
<tr><td>Students will:
(N-VM-1) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).</td><td>Instructional Focus:
Vector Addition (6-6)
Two-Dimensional Vectors (10.1, 10.2)
Vectors in Space (10.3)
Scalar Products and Projection of Vectors (10.4)
Planes in Space (10.5)
Vector Product of Two Vectors (10.6)
Direction Angles and Direction Cosines (10.7)
Vector Equations and Lines in Space (10.8)</td></tr>
</table>

- (N-VM-2) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
- (N-VM-3) Solve problems involving velocity and other quantities that can be represented by vectors.
- (N-VM-4) Add and subtract vectors.
- (N-VM-5) Multiply a vector by a scalar.

Sample Assessments:

1) Find the component form and magnitude of the vector **v** that has the initial point (-2, 3) and terminal point (-7, 9).

2) Let **u**=<1, 2> and **v**=<3, 1>. Find each of the following vectors: a) **u**+**v** b) **u**-**v** c) 2**u**-3**v**

3) Find a unit vector **u** in the direction of **v**=<7, -3> and verify that the result has magnitude 1.

4) Find the direction angle: **v**=-6**i**+6**j**

5) Find the dot product: <3, 4>•<2, -3>

6) Find the angle between **u**=<3, 0> and **v**=<1, 6>.

7) To slide an object across a floor, a person pulls a rope with a constant force of 25 pounds at a constant angle of 30 degrees above the horizontal. Find the work done if the object is dragged 40 feet.

8) The vector $\mathbf{u} = \langle 4000, 5700 \rangle$ gives the number of units of two models of laptops produced by a company. The vector $\mathbf{v} = \langle 1550, 1300 \rangle$ gives the prices (in dollars) of the two models of laptops, respectively. Identify the vector operation used to increase revenue by 5.5%.

9) A force of 50 pounds is exerted along a rope attached to a crate at an angle of $60°$ above the horizontal. The crate is moved 30 feet. How much work has been accomplished?

Instructional Strategies:

Interdisciplinary Connections

Students will discuss how vectors are used in physics through many different applications involving velocity,

	force, tension, and work problems. The students will also discuss the use of vectors in revenue problems and navigation problems. Technology Integration The students will use the following applet to visualize a resultant vector. http://phet.colorado.edu/sims/vector-addition/vector-addition_en.html Global Perspectives The students will research a specific real world application of vectors to share with the class. One example includes selecting a well-known domestic or international airport and examining the effects of wind on the flight of airplanes trafficking that airport. Factors should include wind direction and speed and the impact on the flight path of the planes arriving to and departing from the airport.

Unit 6 – Conic Sections

Standard: Expressing Geometric Properties with Equations (G-GPE)	
Big Ideas: *Course Objectives / Content Statement(s)* • Students will learn and write the standard form equations for circles, parabolas, ellipses, and hyperbolas. • Students will identify the important and defining geometric characteristics of each conic section. • Students will be able to represent conic sections using Cartesian equations and parametric equations using trigonometry.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
	Students will understand that…
What are the geometric definitions of conic sections based on?	The four conic sections can be generated by slicing a cone with a plane. In addition, they can all be defined as a locus of points with different conditions based on distance from fixed points and/or lines.
What information can we learn about a conic section from its standard form?	The center, foci, directrices, vertices, eccentricity, and other critical features of conic sections can be determined by a few simple parameters in the conic's equation.
How are conic sections related to trigonometry?	The equations for conic sections can be written in parametric form using the trigonometric functions.
Areas of Focus: Proficiencies **(Cumulative Progress Indicators)**	**Examples, Outcomes, Assessments**
Students will:	Instructional Focus:
▪ (G-GPE-1) derive the equation of a circle given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation ▪ (G-GPE-2) derive the equation of a parabola given a focus and directrix ▪ (G-GPE-3) derive the equations	1. Introduction to Conic Sections (12.1) 2. Parametric and Cartesian Equations of the Conic Sections (12.2) 3. Quadratic Surfaces and Inscribed Figures (12.3) 4. Analytic Geometry of the Conic Sections (12.4) 5. Parametric and Cartesian Equations for Rotated Conics (12.5) 6. Applications of Conic Sections (12.6)

of ellipses and hyperbolas given the foci, and using the fact that the sum or differences from the foci is constant	Sample Assessments: Instructional Strategies: Interdisciplinary Connections Technology Integration Media Literacy Integration Global Perspectives

Unit 7: Polar Coordinates and Complex Numbers

<table>
<tr><th colspan="2">Standard: The Complex Number System (N-CN)</th></tr>
<tr><td colspan="2">Big Ideas: Course Objectives / Content Statement(s)

• Students will identify and write equations in polar form for curves and conic sections.
• Students will learn to express complex numbers in polar form.</td></tr>
<tr><th>Essential Questions

What provocative questions will foster inquiry, understanding, and transfer of learning?</th><th>Enduring Understandings

What will students understand about the big ideas?</th></tr>
<tr><td>How can points in space be identified other than indicating horizontal and vertical motion from the origin?

How can the polar and rectangular systems be exchanged for a given curve?

How are complex numbers plotted? Why is using a polar format logical?</td><td>Students will understand that…

Points in space can be uniquely identified by the distance from the origin and the angle from the positive x-axis.

A simple set of identities, based on the definition of the trigonometric functions, will allow students to translate from rectangular to polar equations, and vise versa.

Complex numbers are plotted with the real value on the x-axis and the imaginary value on the y-axis. Complex numbers can be written in “trigonometric form” by simply substituting in the polar coordinates of the point.</td></tr>
<tr><th>Areas of Focus: Proficiencies

(Cumulative Progress Indicators)</th><th>Examples, Outcomes, Assessments</th></tr>
<tr><td>Students will:

▪ (N-CN-1) Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.

▪ (N-CN-2) Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</td><td>Instructional Focus:

1. Polar Coordinates (13.1)
2. Polar Equations of Conics and Other Curves (13.2)
3. Intersections of Polar Curves (13.3)
4. Complex Numbers in Polar Form (13.4)
5. Parametric Equations for Moving Objects (13.5)</td></tr>
</table>

▪ (N-CN-3)Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. ▪ (N-CN-4)Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. ▪ (N-CN-5)Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, (–1 + √3 i)3 = 8 because (–1 + √3 i) has modulus 2 and argument 120°.	Sample Assessments: 1) Write the complex number z=6-6i in trigonometric form. 2) Let $z_1 = 3\left(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}\right)$ and $z_2 = 4\left(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6}\right)$. Find the product $z_1 z_2$ and the quotient $\frac{z_1}{z_2}$ of the complex numbers. 3) Ex: Use DeMoivre's Theorem to find $(1+i)^6$. Instructional Strategies: Technology Integration Media Literacy Integration Global Perspectives

Unit 8: Discrete Math and an Introduction to Calculus

Standard: Arithmetic with Polynomials (A-APR)	
Big Ideas: *Course Objectives / Content Statement(s)* • Students will use explicit and recursive equations to analyze finite and infinite sequences and series. • Students will explore the potential convergence of infinite series. • Students will be introduced to the concepts of limits and instantaneous rate of change.	
Essential Questions *What provocative questions will foster inquiry, understanding, and transfer of learning?*	**Enduring Understandings** *What will students understand about the big ideas?*
How do you find the *n*th term or partial sum of an arithmetic sequence?	Students will understand that… The nth term of a sequence may be able to be represented explicitly or recursively. Partial sums of arithmetic sequences can be found by adding the outermost terms and counting the number of such pairs in the sequence.
How do you find terms and sums of geometric sequences?	The students will understand how the formulas for finding *n*th terms and sums of geometric sequences and series are derived, and be able to use them to solve problems.
What patterns can you observe in the expansion of a binomial $(x+y)^n$?	The students will understand the connection between a binomial expansion and Pascal's Triangle. They will learn the formula for binomial coefficients.
What do a convergent infinite geometric series and the derivative of a function at a point have in common?	Both convergent infinite geometric series and the derivative of a function at a point requires the computation of a limit.
Areas of Focus: Proficiencies **(Cumulative Progress Indicators)**	**Examples, Outcomes, Assessments**
Students will:	Instructional Focus: 1. Sequences and Series (14.1) 2. Arithmetic, Geometric Sequences (14.2) 3. Series and Partial Sums (14.3)
▪ (A-SSE-4) Derive the formula for the sum of a finite geometric series (when the common ratio is	

not 1), and use the formula to solve problems. ▪ (A-APR-5) Know and apply the Binomial Theorem for the expansion of $(x+y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. ▪ (F-BF-2) Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.	4. Discontinuities, Limits, and Partial Fraction Decomposition of Rational Functions (15.4) 5. The Derivative (15.5) Sample Assessments: 1) Find the *n*th term of the arithmetic sequence whose common difference is 5 and whose first term is -1. 2) Ex: Find a formula for the *n*th term of the geometric sequence 6, -2, $\frac{2}{3}$,… What is the tenth term? 3) Find the coefficient of the term a^6b^5 in the expansion of $(2a-5b)^{11}$. 4) Ex: Expand and simplify $(3-2i)^6$. 5) Ex: Use a table to estimate $\lim_{x\to 0}\frac{x}{\sqrt{x+1}-1}$. 6) Ex: Find each limit: a) $\lim_{x\to 2}x^3$ b) $\lim_{x\to 4}8x$ c) $\lim_{x\to 16}\sqrt[4]{x}$ 7) Ex: Find the limit: $\lim_{x\to 2}\frac{x^2+2x-8}{x-2}$ 8) Ex: Find the limit: $\lim_{x\to 0}\frac{\sqrt{x+9}-3}{x}$ 9) Ex: Find the slope of the graph of $f(x)=x^3$ at the point (2, 8). 10) Ex: Find the derivative of $f(x)=4x^2-5x$.

	Instructional Strategies: Technology Integration The students will use list editor and sum functions on the graphing calculators to verify the sum formulas that they derive. Media Literacy Integration The students will choose two different banks, then research information about the Certificate of Deposit accounts they offer. They will highlight important information in the account Terms and Conditions, then calculate the account balance if $5000 is deposited and left in the account for 2 years. Global Perspectives Math History- The students will learn about the mathematician Carl Friedrich Gauss. He famously "invented" the formula for finding the sum of an arithmetic sequence when he was 10 years old.