# Readington Township Public Schools

# Pre-Algebra
# (8th grade and Advanced 7th)

**Authored by:** Jennifer M. Sabo

**Reviewed by:** Sarah Pauch
Supervisor of Math, Science, and Technology

**Approval Date: September 25, 2018**



**Readington Township Public Schools**
**Whitehouse Station, NJ 08889**
**www.readington.k12.nj.us**

## I. OVERVIEW

This Pre-Algebra course is the mathematics class for eighth graders working on grade level. It is also the course for the Advanced 7th grade students, placed into the course by district criteria. This course is directly aligned with the New Jersey Student Learning Standards ("NJSLS") for grade 8. Through their work in this course, students will understand and apply their knowledge in real world applications. Focus will be on the content as specified in the NJSLS, as well as the NJSLS Practice Standards. The Practice Standards focus on the development of competencies used by mathematicians in all grades and throughout life.

Students in this course will understand slope of a linear relationship and relate linear equations to lines in the coordinate plane. They will write and solve linear equations, including pairs of linear equations. Students will understand that functions are rules that assign a unique output number to each input number. They will then use linear functions to model relationships. Students will analyze statistical relationships and use lines of best-fit to determine future outcomes in real-life situations. Students will work with positive and negative exponents, square root and cube root symbols, and scientific notation. They will understand congruence and similarity of geometric figures.

## II. STUDENT OUTCOMES (Linked to New Jersey Student Learning Standards for Mathematics)

### THE NUMBER SYSTEM (8.NS)

**Know that there are numbers that are not rational, and approximate them by rational numbers.**

1. Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion, which repeats eventually into a rational number.
2. Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., $\pi^2$).

### EXPRESSIONS AN EQUATIONS (8.EE)

**Work with radicals and integer exponents.**

1. Know and apply the properties of integer exponents to generate equivalent numerical expressions.
2. Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where *p* is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
3. Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.
4. Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology.

**Understand the connections between proportional relationships, lines, and linear equations.**

5. Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways.
6. Use similar triangles to explain why the slope *m* is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at *b*.

**Analyze and solve linear equations and pairs of simultaneous linear equations.**

7. Solve linear equations in one variable.
   a. Give examples of linear equations in one variable with one solution, infinitely many solutions, or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where *a* and *b* are different numbers).

b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8. Analyze and solve pairs of simultaneous linear equations.
   a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
   b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.
   c. Solve real-world and mathematical problems leading to two linear equations in two variables.

**FUNCTIONS (8.F)**

**Define, evaluate, and compare functions.**

1. Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
2. Compare properties (e.g. rate of change, intercepts, domain and range) of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
3. Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

**Use functions to model relationships between quantities.**

4. Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two $(x, y)$ values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
5. Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

**GEOMETRY (8.G)**

**Understand congruence and similarity using physical models, transparencies, or geometry software.**

1. Verify experimentally the properties of rotations, reflections, and translations:
   a. Lines are transformed to lines, and line segments to line segments of the same length.
   b. Angles are transformed to angles of the same measure.
   c. Parallel lines are transformed to parallel lines.
2. Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.
3. Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.
4. Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.
5. Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

**Understand and apply the Pythagorean Theorem.**

6. Explain a proof of the Pythagorean Theorem and its converse.
7. Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
8. Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**

9. Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

**STATISTICS AND PROBABILITY (8.SP)**

**Investigate patterns of association in bivariate data.**

1. Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
2. Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit (e.g. line of best fit) by judging the closeness of the data points to the line.
3. Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.
4. Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between two variables.

## III. ESSENTIAL QUESTIONS AND CONTENT

### The Number System

**Numbers that are not rational are irrational; every number has a decimal expansion.**

- How can decimal expansions of rational and irrational numbers be compared?
- How can a rational number be represented with its decimal expansion, showing that it eventually repeats?
- How can a decimal expansion (which eventually repeats) be converted into a rational number?

**Rational approximation of irrational numbers.**

- How can irrational numbers be compared using their rational approximations?
- How are rational approximations located on a number line?
- How are the values of expressions containing irrational numbers estimated?

### EXPRESSIONS AND EQUATIONS (8.EE)

**Exponents as simplified representation of repeated multiplication.**

- What are the properties of exponents?
- How are the properties of exponents applied to numerical expressions?
- How can equivalent numerical expressions be generated using positive and negative integer exponents?

**Square roots, cube roots, perfect squares and perfect cubes; Inverse relationship between powers and square roots.**

- What are the values of square roots of small perfect squares?
- What are the solutions to equations of the form $x^2 = p$, where $p$ is a positive rational number?
- How can the square root symbol be used to represent solutions to equations of the form $x^2 = p$?
- What are the values of cube roots of small perfect cubes?
- What are the solutions to equations of the form $x^3 = p$, where $p$ is a positive rational number?
- How can the cube root symbol be used to represent solutions to equations of the form $x^3 = p$?
- Why is the $\sqrt{2}$ an irrational number?

**Very large and very small quantities can be approximated with numbers expressed in the form of a single digit times an integer power of 10.**

- How can very large or very small quantities be estimated with numbers expressed in the form of a single digit times an integer power of 10?
- How can numbers written in the form of a single digit times an integer power of 10 be compared and how can it be expressed how many times as much one is than the other?

**Operations with scientific notation**

- How can numbers expressed in scientific notation be multiplied and divided?
- How can numbers expressed in scientific notation be added and subtracted?
- How can scientific notation and units of appropriate size be used to represent measurements of very large or very small quantities?
- How is scientific notation generated by technology interpreted?

**Quantitative relationships can be represented in different ways.**

- How are proportional relationships graphed?
- How is the unit rate interpreted as the slope of a graph?
- How can two different proportional relationships that are represented in different ways (table of values, equation, graph, verbal description) be compared?

**Slope and similar triangles; Slope-intercept form of an equation.**

- How can it be proved using similar triangles that the slope $m$, is the same between any two distinct points on a non-vertical line?
- How can the equation $y = mx$ be derived from two points for a line that passes through the origin?
- How can the equation $y = mx + b$ be derived from two points for a line that intercepts the vertical axis at $b$?

**Linear equations may have a single solution, no solution or infinite solutions.**

- What are some examples of linear equations in one variable with one solution ($x = a$), infinitely many solutions ($a = a$), or no solutions ($a = b$)?
- How can a given equation be transformed using the properties of equality into simpler forms?
- How can a given equation be transformed into an equivalent equation of the form $x = a$, $a = a$ or $a = b$ ($a$ *and b are different numbers*)?
- How is a linear equation solved that has fractional coefficients? (including equations that require the use of the distributive property and combining like terms)

**Simultaneous linear equations may have a single solution, no solution or infinite solutions; Solutions to a system of two linear equations in two variables corresponds to the intersection point of their graphs.**

- How is a system of linear equations in two variables solved algebraically?
- How is the solution of a linear system of two equations estimated by graphing?
- How are simple cases of linear systems of two equations solved by inspection?
- How are real-world and mathematical problems leading to two linear equations in two variables solved?

## FUNCTIONS (8.F)

**A function is a rule in which each input has exactly one output.**

- What is function language and how is it used?
- How can a function be described as a single output for each input?
- How can it be determined whether a non-numerical relationship is a function?
- How can a function be described as a set of ordered pairs?
- How can inputs and outputs be read from a graph?
- How can an ordered pairs be described as containing an input and the corresponding output?

**Functions (quantitative relationships) can be represented in different ways; Functions/linear functions have properties.**

- How can functions be analyzed when represented algebraically, as a table of values, and as a graph?
- How can functions be interpreted when represented by a verbal description?
- How are the properties of two functions compared when each are represented in a different way?

**A linear function is defined by the equation $y = mx + b$; the graph of a linear function is a straight line.**

- How can the analysis of tables of values, graphs, and equations be used to classify a function as linear or non-linear?
- How can it be determined if equations presented in forms other than $y = mx + b$ (for example$3y - 2x = 7$) define a linear function?
- What are examples of equations that are non-linear functions?
- How can it be shown that a function is not linear using a pair of points?

**Two (x, y) values can be used to construct a function.**

- How can the rate of change and initial value of a function from a description of a relationship be determined?

- How can the rate of change and initial value of a function from two (x, y) values by reading from a table of values be determined?
- How can the rate of change and initial value of a function from two (x, y) values by reading these from a graph be determined?
- How can a function be constructed to model a linear relationship?
- How can the rate of change and initial value of a linear function in context be interpreted?

**Graphs of functional relationships.**
- How can the graph of a functional relationship be analyzed?
- What are qualitative descriptions of graphs? (e.g. where increasing or decreasing, linear or non-linear)
- How can the graph of a function be sketched given a verbal description based on qualitative features?

## GEOMETRY (8.G)

**Two-dimensional objects under rigid motion transformations (rotation, reflection, and translation) remain unchanged.**
- Do performing rotations, reflections, and translations on lines result in a line?
- Do performing rotations, reflections, and translations on line segments result in line segments with un-altered lengths?
- Do performing rotations, reflections, and translations on angles result in angles with un-altered measures?
- Does performing rotations, reflections, and translations on parallel lines result in parallel lines?
- Does the measure of a two-dimensional object remain unchanged when under a rigid motion transformation (rotation, reflection, and translation)?

**Two-dimensional figures are congruent if the second can be obtained from the first by a sequence of rotations, reflections, and translations.**
- How can two figures be identified as congruent using a sequence of transformations?

**The effects of transformations on two-dimensional figures using coordinates.**
- What results after applying dilations with a scale factor great than, less than, and equal to 1 on the coordinates of a two-dimensional figure?
- What results after applying translations, rotations, and reflections to the coordinates of a two-dimensional figure?

**Two-dimensional figures are similar if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; congruent figures are also similar.**
- What transformation or sequence of transformations can be performed to describe the similarity between two given similar two-dimensional figures?

**Angle sum and exterior angles of triangles; Parallel lines cut by a transversal.**
- How can the interior angle sum of a triangle be justified?
- What are the facts about the exterior angles of a triangle?
- What are the facts about the angles created when parallel lines are cut by a transversal?
- How can the angle-angle criterion for similarity of triangles be established?

**Pythagorean Theorem; the converse of the Pythagorean Theorem**
- How can a proof of the Pythagorean Theorem be explained?
- How can a proof of the converse of the Pythagorean Theorem be explained?

**Apply the Pythagorean Theorem.**
- How can the side lengths of two-dimensional right triangles in real world and mathematical problems be determined using the Pythagorean Theorem?
- How can the side lengths of three-dimensional right triangles in real world and mathematical problems be determined using the Pythagorean Theorem?

**Find the distance between two coordinate points.**
- How can the distance between two points in a coordinate plane be determined using the Pythagorean Theorem?

**Volume of cones, cylinders, and spheres.**

- How can the volumes of cones, cylinders, and spheres be used to solve real-world problems?
- How can a single unknown dimension of cones, cylinders, and spheres be found using volume formulas in real-world problems?

### STATISTICS AND PROBABILITY (8.SP)

**Association in bivariate measurement data**

- How is a scatter plot constructed and interpreted?
- How are patterns of association between two quantities represented by a scatter plot analyzed?
- How are the terms clustering, outliers, positive or negative association, linear or nonlinear association used when describing patterns of association in a scatter plot?

**Approximately linear relationships between quantitative variables are modeled using straight lines.**

- How can a line of best fit be used on a scatter plot to suggest a linear association?
- How can the model's fit be assessed by judging the closeness of the data points to the line of best fit?

**Use the equation of a linear model with bivariate data.**

- How can the slope and intercept be interpreted when given the equation for a linear model (line of best fit)?
- How can problems in the context of measurement data be solved when given the equation for a linear model?

**Analyzing categorical data using two-way tables.**

- How is a two-way frequency table containing data on two categorical variables constructed and interpreted?
- How is a two-way relative frequency table containing data on two categorical variables constructed and interpreted?
- How can any association between two categorical variables using relative frequencies calculated for rows or columns be described?

## IV. STRATEGIES

The curriculum will be instructed through a variety of strategies, based in research on middle school learning and educational best practices. Students will be engaged in meaningful lessons and activities using guided and independent practice and cooperative learning. Students will participate in hands-on activities, use manipulatives or technology where appropriate, and participate actively in class discussions.

Teachers will encourage students to employ a number of problem solving strategies, relevant to the situations they are in. They will demonstrate evidence of understanding through modeling, verbal descriptions and oral presentations. Students may also use tools of technology where needed to better enhance their ability to complete and defend their mathematical reasoning.

## V. EVALUATION

Teacher observations
Homework assignments
Notebooks
Student projects
Unit tests and quizzes
Benchmark unit assessments
Performance-based assessments

## VI. REQUIRED RESOURCES

*Big Ideas Math 8* textbook (Ron Larson and Laurie Boswell; published by Big Ideas Learning)
*Record and Practice Journal*

The following resources can also be used as a reference or may be used by the course instructor.
Project Based Assignment Resources – Including:
Illustrative Mathematics (www.illustratviemathematics.org)

The MAP Project (www.map.mathshell.org/materials/index.php)
Gizmos

## VII. SCOPE AND SEQUENCE

### The Number System
- Understanding Rational and Irrational Numbers (10 days)

### Expressions and Equations
- Radical and Integer Exponents (25 days)
  - Properties of integer exponents
  - Square roots and cube roots
  - Scientific notation
  - Operations and Scientific notation

### Functions
- Understanding Functions (15 days)
  - Define and identify functions
  - Compare functions
  - Identify and interpret linear functions
- Functions to Model Relationships (10 days)
  - Analyze functions
  - Graphing functional relationships

### Expressions and Equations
- Proportional Relationships and Linear Equations (15 days)
  - Representing proportional relationships
  - Slope-intercept equation of a line
- Solving Linear Equations and Pairs of Simultaneous Linear Equations (25 days)
  - Solving linear equations with rational coefficients
  - Solutions of linear equations
  - Understanding systems of equations
  - Solving systems of equations algebraically*
  - Solve real-world problems using systems of equations*

**if time permits will be completed as an extension for Pre-Algebra 7th*

### Geometry
- Congruence and Similarity (20 days)
  - Properties of transformations
  - Transformations and congruence
  - Transformations and similarity
  - Angle relationships
  - Angle relationships in triangles
- Pythagorean Theorem (15 days)
  - Understanding the Pythagorean Theorem
  - Solving problems using the Pythagorean Theorem
  - Distance in the coordinate plane
- Volume of Cylinders, Cones, and Spheres (10 days)
  - Understand the volume of cylinders, cones, and spheres
  - Solve real-world problems with cylinders, cones, and spheres.

### Statistics and Probability
- Bivariate Data (15 days)
  - Scatter plots
  - Scatter plots and linear models

- Solving problems with linear models of scatter plots
- Categorical data in frequency tables