Some practice problems

(1) For $f(x)=x^2-4x+3,$ tell whether the function is increasing ($\uparrow$) or decreasing ($\downarrow$) at: (a) x = -2, (b) x = 3.

(2) Find all Critical values of the function: $f(x)=x^4-2x^2.$

(3) Find all relative extrema for the function using the first derivative test (1DT):
$f(x)=\frac{1}{3}x^3-9x+2.$

(4) Find all relative extrema for the function using the second derivative test (2DT):
$f(x)=x^3-3x^2-24x+2$.

(5) Tell whether the function $f(x)=x^4-12x^2$ is concave up or concave down at: (a) x = 1, (b) x = 2.

(6) Find all points of inflection of the function: $f(x)=x^3-6x^2-8x+2$.

(7) Find the extrema and points of inflection of the function, and graph - label each max, min, and POI.
$f(x)=\frac{1}{3}x^3-x^2-3x+2$.

Solutions:

(1) yeah, alright - the crucial concept here is that the function is increasing @ x = c if the derivative $f'(x) > 0,$ and decreasing when $f'(x) < 0$. So we need to find the derivative using the power rule:

$$
\begin{aligned}
&f(x) = x^2 - 4x + 3,\\
&f'(x) = 2x - 4,\\
&f'(-2) = 2(-2) - 4 = -8, \; so\, f'(-2) < 0, \;\; f \downarrow\\
&f'(3) = 2(3) - 4 = -8, \; so\, f'(3) > 0, \;\; f \uparrow
\end{aligned}
$$

(2) Concept: you find the CN's of a polynomial by setting the derivative (that's f'(x) for the sleepers) equal to zero and solving:

$$
\begin{aligned}
&f(x) = x^4 - 2x^2.\\
&f'(x) = 4x^3 - 4x = 0, \quad (\textit{factor out } 4x),\\
&\qquad\quad 4x(x^2 - 1) = 0, \;\; (\textit{set the factors} = 0),\\
&4x = 0, x^2 - 1 = 0,\\
&x = \{0, \pm 1\},
\end{aligned}
$$

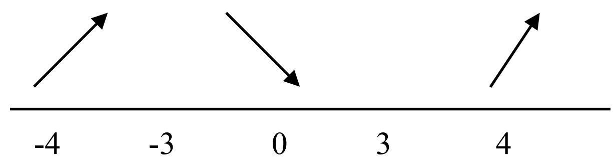
these are the CN's.

(3) Roughly speaking, locating extrema is a two-step process - find the CN's, and test them. Here, we have specified the use of the 1DT.

Here we go: set the derivative equal to 0:

$$
\begin{aligned}
&f(x) = \frac{1}{3}x^3 - 9x + 2.\\
&f'(x) = x^2 - 9 = 0,\\
&\qquad x^2 = 9,\\
&\qquad x = \pm\sqrt{9},\\
&\qquad x = \pm 3.
\end{aligned}
$$

These are the CN's - now we're going to test them - set up a number line with -3, and 3, and pick some numbers to the left and right of both of them: (circle the -3 and 3 below)



$f'(-4) = (-4)^2 - 9$	$f'(0) = (0)^2 - 9$	$f'(4) = (4)^2 - 9$
$= 7 > 0$,	$= -9 < 0$,	$= 7 > 0$
(Increasing)	(decreasing)	(Increasing)

Thus we have a maximum @ $(-3, f(-3)) = (-3, \frac{1}{3}(-3)^3 - 9(-3) + 2) = (-3, 20)$.

And we have a minimum @ $(3, f(3)) = (3, \frac{1}{3}(3)^3 - 9(3) + 2) = (3, -16)$.

(4) Here, we follow the same steps as the last time, but we test with the 2DT instead of the 1DT:

Find the critical numbers:

$$f(x) = x^3 - 3x^2 - 24x + 2$$
$$f'(x) = 3x^2 - 6x - 24 = 0, \qquad (\textit{factor out } 3)$$
$$3(x^2 - 2x - 8) = 0,$$
$$3(x-4)(x+2) = 0,$$
$$x - 4 = 0, x + 2 = 0,$$
$$x = 4,\ x = -2.$$

Now we use the second derivative test, which starts with finding the second derivative:

$f''(x) = 6x - 6$, (now plug the CN's into the second derivative:)

$f''(4) = 6(4) - 6 = 18 > 0$, (there is a min. @ (4,$f$(4)).)

$f''(-2) = 6(-2) - 6 = -18 < 0$, (there is a max. @ (-2,f(-2)).)

(5) Concept: the function is concave up where the second derivative is positive, and it is concave down where the second derivative is negative. Thus, to resolve the problem at and, we need to start by finding the second derivative.

$$f(x) = x^4 - 12x^2$$
$$f'(x) = 4x^3 - 24x,$$

$f''(x) = 12x^2 - 24$, (now plug in the values in (a) and (b):)

(a) $f''(1) = 12(1)^2 - 24 = -12 < 0$, ($f \cap$ @ x = 1),

(b) $f''(2) = 12(2)^2 - 24 = 24 > 0$, ($f \cup$ @ x = 2).

(6) Concept: candidate POI occur where f''(x)=0. Then we have to test the concavity to the left and right of the candidate (using the second derivative, as in the last question), to make sure it is, indeed, a POI.

$$f(x) = x^3 - 6x^2 - 8x + 2$$

So, find the second derivative and set it equal to 0:

$$f'(x) = 3x^2 - 12x - 8,$$
$$f''(x) = 6x - 12 = 0,$$
$$x = 2.$$

Set up a table for the candidate, selecting values to the left and right on the number line: (circle 2 in the table below)

$\cup$ $\cap$

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0 2 3

$$f''(0) = 6(0) - 12 = -12 < 0, \quad f''(3) = 6(3) - 12 = 6 > 0.$$

(concave down) (concave up),

The concavity differs on both sides of x = 2, so we do have a POI @
$(2, f(2)) = (2, 2^3 - 6(2)^2 - 8(2) + 2) = (2, -30).$

(7) Concepts: To find the VA's, we set the denominator equal to zero and solve the resulting equation. To find the HA's, we have to compare the degrees of the numerator and denominator, as per page (whatever?).

(a) $f(x) = \dfrac{2x}{x-3}$,

VA: x - 3 = 0,
x = 3

HA: the top and bottom both have degree 1(they're equal), so the HA is at the quotient of the lead coefficients y = 2/1, y = 2

(b) $l(x) = \dfrac{3x}{x^2 - 4}$

$$x^2 - 4 = 0,$$
$$x^2 = 4,$$

VA: $x = \pm 2$

HA: The denominator has degree 2, the numerator degree 1, thus the denominator has bigger degree, and the HA is the x-axis.

(c) $\Omega(x) = \dfrac{x^3}{x^2 - 3x + 2}$

$(x-2)(x-1)=0$

$x-2=0, x-1=0,$

VA: $x=2, x=1.$

HA: The numerator has bigger degree, so the function has no HA.

(8) $f(x)=\frac{1}{3}x^3-x^2-3x+2$, start by finding the CN's:

$$f'(x)=x^2-2x-3=0,$$
$$(x-3)(x+1)=0,$$
$$x-3=0, x+1=0,$$
$$x=3, \quad x=-1.$$

Now we need to test for extrema - I'm going to use the 2DT (you could also use the 1DT):

$f''(x)=2x-2, \quad (\textit{plug in the CN's}:)$

$f''(3)=2(3)-2=4>0,$ (there's a min. @ (3,*f*(3))).

$f''(-1)=2(-1)-2=-4<0,$ (there's a max @ (-1,*f*(-1)).

min: $(3, f(3))=(3, \frac{1}{3}(3)^3-3^2-3(3)+2)=(3,-7)$

max: $(-1, f(-1))=(-1, \frac{1}{3}(-1)^3-(-1)^2-3(-1)+2)=(-1, \frac{11}{3})$

Now , to the POI - set the second derivative equal to zero:

$f''(x)=2x-2=0,$

$x=1 \ (\textit{candidate POI})$

Test it with the number line (checking the concavity to the left and right of x = 2 - circle 2 below):

$\cap$ $\cup$

0 1 3

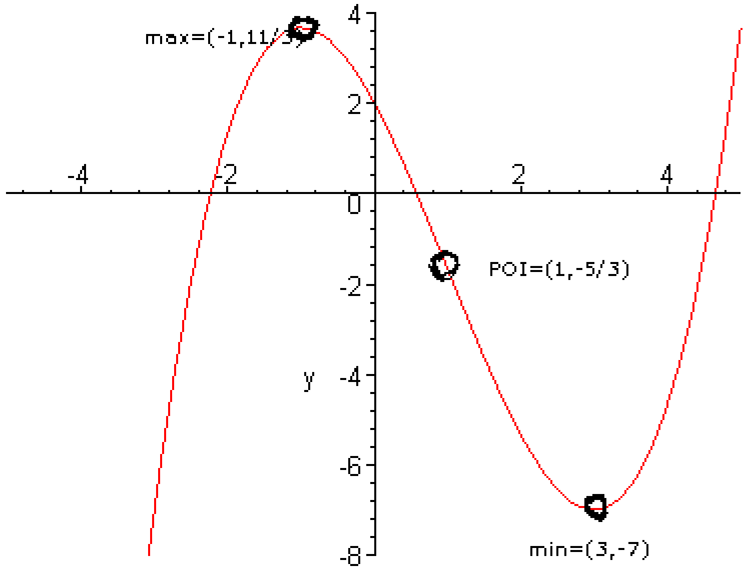
$f''(0)=2(0)-2=-2<0,$ $f''(3)=2(3)-2=4>0,$

(concave down) (concave up)

So, we seem to have a POI @

$(1, f(1))=(1, \frac{1}{3}(1)^3-(1)^2-3(1)+2)=(1, -\frac{5}{3})$.

Graph:


max=(-1,11/3)
POI=(1,-5/3)
min=(3,-7)
y
4
2
0
-2
-4
-6
-8
-4
-2
2
4