

Practice Problems for Derivatives Quiz 1

AP Calculus

Name:

Answers

The quiz will be about 40 points and you will NOT be using a calculator! Be sure you know limit definition of the derivative and all the "short cut" rules including product rule, quotient rule, chain rule, how to take the derivative of an exponential function and natural log function, how to take the derivative of the six trig functions, how to find the derivative of the inverse of a function (including inverse trig functions), and how to find the derivative of an absolute value function. You will have access to a unit circle for this quiz, but this is the last quiz where you will have that access!

For each multiple choice problem, SHOW YOUR WORK and circle the best answer.

- 1) A hanging spring with a weight at the bottom is compressed and then released so that it bounces up and down. A cosine function models the motion of the spring, with y as vertical displacement and x as time. What is the instantaneous rate of change of the spring given the sinusoidal model below.

$$y = 4 \cos\left(\frac{5\pi}{2}x\right)$$

(A) $4\pi \cos\left(\frac{5\pi}{2}x\right)$

(B) $4\sin\left(\frac{5\pi}{2}x\right)$

(C) $-4\sin\left(\frac{5\pi}{2}x\right)$

(D) $-10\pi \sin\left(\frac{5\pi}{2}x\right)$

(E) $10\pi \sin\left(\frac{5\pi}{2}x\right)$

$$y' = 4 \cdot -\sin\left(\frac{5\pi}{2}x\right) \cdot \frac{5\pi}{2}$$

- 2) If $y = \tan^5(2x)$ then $\frac{dy}{dx}$ is:

$$(\tan(2x))^5$$

(A) $\sec^{10}(2x)$

(B) $10\sec^{10}(2x)$

(C) $5\tan^4(2x)$

(D) $10\tan^4(2x)$

(E) $10\tan^4(2x)\sec^2(2x)$

$$y' = 5(\tan(2x))^4 \cdot \sec^2(2x) \cdot 2$$

- 3) If $f(x) = \sqrt[3]{x^3 - x}$ then $f'(x)$ is:

$$(x^3 - x)^{1/3}$$

(A) $\frac{1}{3(x^3 - x)}$

(B) $\frac{1}{3(x^3 - x)^{2/3}}$

(C) $\frac{3x^2 - 1}{3(x^3 - x)^{2/3}}$

(D) $\frac{x^2 - 1}{(x^3 - x)^{2/3}}$

(E) None of these

$$f'(x) = \frac{1}{3}(x^3 - x)^{-2/3}(3x^2 - 1)$$

4) The $\lim_{h \rightarrow 0} \frac{\ln(x-3+h) - \ln(x-3)}{h}$ is

← This is limit def. of deriv!

(A) $\ln(x+3)$

(B) $\ln(x-3)$

(C) $\frac{1}{\ln(x-3)}$

(D) $\frac{1}{(x+3)}$

(E) $\frac{1}{(x-3)}$

$f(x) = \ln(x-3)$ so $f'(x) = \frac{1}{x-3} \cdot 1$

5) Find the derivative of each function below using the limit definition of the derivative.

a) $f(x) = 3x - x^2$

b) $g(x) = \frac{3}{x}$

c) $h(x) = \sqrt{2x+1}$

See last page of answer key

6) Find the derivative of $f(x) = \frac{e^x}{\sin(x)}$. Then evaluate $f'(\frac{\pi}{2}) = \boxed{e^{\frac{\pi}{2}}}$

$f'(x) = \frac{e^x \cdot \sin x - e^x \cdot \cos x}{(\sin x)^2}$ $f'(\frac{\pi}{2}) = \frac{e^{\frac{\pi}{2}} \cdot \sin \frac{\pi}{2} - e^{\frac{\pi}{2}} \cdot \cos \frac{\pi}{2}}{(\sin \frac{\pi}{2})^2}$

7) Find the derivative of $g(x) = \sqrt{\ln x}$. Then evaluate $g'(e)$

$g'(x) = \frac{1}{2} (\ln x)^{-1/2} \cdot \frac{1}{x}$ $g'(e) = \frac{1}{2e(\ln e)^{1/2}} \Rightarrow \frac{1}{2e \cdot \frac{1}{2}} \Rightarrow \boxed{\frac{1}{e}}$

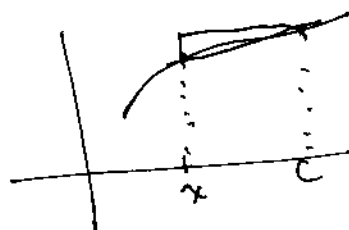
8) Find the derivative of $h(x) = e^{-4x} + 5 \ln(\cos x)$. Then evaluate $h'(\frac{\pi}{4})$

$h'(x) = e^{-4x} \cdot -4 + 5 \cdot \frac{1}{\cos x} \cdot -\sin x$

$h'(\frac{\pi}{4}) = -4e^{-\pi} + -5 \tan(\frac{\pi}{4}) \Rightarrow \boxed{-4e^{-\pi} - 5}$

9) Explain using complete sentences why the expression below represents the derivative of a function. Write in complete sentences and feel free to supply a sketch of a graph to illustrate the situation.

$\lim_{x \rightarrow c} \frac{f(x) - f(c)}{x - c}$



10) Take the derivative of each function (attempt to clean it up when applicable)

a. $f(x) = 3x(7x^2 - 2)^4$

$$\begin{aligned} f'(x) &= 3(7x^2 - 2)^4 + 3x \cdot 4(7x^2 - 2)^3(14x) \\ &= 3(7x^2 - 2)^3(7x^2 - 2 + 56x^2) \\ &= 3(7x^2 - 2)^3(63x^2 - 2) \end{aligned}$$

c. $y = \frac{3x-1}{\sqrt{x}} = \frac{3x}{x^{1/2}} - \frac{1}{x^{1/2}} = 3x^{1/2} - x^{-1/2}$

$$\frac{dy}{dx} = \frac{3}{2}x^{-1/2} + \frac{1}{2}x^{-3/2}$$

e. $f(x) = \ln(x^2 + 1)$

$$\begin{aligned} f'(x) &= \frac{1}{x^2 + 1} \cdot 2x \\ &= \frac{2x}{x^2 + 1} \end{aligned}$$

g) $f(x) = \sqrt{\ln(\sin x) + \ln(\cos x)}$

$$f'(x) = \frac{1}{2} (\ln(\sin x) + \ln(\cos x))^{-1/2} \cdot \left(\frac{1}{\sin x} \cdot \cos x + \frac{1}{\cos x} \cdot (-\sin x) \right)$$

$$= \frac{\cot x - \tan x}{2 (\ln(\sin x) + \ln(\cos x))^{1/2}}$$

b. $f(x) = \sqrt[3]{2x^3 - 1}$

$$\begin{aligned} f'(x) &= \frac{1}{3} (2x^3 - 1)^{-2/3} (6x^2) \\ &= 2x^2 (2x^3 - 1)^{-2/3} \end{aligned}$$

d. $g = \frac{e^x}{7} - \frac{7}{e^x} \Rightarrow \frac{1}{7}e^x - 7e^{-x}$

$$\begin{aligned} g'(x) &= \frac{1}{7}e^x - 7e^{-x} \cdot (-1) \\ &= \frac{1}{7}e^x + 7e^{-x} \end{aligned}$$

f. $y = 2^{\ln x}$

$$\begin{aligned} y' &= (\ln 2) 2^{\ln x} \cdot \frac{1}{x} \\ &= \frac{(\ln 2) 2^{\ln x}}{x} \end{aligned}$$

h. $f(x) = \tan(\sin^3 x)$

$$f'(x) = \sec^2(\sin^3 x) \cdot 3\sin^2 x \cdot \cos x$$

11) Which expression below is the derivative of $f(x) = 2\csc^3(4-x)$? $\Rightarrow 2(\csc(4-x))^3$

a) $-6\csc^2(4-x)\cot(4-x)$

b) $-3\csc(4-x)\cot^2(4-x)$

c) $6\csc^3(4-x)\cot(4-x)$

d) $-6\csc(4-x)\cot(4-x)$

e) $24\csc^2(4-x)\cot^2(4-x)$

$$f'(x) = 6(\csc(4-x))^2 \cdot -\csc(4-x)\cot(4-x) \cdot -1$$

12) Given $f(x) = \sin(2x)\cos(x)$, then $f'(\frac{\pi}{6}) = \dots$

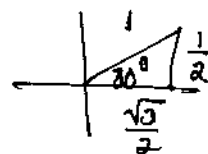
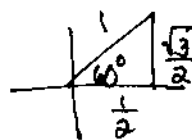
a) $\frac{\sqrt{3}-3}{4}$

b) $\frac{\sqrt{3}}{4}$

c) $\frac{\sqrt{3}}{2}$

d) $\frac{2\sqrt{3}+3}{4}$

e) 0

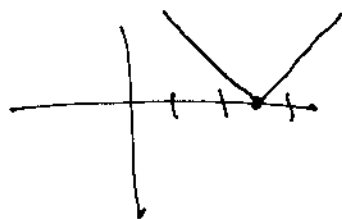


$$f'(x) = \cos(2x) \cdot 2 \cdot \cos x + \sin(2x) \cdot -\sin(x)$$

$$f'(\frac{\pi}{6}) = \cos \frac{\pi}{3} \cdot 2 \cdot \cos \frac{\pi}{6} + \sin \frac{\pi}{3} \cdot -\sin \frac{\pi}{6} \Rightarrow \frac{1}{2} \cdot 2 \cdot \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{2} \cdot -\frac{1}{2}$$

13) Find the derivative of $f(x) = |2x-5|$

$$= \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{4} = \frac{\sqrt{3}}{4}$$



$$f'(x) = \begin{cases} -2, & x < 2.5 \\ 2, & x > 2.5 \end{cases}$$

Note: $f'(x)$ is undefined at $x=2.5$! More on this later!

14) Given that $f(x)$ and $g(x)$ are inverses of each other, find $g'(31)$ if $f(x) = 4x^3 - 1$

$$g \rightarrow x = 4y^3 - 1$$

$$\frac{x+1}{4} = y^3$$

$$g(x) = \left(\frac{x+1}{4}\right)^{\frac{1}{3}}$$

$$g'(x) = \frac{1}{3} \left(\frac{x+1}{4}\right)^{-2/3} \cdot \frac{1}{4}$$

$$g'(31) = \frac{1}{12} (8)^{-2/3} \Rightarrow \frac{1}{12 \cdot 2\sqrt[3]{8}} = \frac{1}{12 \cdot 4} = \frac{1}{48}$$

15) Given that $f(g(x)) = x$, find $g'(7)$ given $f(-2) = 7$ and $f'(-2) = 15$.

$$g'(x) = \frac{1}{f'(g(x))}$$

$$g'(7) = \frac{1}{f'(g(7))}$$

$$= \frac{1}{f'(-2)} \Rightarrow \frac{1}{15}$$

x	$f(x)$	x	$g(x)$
-2	7	7	-2

16) Find $H'(x)$ given that $H(x) = e^{\arccos(\sqrt{x})}$. Then evaluate $H'(\frac{1}{4})$. Leave answers in simplest form (but a radical in the denominator is fine.) (Note: I'll provide the inverse trig derivative rules on the quiz.)

$$H'(x) = (\ln e) e^{\arccos \sqrt{x}} \cdot \frac{-1}{\sqrt{1-(\sqrt{x})^2}} \cdot \frac{1}{2} x^{-\frac{1}{2}}$$

$$H'(\frac{1}{4}) = e^{\arccos \sqrt{\frac{1}{4}}} \cdot \frac{-1}{\sqrt{1-\frac{1}{4}}} \cdot \frac{1}{2\sqrt{\frac{1}{4}}}$$

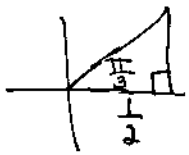
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16 cont)

$$H'(\frac{1}{4}) = e^{\arccos(\frac{1}{2})} \cdot \frac{-1}{\sqrt{\frac{3}{4}}} \cdot \frac{1}{2 \cdot \frac{1}{2}}$$

Note:

$$\arccos(\frac{1}{2}) = \frac{\pi}{3}$$



$$\text{since } \cos(\frac{\pi}{3}) = \frac{1}{2}$$

$$= e^{\frac{\pi}{3}} \cdot -\frac{2}{\sqrt{3}} \cdot 1$$

$$H'(\frac{1}{4}) = \frac{-2e^{\frac{\pi}{3}}}{\sqrt{3}}$$

$$5a) f(x) = 3x - x^2$$

$$\begin{aligned} f'(x) &= \lim_{h \rightarrow 0} \frac{3(x+h) - (x+h)^2 - (3x - x^2)}{h} \\ &= \frac{\cancel{3x} + 3h - \cancel{x^2} - 2xh - h^2 - \cancel{3x} + \cancel{x^2}}{h} \\ &= \frac{3h - 2xh - h^2}{h} \\ &= \downarrow 3 - 2x - h \end{aligned}$$

$$\boxed{f'(x) = 3 - 2x}$$

$$5b) g(x) = \frac{3}{x}$$

$$\begin{aligned} g'(x) &= \lim_{h \rightarrow 0} \frac{\frac{3}{x+h} - \frac{3}{x}}{h} \\ &= \frac{\frac{3x - 3(x+h)}{x(x+h)}}{h} \end{aligned}$$

← get common denom. to combine

$$\begin{aligned} &= \frac{3x - 3x - 3h}{x(x+h)} \div h \\ &= \frac{-3h}{x(x+h)} \cdot \frac{1}{h} \end{aligned}$$

$$= \frac{-3}{x(x+h)}$$

$$\boxed{g'(x) = \frac{-3}{x^2}}$$

← Now it's "safe" to put in "0" for h

$$5c) \quad h(x) = \sqrt{2x+1}$$

Multiply by the
↓ conjugate

$$h'(x) = \lim_{h \rightarrow 0} \frac{\sqrt{2(x+h)+1} - \sqrt{2x+1}}{h} \cdot \frac{(\sqrt{2(x+h)+1} + \sqrt{2x+1})}{(\sqrt{2(x+h)+1} + \sqrt{2x+1})}$$

$$= \frac{2(x+h)+1 - (2x+1)}{h(\sqrt{2(x+h)+1} + \sqrt{2x+1})}$$

$$= \frac{\cancel{2x} + 2h + \cancel{1} - \cancel{2x} - \cancel{1}}{h(\sqrt{2(x+h)+1} + \sqrt{2x+1})}$$

$$= \frac{2}{\sqrt{2(x+0)+1} + \sqrt{2x+1}}$$

$$= \frac{2}{2\sqrt{2x+1}}$$

$$\boxed{h'(x) = \frac{1}{\sqrt{2x+1}}}$$