Practice #2 for Quiz 3 Related Rates, Extrema and Inflection Points, Motion…
AP Calculus

Find all inflection points of each function below.

1) $f(x) = x^4 - 2x$ 2) $g(x) = \cos^2(x)$

See next pages for solutions →

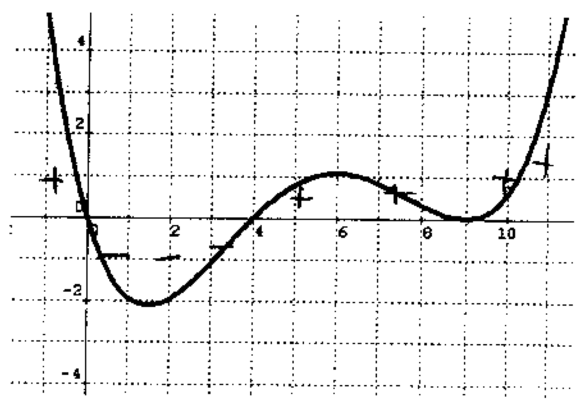
Find the critical points and any relative extrema for the following functions. Also find the absolute maxima and absolute minima. Justify your answers with clear work AND a sentence.

3) $f(\theta) = \theta - 2\cos\theta$ on the interval $[0, 2\pi]$

4) $g(x) = \dfrac{e^x}{x}$ for x > 0. →

Use the graph on the interval [-1, 11] to answer each question.

If an answer cannot be determined with the given information, state this to be the fact. If you need to do some estimating of values, that's fine.



5) Assume the graph is $f'(x)$

a) Determine the interval(s) on which $f(x)$ is increasing and clearly state how you know that.

$f(x)$ is incr on $(-1, 0) \cup (4, 11)$ since $f'(x)$ is positive

b) Find all relative (local) maxima and minima of $f(x)$ and clearly state how you know that.

rel max at. $x = 0$ since $f'(x)$ is + then 0, then neg.

rel min at $x = 4$ since $f'(x)$ is neg. then positive

c) Find all inflection points of $f(x)$ and clearly state how you know that.

Infl points of $f(x)$ occur when $f'(x)$ has min,s & max,s

i.e when $f''(x)$ changes sign at $x \approx 1.5$, $x = 6$, $x = 9$.

Practice #2 for Quiz 3
2017

Find inflection points

$f(x) = x^4 - 2x$

1) $f'(x) = 4x^3 - 2$

$f''(x) = 12x^2$

$f''(-1) = +$ $\quad f''(1) = +$

f'': -1, 0, 1

$12x^2 = 0$

$x = 0$

$f(x)$ has No inflection points since sign of $f''(x)$ doesn't change. This means concavity doesn't change.

2) $g(x) = (\cos x)^2$

$g'(x) = 2\cos x \cdot -\sin x = -2\cos x \cdot \sin x$

$g''(x) = -2 \cdot -\sin x \cdot \sin x + -2\cos x \cdot \cos x$

$0 = 2\sin^2 x - 2\cos^2 x$

$2\sin^2 x = 2\cos^2 x$

$\pm \sin x = \pm \cos x$

$g''(\frac{\pi}{6}) = -$ $\quad g''(\frac{\pi}{3}) = +$ $\quad -$

$\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, $\frac{3\pi}{4}$

$x = \ldots \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}, \ldots$

All multiples of $\frac{\pi}{4}$ are inflection points of $g(x)$

i.p. at $\frac{\pi}{4}z$ where z is an integer.

3) $f(\theta) = \theta - 2\cos\theta$ on $[0, 2\pi]$

Find c.p. + extrema, abs. + rel.

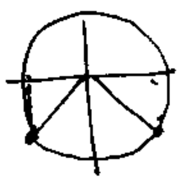
$$f'(\theta) = 1 + 2\sin\theta$$

$$0 = 1 + 2\sin\theta$$

$$-1 = 2\sin\theta$$

$$-\frac{1}{2} = \sin\theta$$

C.P. $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$



$f'(\pi) = +$ $\quad f'(\frac{3\pi}{2}) = -$ $\quad f'(2\pi) = +$

f': $\pi \quad \frac{7\pi}{6} \quad \frac{3\pi}{2} \quad \frac{11\pi}{6} \quad 2\pi$

Rel max at $\theta = \frac{7\pi}{6}$

Rel min at $\theta = \frac{11\pi}{6}$

θ	$f(\theta)$
0	-2
$\frac{7\pi}{6}$	$\frac{7\pi}{6} - 2\cos(\frac{7\pi}{6}) \approx 5.397$
$\frac{11\pi}{6}$	$\frac{11\pi}{6} - 2\cos(\frac{11\pi}{6}) \approx 4.028$
2π	$2\pi - 2\cos(2\pi) \approx 4.283$

Absolute max $= 5.397$

Absolute min $= -2$

4) $g(x) = \dfrac{e^x}{x}$ for $x > 0$ Find c.p. & any extrema

$$g'(x) = \frac{e^x \cdot x - e^x \cdot 1}{x^2}$$

$0 = e^x(x-1)$

$e^x = 0$ or $x - 1 = 0$

C.P at $x = 1$

$g'\left(\tfrac{1}{2}\right) = -$ $\quad$ $g'(2) = +$

g' $\quad$ 1

- $g(x)$ has a relative min. at $x = 1$ since $g'(x)$ is neg. on $(0, 1)$ & positive on $(1, \infty)$

- $g(x)$ has an absolute min of e, at $x = 1$.

  $g(1) = \dfrac{e^1}{1} = e$

- $g(x)$ has NO absolute max, since $g'(x)$ is $+$ on $(1, \infty)$.

6) Use well organized and well communicated mathematics to answer the following. For each question, be sure to explain and justify your answer clearly.

The following function $p(t) = 3\ln(t^4) - 0.5t + 6$ describes the position of a particle moving along a line over the time interval [1, 12].

a) What is the velocity function for this particle? Simplify the function as much as you can.

$$v(t) = p'(t) = 3 \cdot \frac{1}{t^4} \cdot 4t^3 - \frac{1}{2} = \frac{12}{t} - \frac{1}{2}$$

b) Is the particle speeding up or slowing down at t = 2? Supply justification.

$v(2) = 5.5$

$a(t) = -12t^{-2}$

$a(2) = \frac{-12}{(2)^2} = -3$

Particle is slowing down at $t = 2$ since $v(2)$ is + and $a(2)$ is negative.

c) Is the velocity of the particle increasing or decreasing at t = 2?

$a(2) = -3$

Velocity is decreasing at $t = 2$ since $a(2)$ is negative.

d) Determine the exact time that this particle turns around on [1,12]

$v(t) = 0 \implies \frac{12}{t} - \frac{1}{2} = 0$

$t = 24$

$v(t)$: + (20) | 24 | − (26)

C.P. is outside domain... Particle goes right over [1, 12]... The particle doesn't turn around until $t = 24$.

e) Find the displacement of the particle over the interval [1, 12].

$p(12) - p(1) = \boxed{24.319}$

$29.818879 - 5.5$