Warm Up

Simplify each radical.

1. $\sqrt{12}$ $2\sqrt{3}$ **2.** $\sqrt{50}$ $5\sqrt{2}$ **3.** $\sqrt{75}$ $5\sqrt{3}$

Find the distance between each pair of points. Write your answer in simplest radical form.

4. *C* (1, 6) and *D* (–2, 0) $3\sqrt{5}$

5. *E*(–7, –1) and *F*(–1, –5) $2\sqrt{13}$

Objectives

Apply similarity properties in the coordinate plane.

Use coordinate proof to prove figures similar.

A **dilation** is a transformation that changes the size of a figure but not its shape.

The **preimage** and the **image** are *always* similar.

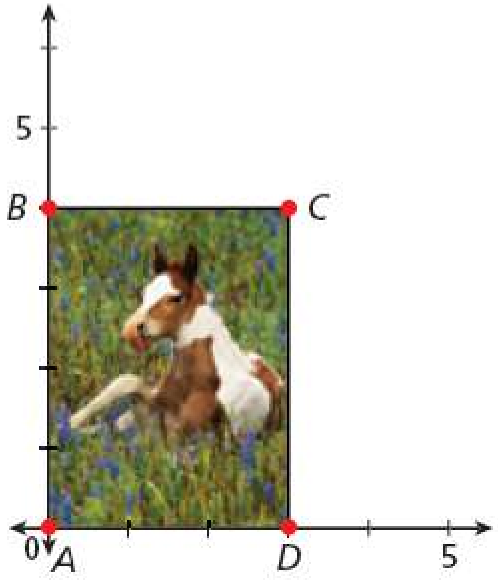
A **scale factor** describes how much the figure is enlarged or reduced.

Helpful Hint

If the scale factor of a dilation is greater than 1 ($k > 1$), it is an *enlargement*. If the scale factor is less than 1 ($k < 1$), it is a *reduction*.

Example 1: Computer Graphics Application

Draw the border of the photo after a dilation with scale factor $\frac{5}{2}$.



Example 1 Continued

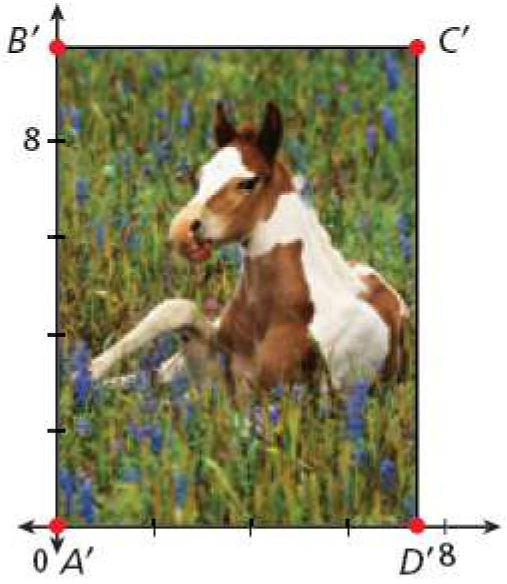
Step 1 Multiply the vertices of the photo *A*(0, 0), *B*(0, 4), *C*(3, 4), and *D*(3, 0) by $\frac{5}{2}$.

Rectangle *ABCD*		Rectangle *A′B′C′D′*
$A(0,0) \longrightarrow$	$A'\left(0 \bullet \frac{5}{2}, 0 \bullet \frac{5}{2}\right)$	$\longrightarrow A'(0,0)$
$B(0,4) \longrightarrow$	$B'\left(0 \bullet \frac{5}{2}, 4 \bullet \frac{5}{2}\right)$	$\longrightarrow B'(0,10)$
$C(3,4) \longrightarrow$	$C'\left(3 \bullet \frac{5}{2}, 4 \bullet \frac{5}{2}\right)$	$\longrightarrow C'(7.5,10)$
$D(3,0) \longrightarrow$	$D'\left(3 \bullet \frac{5}{2}, 0 \bullet \frac{5}{2}\right)$	$\longrightarrow D'(7.5,0)$

Example 1 Continued

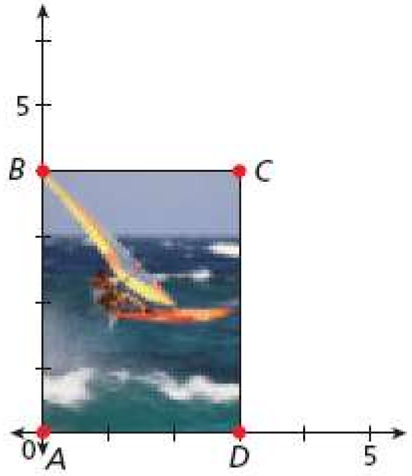
Step 2 Plot points $A'(0, 0)$, $B'(0, 10)$, $C'(7.5, 10)$, and $D'(7.5, 0)$.

Draw the rectangle.



Check It Out! Example 1

What if...? Draw the border of the original photo after a dilation with scale factor $\frac{1}{2}$.



Check It Out! Example 1 Continued

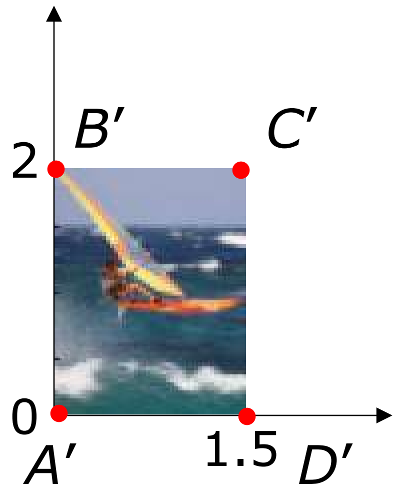
Step 1 Multiply the vertices of the photo $A(0, 0)$, $B(0, 4)$, $C(3, 4)$, and $D(3, 0)$ by $\frac{1}{2}$.

Rectangle $ABCD$		Rectangle $A'B'C'D'$
$A(0,0) \longrightarrow$	$A'\left(0 \cdot \frac{1}{2}, 0 \cdot \frac{1}{2}\right) \longrightarrow$	$A'(0,0)$
$B(0,4) \longrightarrow$	$B'\left(0 \cdot \frac{1}{2}, 4 \cdot \frac{1}{2}\right) \longrightarrow$	$B'(0,2)$
$C(3,4) \longrightarrow$	$C'\left(3 \cdot \frac{1}{2}, 4 \cdot \frac{1}{2}\right) \longrightarrow$	$C'(1.5,2)$
$D(3,0) \longrightarrow$	$D'\left(3 \cdot \frac{1}{2}, 0 \cdot \frac{1}{2}\right) \longrightarrow$	$D'(1.5,0)$

Check It Out! Example 1 Continued

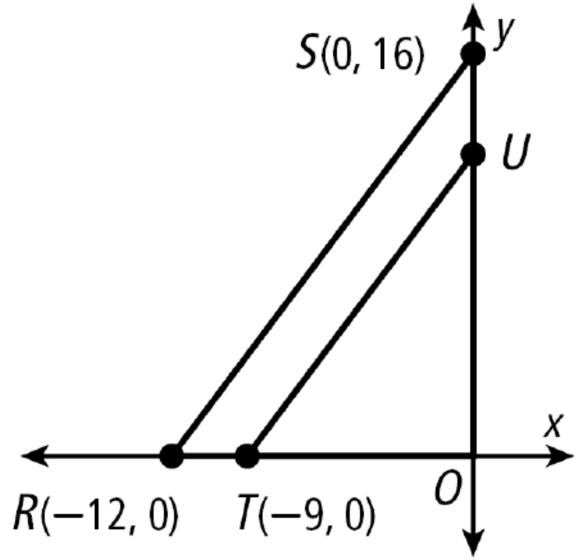
Step 2 Plot points *A′*(0, 0), *B′*(0, 2), *C′*(1.5, 2), and *D′*(1.5, 0).

Draw the rectangle.



Example 2: Finding Coordinates of Similar Triangle

Given that ΔTUO ~ ΔRSO, find the coordinates of U and the scale factor.



Since ΔTUO ~ ΔRSO,

$$\frac{RO}{TO} = \frac{OY}{OU}$$

$$\frac{12}{9} = \frac{16}{OU}$$ *Substitute 12 for RO, 9 for TO, and 16 for OY.*

$12OU = 144$ *Cross Products Prop.*

$OU = 12$ *Divide both sides by 12.*

Example 2 Continued

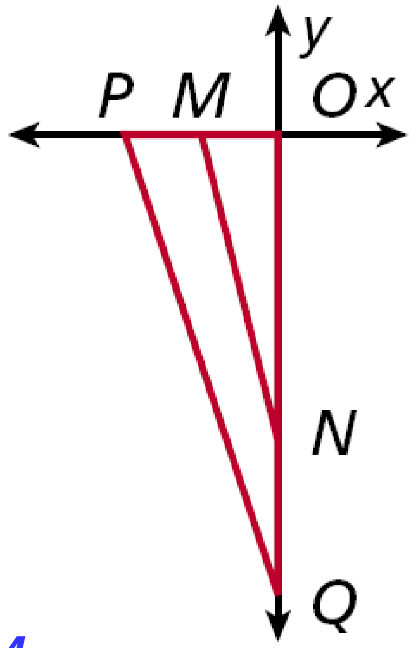
U lies on the *y*-axis, so its *x*-coordinate is 0. Since *OU* = 12, its *y*-coordinate must be 12. The coordinates of *U* are (0, 12).

$$(0,16) \longrightarrow \left(0 \bullet \frac{3}{4}, 16 \bullet \frac{3}{4}\right) \longrightarrow (0,12)$$

So the scale factor is $\frac{3}{4}$.

Check It Out! Example 2

Given that $\Delta MON \sim \Delta POQ$ and coordinates $P(-15, 0)$, $M(-10, 0)$, and $Q(0, -30)$, find the coordinates of N and the scale factor.



Since $\Delta MON \sim \Delta POQ$,

$$\frac{OM}{OP} = \frac{ON}{OQ}$$

$$\frac{10}{15} = \frac{ON}{30}$$ *Substitute 10 for OM, 15 for OP, and 30 for OQ.*

$$15\,ON = 300$$ *Cross Products Prop.*

$$ON = 20$$ *Divide both sides by 15.*

Check It Out! Example 2 Continued

N lies on the *y*-axis, so its *x*-coordinate is 0. Since *ON* = 20, its *y*-coordinate must be –20. The coordinates of *N* are (0, –20).

$$(0, -30) \longrightarrow \left(0 \bullet \frac{2}{3}, -30 \bullet \frac{2}{3}\right) \longrightarrow (0, -20)$$

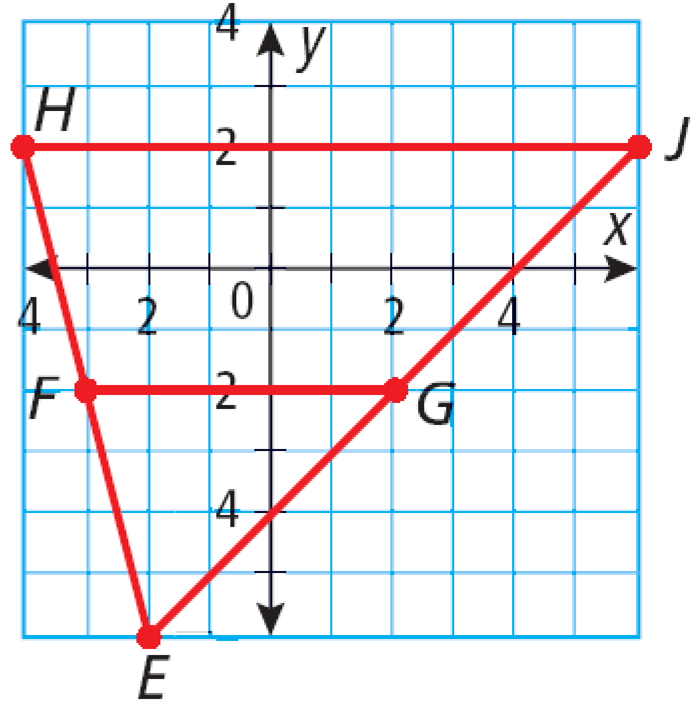
So the scale factor is $\frac{2}{3}$.

Example 3: Proving Triangles Are Similar

Given: $E(-2, -6)$, $F(-3, -2)$, $G(2, -2)$, $H(-4, 2)$, and $J(6, 2)$.

Prove: $\Delta EHJ \sim \Delta EFG$.

Step 1 Plot the points and draw the triangles.



Example 3 Continued

Step 2 Use the Distance Formula to find the side lengths.

$$EH = \sqrt{[-4-(-2)]^2 + [2-(-6)]^2} = \sqrt{68} = 2\sqrt{17}$$

$$EJ = \sqrt{[6-(-2)]^2 + [2-(-6)]^2} = \sqrt{128} = 8\sqrt{2}$$

$$EF = \sqrt{[-3-(-2)]^2 + [-2-(-6)]^2} = \sqrt{17}$$

$$EG = \sqrt{[2-(-2)]^2 + [-2-(-6)]^2} = \sqrt{32} = 4\sqrt{2}$$

Example 3 Continued

Step 3 Find the similarity ratio.

$$\frac{EH}{EF} = \frac{2\sqrt{17}}{\sqrt{17}} = \frac{2}{1} = 2$$

$$\frac{EJ}{EF} = \frac{8\sqrt{2}}{4\sqrt{2}} = \frac{4}{2} = 2$$

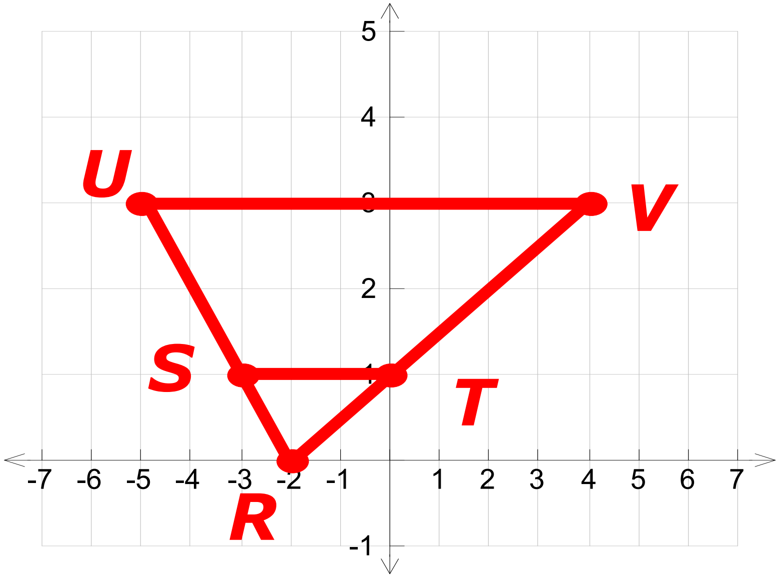
Since $\frac{EH}{EF} = \frac{EJ}{EG}$ and $\angle E \cong \angle E$, by the Reflexive Property, $\Delta EHJ \sim \Delta EFG$ by SAS ~ .

Check It Out! Example 3

Given: $R(-2, 0)$, $S(-3, 1)$, $T(0, 1)$, $U(-5, 3)$, and $V(4, 3)$.

Prove: $\Delta RST \sim \Delta RUV$

Step 1 Plot the points and draw the triangles.



Check It Out! Example 3 Continued

Step 2 Use the Distance Formula to find the side lengths.

$$RS = \sqrt{[-3-(-2)]^2 + [1-(0)]^2} = \sqrt{2}$$

$$RT = \sqrt{[0-(-2)]^2 + [1-(0)]^2} = \sqrt{5}$$

$$RU = \sqrt{[-5-(-2)]^2 + [3-(0)]^2} = \sqrt{18} = 3\sqrt{2}$$

$$RV = \sqrt{[4-(-2)]^2 + [3-(0)]^2} = \sqrt{45} = 3\sqrt{5}$$

Check It Out! Example 3 Continued

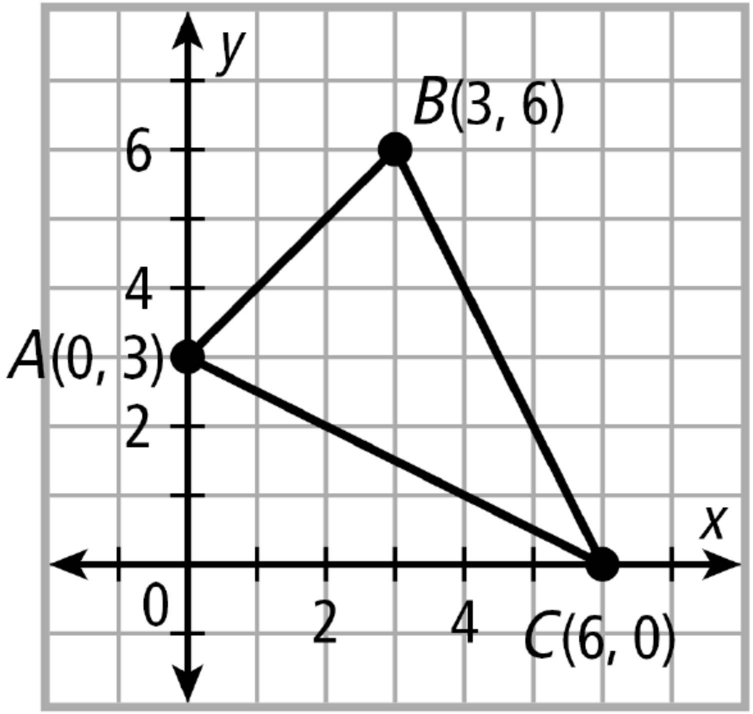
Step 3 Find the similarity ratio.

$$\frac{RS}{RU} = \frac{\sqrt{2}}{3\sqrt{2}} = \frac{1}{3} \qquad \frac{RT}{RV} = \frac{\sqrt{5}}{3\sqrt{5}} = \frac{1}{3}$$

Since $\frac{RS}{RU} = \frac{RT}{RV}$ and $\angle R \cong \angle R$, by the Reflexive Property, $\triangle RST \sim \triangle RUV$ by SAS ~ .

Example 4: Using the SSS Similarity Theorem

Graph the image of Δ*ABC* after a dilation with scale factor $\frac{2}{3}$.



Verify that Δ*A'B'C'* ~ Δ*ABC*.

Example 4 Continued

Step 1 Multiply each coordinate by $\frac{2}{3}$ to find the coordinates of the vertices of $\Delta A'B'C'$.

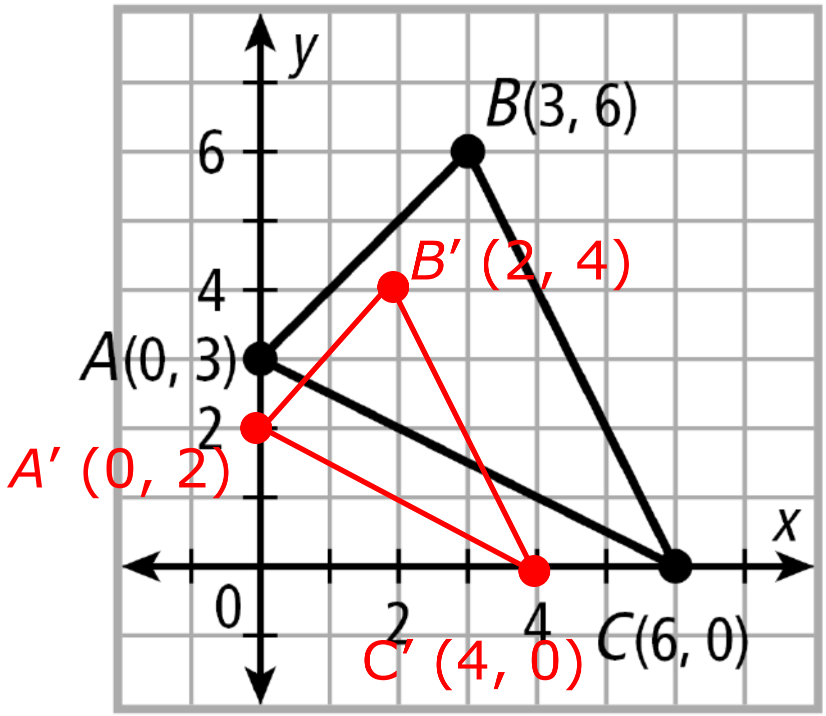
$$A(0, 3) \longrightarrow A'\left(0 \bullet \frac{2}{3}, 3 \bullet \frac{2}{3}\right) \longrightarrow A'(0, 2)$$

$$B(3, 6) \longrightarrow B'\left(3 \bullet \frac{2}{3}, 6 \bullet \frac{2}{3}\right) \longrightarrow B'(2, 4)$$

$$C(6, 0) \longrightarrow C'\left(6 \bullet \frac{2}{3}, 0 \bullet \frac{2}{3}\right) \longrightarrow C'(4, 0)$$

Example 4 Continued

Step 2 Graph $\Delta A'B'C'$.



Example 4 Continued

Step 3 Use the Distance Formula to find the side lengths.

$$AB = \sqrt{(3-0)^2 + (6-3)^2}$$
$$= \sqrt{18} = 3\sqrt{2}$$

$$A'B' = \sqrt{(2-0)^2 + (4-2)^2}$$
$$= \sqrt{8} = 2\sqrt{2}$$

$$BC = \sqrt{(6-3)^2 + (0-6)^2}$$
$$= \sqrt{45} = 3\sqrt{5}$$

$$B'C' = \sqrt{(4-2)^2 + (0-4)^2}$$
$$= \sqrt{20} = 2\sqrt{5}$$

$$AC = \sqrt{(6-0)^2 + (0-3)^2}$$
$$= \sqrt{45} = 3\sqrt{5}$$

$$A'C' = \sqrt{(4-0)^2 + (0-2)^2}$$
$$= \sqrt{20} = 2\sqrt{5}$$

Example 4 Continued

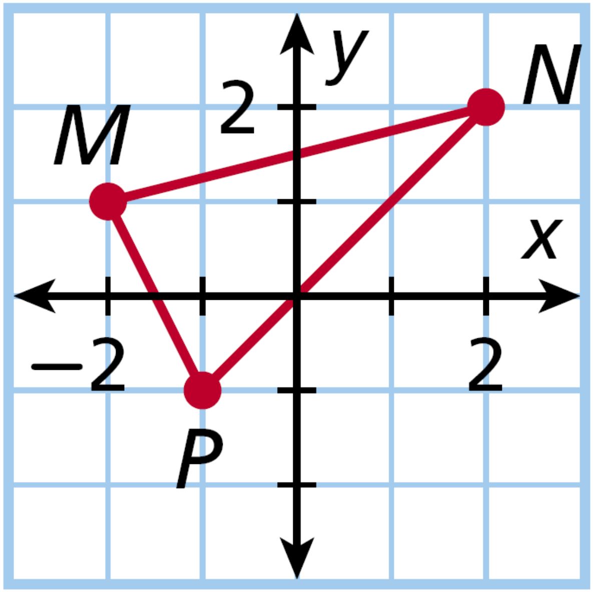
Step 4 Find the similarity ratio.

$$\frac{A'B'}{AB} = \frac{2\sqrt{2}}{3\sqrt{2}} = \frac{2}{3}, \frac{B'C'}{BC} = \frac{2\sqrt{5}}{3\sqrt{5}} = \frac{2}{3}, \frac{A'C'}{AC} = \frac{2\sqrt{5}}{3\sqrt{5}} = \frac{2}{3}$$

Since $\frac{A'B'}{AB} = \frac{B'C'}{BC} = \frac{A'C'}{AC}$, $\Delta ABC \sim \Delta A'B'C'$ by SSS ~.

Check It Out! Example 4

Graph the image of Δ*MNP* after a dilation with scale factor 3.
Verify that Δ*M'N'P'* ~ Δ*MNP*.



Check It Out! Example 4 Continued

Step 1 Multiply each coordinate by 3 to find the coordinates of the vertices of Δ*M'N'P'*.

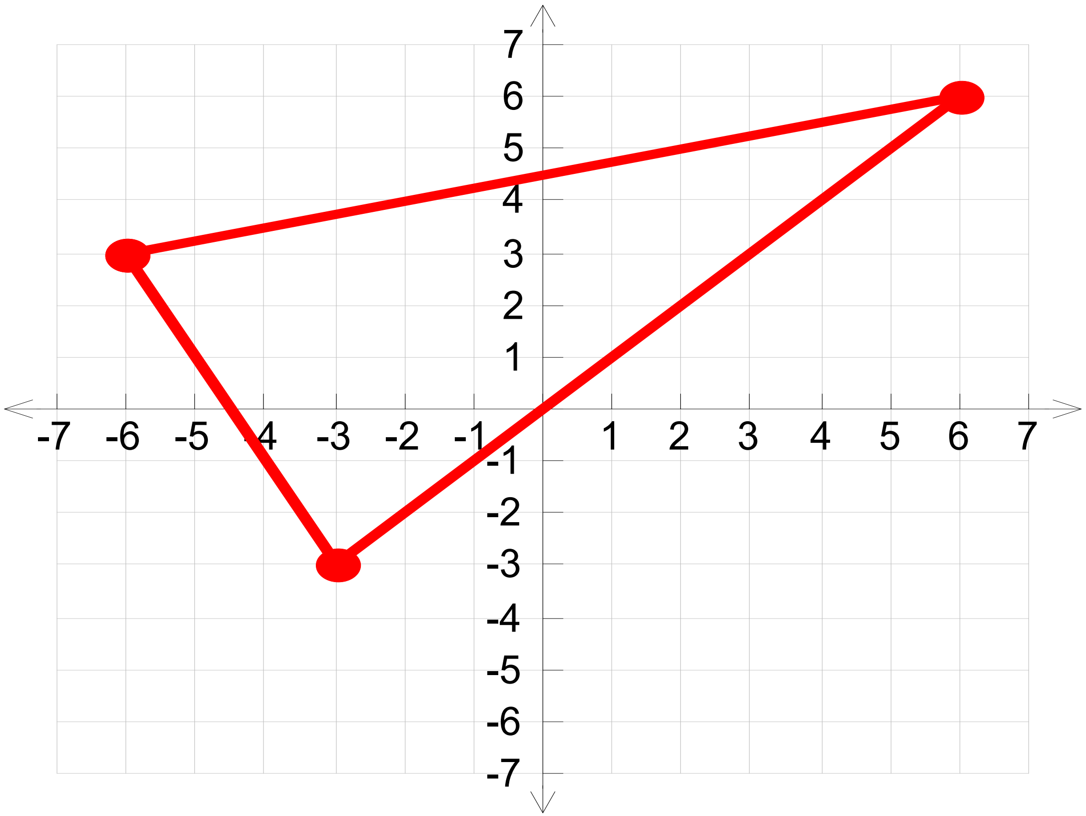
$$M(-2, 1) \longrightarrow M'(-2 \bullet 3, 1 \bullet 3) \longrightarrow M'(-6, 3)$$

$$N(2, 2) \longrightarrow N'(2 \bullet 3, 2 \bullet 3) \longrightarrow N'(6, 6)$$

$$P(-1, -1) \longrightarrow P'(-1 \bullet 3, -1 \bullet 3) \longrightarrow P'(-3, -3)$$

Check It Out! Example 4 Continued

Step 2 Graph Δ*M′N′P′*.



Check It Out! Example 4 Continued

Step 3 Use the Distance Formula to find the side lengths.

$$MP = \sqrt{(-1-(-2))^2 + (-1-1)^2} = \sqrt{5}$$

$$M'P' = \sqrt{(-3-(-6))^2 + (-3-3)^2} = 3\sqrt{5}$$

$$MN = \sqrt{(2-(-2))^2 + (2-1)^2} = \sqrt{17}$$

$$M'N' = \sqrt{(6-(-6))^2 + (6-3)^2} = 3\sqrt{17}$$

$$PN = \sqrt{(2-(-1))^2 + (2-(-1))^2} = 3\sqrt{2}$$

$$P'N' = \sqrt{(6-(-3))^2 + (6-(-3))^2} = 9\sqrt{2}$$

Check It Out! Example 4 Continued

Step 4 Find the similarity ratio.

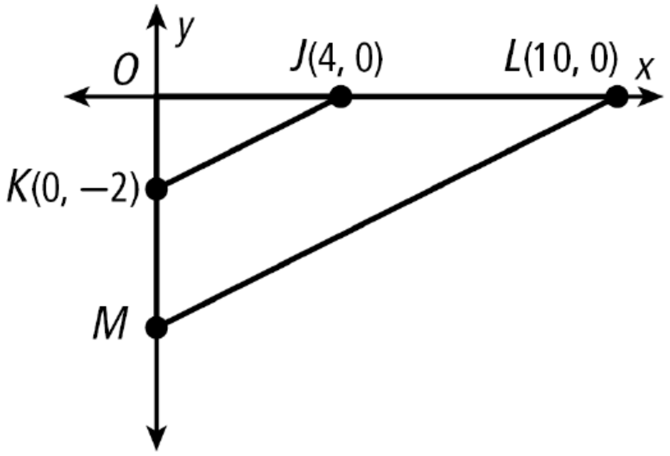
$$\frac{M'P'}{MP} = \frac{3\sqrt{5}}{\sqrt{5}} = 3, \frac{M'N'}{MN} = \frac{3\sqrt{17}}{\sqrt{17}} = 3, \frac{P'N'}{PN} = \frac{9\sqrt{2}}{3\sqrt{2}} = 3$$

Since $\frac{M'P'}{MP} = \frac{M'N'}{MN} = \frac{P'N'}{PN}$, $\Delta MNP \sim \Delta M'N'P'$ by SSS ~.

Lesson Quiz: Part I

1. Given $X(0, 2)$, $Y(-2, 2)$, and $Z(-2, 0)$, find the coordinates of X', Y', and Z' after a dilation with scale factor –4. $X'(0, -8)$; $Y'(8, -8)$; $Z'(8, 0)$

2. $\Delta JOK \sim \Delta LOM$. Find the coordinates of M and the scale factor.



$M(0, -5); \frac{5}{2}$

Lesson Quiz: Part II

3. Given: $A(-1, 0)$, $B(-4, 5)$, $C(2, 2)$, $D(2, -1)$, $E(-4, 9)$, and $F(8, 3)$

Prove: $\Delta ABC \sim \Delta DEF$

$AB = \sqrt{34}$, $AC = \sqrt{13}$, and $BC = 3\sqrt{5}$.

$DE = 2\sqrt{34}$, $DF = 2\sqrt{13}$, and $EF = 6\sqrt{5}$.

Therefore, $\frac{AB}{DE} = \frac{AC}{DF} = \frac{BC}{EF} = \frac{1}{2}$ and $\Delta ABC \sim \Delta DEF$ by SSS ~.

Homework

Worksheet 7.6