

7-3 Triangle Similarity: AA, SSS, and SAS

Warm Up

Solve each proportion.

$$1. \frac{6}{11} = \frac{8}{b}$$

$$b = \frac{44}{3} \text{ or } 14\frac{2}{3}$$

$$\frac{5}{z} = \frac{z}{20}$$

$$z = \pm 10$$

$$\frac{3}{10} = \frac{6}{x+12}$$

$$x = 8$$

4. If $\triangle QRS \sim \triangle XYZ$, identify the pairs of congruent angles and write 3 proportions using pairs of corresponding sides.

$$\angle Q \cong \angle X; \angle R \cong \angle Y; \angle S \cong \angle Z;$$

$$\frac{QR}{XY} = \frac{RS}{YZ}; \frac{RS}{YZ} = \frac{QS}{XZ}; \frac{QS}{XZ} = \frac{QR}{XY}$$

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Objectives

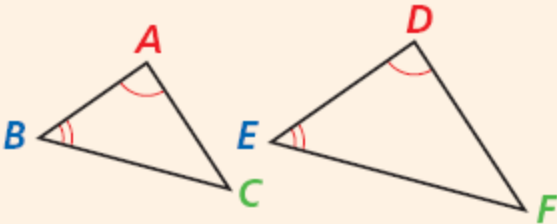
Prove certain triangles are similar by using AA, SSS, and SAS.

Use triangle similarity to solve problems.

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There are several ways to prove certain triangles are similar. The following postulate, as well as the SSS and SAS Similarity Theorems, will be used in proofs just as SSS, SAS, ASA, HL, and AAS were used to prove triangles congruent.

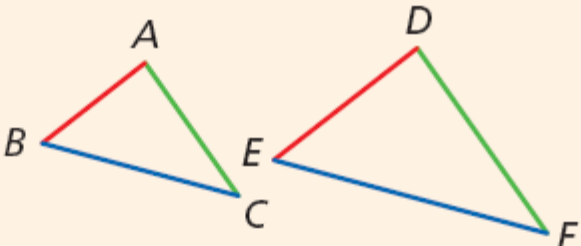
Postulate 7-3-1 Angle-Angle (AA) Similarity

POSTULATE	HYPOTHESIS	CONCLUSION
If two angles of one triangle are congruent to two angles of another triangle, then the triangles are similar.		$\triangle ABC \sim \triangle DEF$

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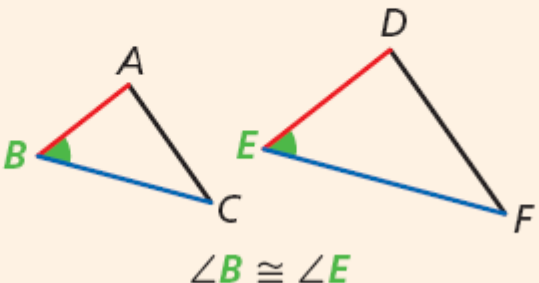
Theorem 7-3-2

Side-Side-Side (SSS) Similarity

THEOREM	HYPOTHESIS	CONCLUSION
If the three sides of one triangle are proportional to the three corresponding sides of another triangle, then the triangles are similar.		$\triangle ABC \sim \triangle DEF$

Theorem 7-3-3

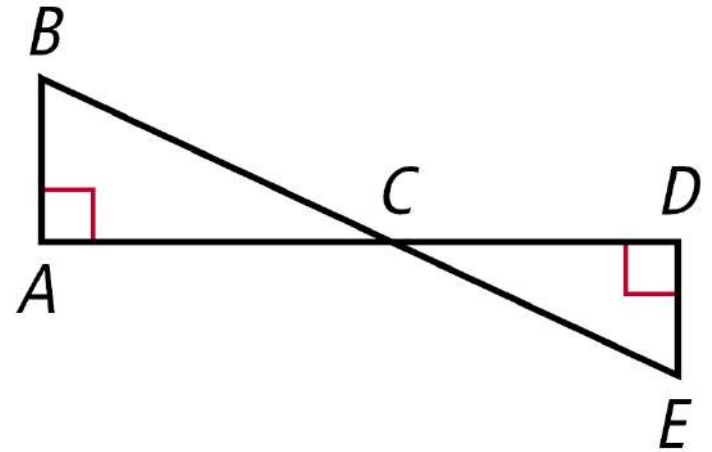
Side-Angle-Side (SAS) Similarity

THEOREM	HYPOTHESIS	CONCLUSION
If two sides of one triangle are proportional to two sides of another triangle and their included angles are congruent, then the triangles are similar.	 $\angle B \cong \angle E$	$\triangle ABC \sim \triangle DEF$

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Example 1: Using the AA Similarity Postulate

Explain why the triangles are similar and write a similarity statement.

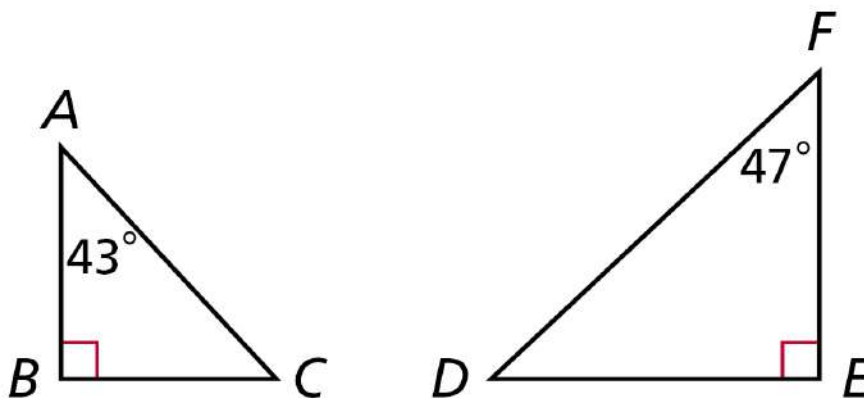


Since $\overline{AC} \parallel \overline{DE}$, $\angle B \cong \angle E$ by the Alternate Interior Angles Theorem. Also, $\angle A \cong \angle D$ by the Right Angle Congruence Theorem. Therefore $\triangle ABC \sim \triangle DEC$ by AA~.

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Check It Out! Example 1

Explain why the triangles are similar and write a similarity statement.



By the Triangle Sum Theorem, $m\angle C = 47^\circ$, so $\angle C \cong \angle F$. $\angle B \cong \angle E$ by the Right Angle Congruence Theorem. Therefore, $\triangle ABC \sim \triangle DEF$ by AA \sim .

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Example 2A: Verifying Triangle Similarity

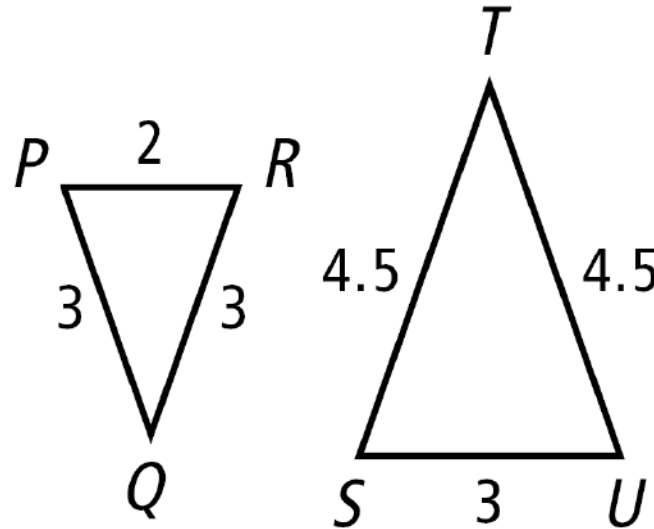
Verify that the triangles are similar.

$\triangle PQR$ and $\triangle STU$

$$\frac{PQ}{ST} = \frac{3}{4.5} = \frac{2}{3}$$

$$\frac{QR}{TU} = \frac{3}{4.5} = \frac{2}{3}$$

$$\frac{PR}{SU} = \frac{2}{3}$$



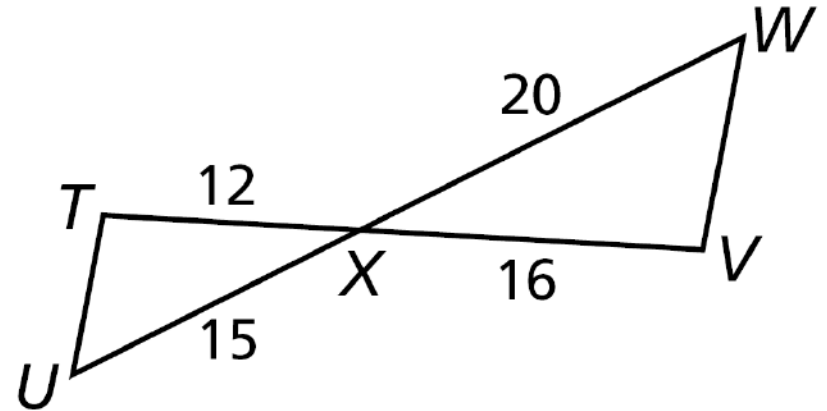
Therefore $\triangle PQR \sim \triangle STU$ by SSS \sim .

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Check It Out! Example 2

Verify that $\triangle TXU \sim \triangle VXW$.

$\angle TXU \cong \angle VXW$ by the Vertical Angles Theorem.



$$\frac{TX}{VX} = \frac{12}{16} = \frac{3}{4} \quad \frac{XU}{XW} = \frac{15}{20} = \frac{3}{4}$$

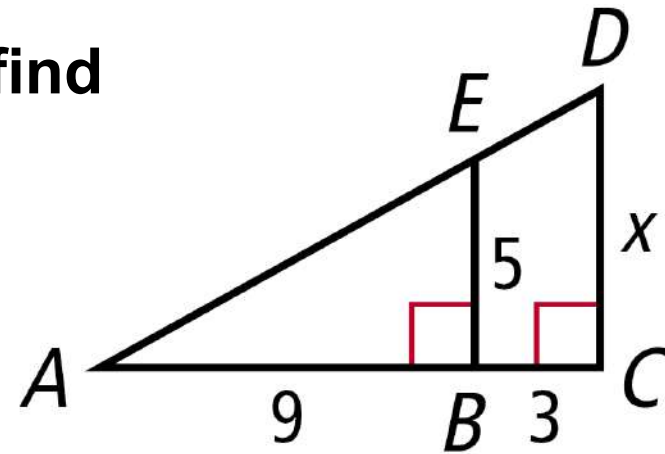
Therefore $\triangle TXU \sim \triangle VXW$ by SAS \sim .

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Example 3: Finding Lengths in Similar Triangles

Explain why $\triangle ABE \sim \triangle ACD$, and then find CD .

Step 1 Prove triangles are similar.



$\angle A \cong \angle A$ by Reflexive Property of \cong , and $\angle B \cong \angle C$ since they are both right angles.

Therefore $\triangle ABE \sim \triangle ACD$ by AA \sim .

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Example 3 Continued

Step 2 Find CD .

$$\frac{CD}{BE} = \frac{CA}{BA} = \frac{CB + BA}{BA}$$

Corr. sides are proportional. Seg. Add. Postulate.

$$\frac{x}{5} = \frac{3+9}{9}$$

Substitute x for CD , 5 for BE , 3 for CB , and 9 for BA .

$$x(9) = 5(3 + 9)$$

Cross Products Prop.

$$9x = 60$$

Simplify.

$$x = \frac{60}{9} = 6\frac{2}{3}$$

Divide both sides by 9.

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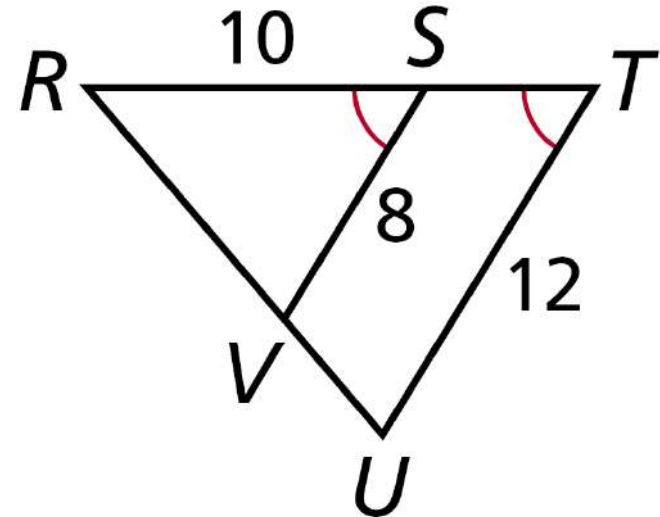
Check It Out! Example 3

Explain why $\triangle RSV \sim \triangle RTU$ and then find RT .

Step 1 Prove triangles are similar.

It is given that $\angle S \cong \angle T$.
 $\angle R \cong \angle R$ by Reflexive Property of \cong .

Therefore $\triangle RSV \sim \triangle RTU$ by AA \sim .



7-3 Triangle Similarity: AA, SSS, and SAS

Check It Out! Example 3 Continued

Step 2 Find RT .

$$\frac{RT}{RS} = \frac{TU}{SV}$$

Corr. sides are proportional.

$$\frac{RT}{10} = \frac{12}{8}$$

Substitute RS for 10, 12 for TU, 8 for SV.

$$RT(8) = 10(12)$$

Cross Products Prop.

$$8RT = 120$$

Simplify.

$$RT = 15$$

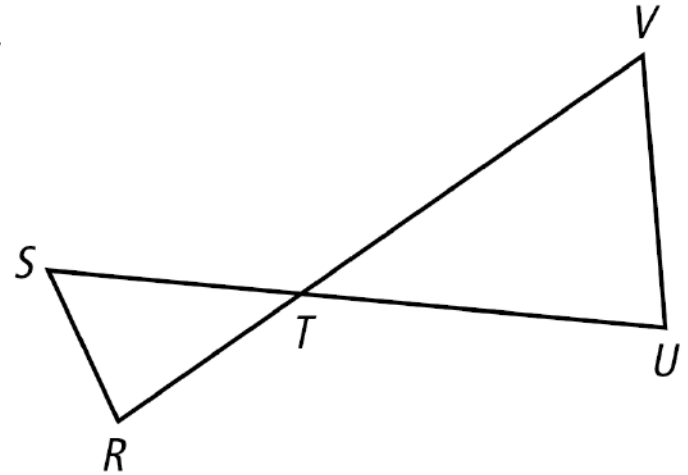
Divide both sides by 8.

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Example 4: Writing Proofs with Similar Triangles

Given: $3UT = 5RT$ and $3VT = 5ST$

Prove: $\triangle UVT \sim \triangle RST$



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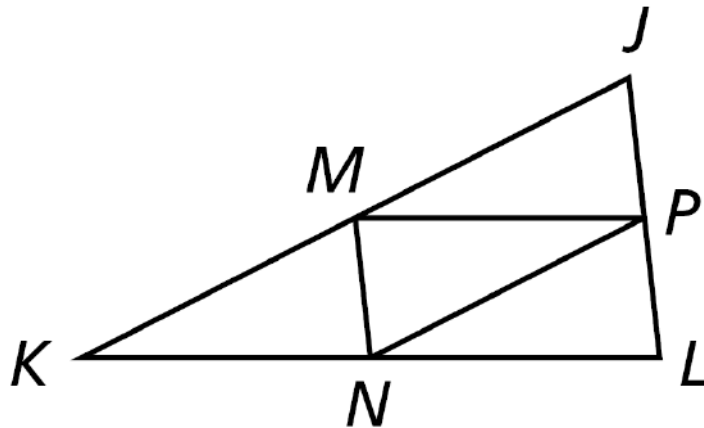
Example 4 Continued

Statements	Reasons
1. $3UT = 5RT$	1. Given
2. $\frac{UT}{RT} = \frac{5}{3}$	2. Divide both sides by $3RT$.
3. $3VT = 5ST$	3. Given.
4. $\frac{VT}{ST} = \frac{5}{3}$	4. Divide both sides by $3ST$.
5. $\angle RTS \cong \angle VTU$	5. Vert. \angle s Thm.
6. $\triangle UVT \sim \triangle RST$	6. SAS ~ Steps 2, 4, 5

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Check It Out! Example 4

Given: M is the midpoint of \overline{JK} . N is the midpoint of \overline{KL} , and P is the midpoint of \overline{JL} .



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Check It Out! Example 4 Continued

Statements	Reasons
1. M is the mdpt. of JK , N is the mdpt. of KL , and P is the mdpt. of JL .	1. Given
2. $MP = \frac{1}{2}KL$, $MN = \frac{1}{2}JL$, $NP = \frac{1}{2}KJ$	2. Δ Midsegs. Thm
3. $\frac{MP}{KL} = \frac{MN}{JL} = \frac{NP}{KJ} = \frac{1}{2}$	3. Div. Prop. of =.
4. $\triangle JKL \sim \triangle NPM$	4. SSS ~ Step 3

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You learned in Chapter 2 that the Reflexive, Symmetric, and Transitive Properties of Equality have corresponding properties of congruence. These properties also hold true for similarity of triangles.

Properties of Similarity

Reflexive Property of Similarity

$\triangle ABC \sim \triangle ABC$ (Reflex. Prop. of \sim)

Symmetric Property of Similarity

If $\triangle ABC \sim \triangle DEF$, then $\triangle DEF \sim \triangle ABC$. (Sym. Prop. of \sim)

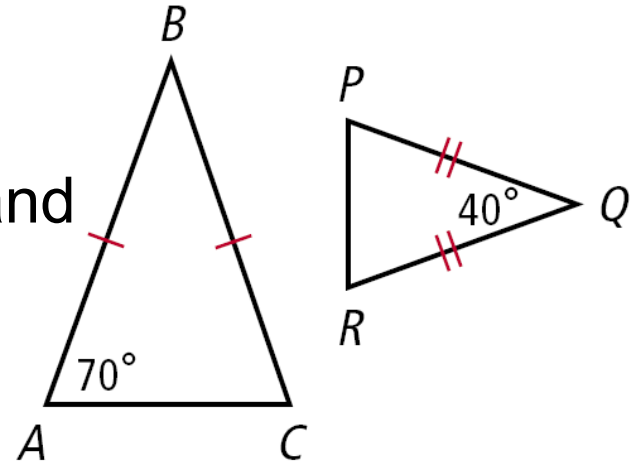
Transitive Property of Similarity

If $\triangle ABC \sim \triangle DEF$ and $\triangle DEF \sim \triangle XYZ$, then $\triangle ABC \sim \triangle XYZ$.
(Trans. Prop. of \sim)

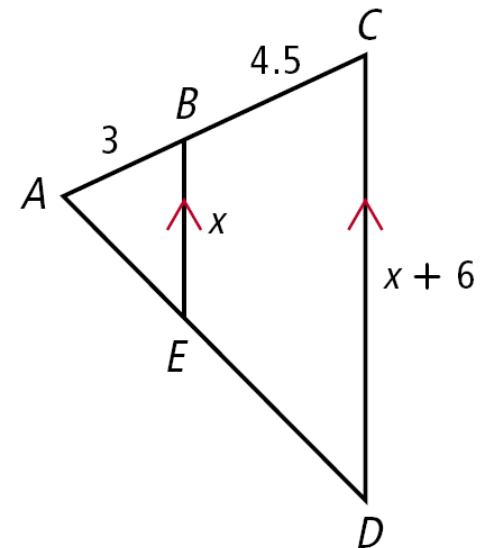
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Lesson Quiz

1. Explain why the triangles are similar and write a similarity statement.



2. Explain why the triangles are similar, then find BE and CD .



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Lesson Quiz

1. By the Isosc. \triangle Thm., $\angle A \cong \angle C$, so by the def. of \cong , $m\angle C = m\angle A$. Thus $m\angle C = 70^\circ$ by subst. By the \triangle Sum Thm., $m\angle B = 40^\circ$. Apply the Isosc. \triangle Thm. and the \triangle Sum Thm. to $\triangle PQR$. $m\angle R = m\angle P = 70^\circ$. So by the def. of \cong , $\angle A \cong \angle P$, and $\angle C \cong \angle R$. Therefore $\triangle ABC \sim \triangle PQR$ by AA \sim .
2. $\angle A \cong \angle A$ by the Reflex. Prop. of \cong . Since $BE \parallel CD$, $\angle ABE \cong \angle ACD$ by the Corr. \angle s Post. Therefore $\triangle ABE \sim \triangle ACD$ by AA \sim . $BE = 4$ and $CD = 10$.

Homework

Worksheet

Congruent and
similar triangles