

## Potato Differential Equation free response

### AP Calculus

Give yourself 15 minutes to do this problem. Practice showing clear, concise work.

At time  $t = 0$ , a boiled potato is taken from a pot on a stove and left to cool in a kitchen. The internal temperature of the potato is 91 degrees Celsius ( $^{\circ}\text{C}$ ) at time  $t = 0$ , and the internal temperature of the potato is greater than  $27^{\circ}\text{C}$  for all times  $t > 0$ . The internal temperature of the potato at time  $t$  minutes can be modeled by the function  $H$  that satisfies the differential equation  $\frac{dH}{dt} = -\frac{1}{4}(H - 27)$ , where  $H(t)$  is measured in degrees Celsius and  $H(0) = 91$ .

- (a) Write an equation for the line tangent to the graph of  $H$  at  $t = 0$ . Use this equation to approximate the internal temperature of the potato at time  $t = 3$ .
- (b) Use  $\frac{d^2H}{dt^2}$  to determine whether your answer in part (a) is an underestimate or an overestimate of the internal temperature of the potato at time  $t = 3$ .
- (c) For  $t < 10$ , an alternate model for the internal temperature of the potato at time  $t$  minutes is the function  $G$  that satisfies the differential equation  $\frac{dG}{dt} = -(G - 27)^{2/3}$ , where  $G(t)$  is measured in degrees Celsius and  $G(0) = 91$ . Find an expression for  $G(t)$ . Based on this model, what is the internal temperature of the potato at time  $t = 3$  ?