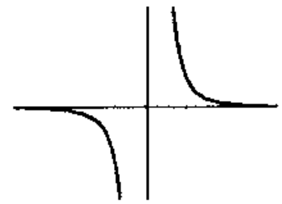
Polynomial Review For QUIZ **Name:** Answers

Introduction to Calculus

There will be at least one question from your last unit on power functions!

1) Fill in the information below for the function graphed:



a) Domain: $x \neq 0$ Range: $y \neq 0$

b) Asymptotes: $x = 0$, $y = 0$

c) End Behavior: as $x \to \infty$, y $\to 0$ $x \to -\infty$, y $\to 0$

For what values of x is the function…

e) Concave up: for $x > 0$ Concave Down: for $x < 0$

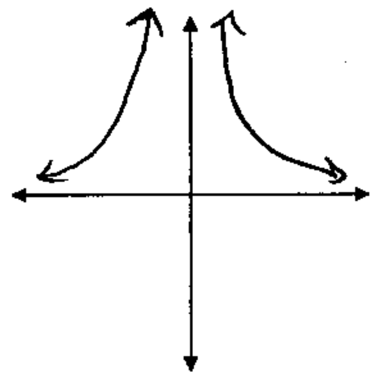
f) Increasing: never Decreasing: over whole domain

g) Write a possible equation for the above power function: $y = x^{-3}$ or $y = \frac{1}{x^3}$

2) Sketch a power function with the following characteristics:
- Increasing for $x < 0$ and decreasing for $x > 0$.
- Concave up for the whole domain.
- End behavior: as $x \to \infty,\ y \to 0$ as $x \to -\infty,\ y \to 0$

Write a possible equation for the power function: $y = x^{-2}$

$y = \frac{1}{x^2}$

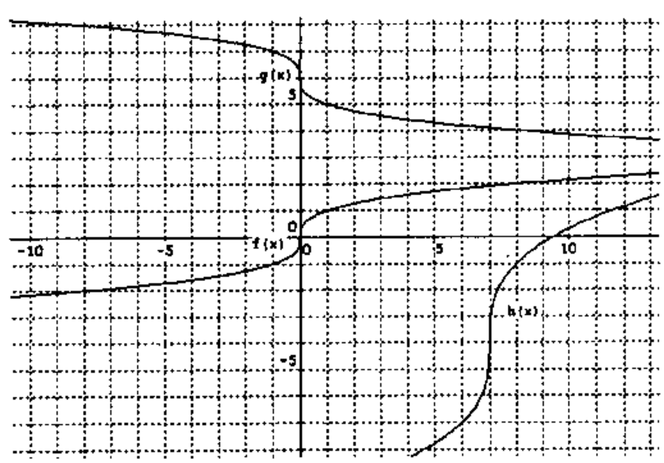


3) Given the power function *f(x)* below, use the rules of shifting and stretching to determine the explicit equations of *f(x)*, *g(x)*, and *h(x)*.

$f(x) = x^{\frac{1}{3}}$

$g(x) = -x^{\frac{1}{3}} + 5$

$h(x) = 3(x-7)^{\frac{1}{3}} - 4$



Know how to find roots (real and imaginary) of polynomials and sketch polynomials.

4a) Write the new function $f(x)$ that is created by shifting $y = x^3$ four units left and 8 units down.

b) Sketch a graph and use the graph to describe the roots of f(x) (how many roots? Real? imaginary?)

c) Write the function in standard form (i.e. expand and simplify…Remember Pascal!).

d) Find all three roots of this function. $\{-2, -5 \pm i\sqrt{3}\}$

a) $f(x) = (x+4)^3 - 8$

b)  1 real root, 2 imag. roots

c) $f(x) = (x+4)^3 - 8$

$= x^3 + 3x^2 \cdot 4 + 3x \cdot 4^2 + 4^3 - 8$

$= x^3 + 12x^2 + 48x + 56$

1
1 1
1 2 1
1 3 3 1

$0 = (x+4)^3 - 8$

$(8)^{1/3} = \left((x+4)^3\right)^{1/3}$

$2 = x + 4$

$\boxed{x = -2}$ real root

d)
$$\begin{array}{r} x^2 + 10x + 28 \\ x+2 \overline{\big) x^3 + 12x^2 + 48x + 56} \\ -(x^3 + 2x^2) \\ 10x^2 + 48x \\ -(10x^2 + 20x) \\ 28x + 56 \\ -(28x + 56) \\ 0 \end{array}$$

find roots of $x^2 + 10x + 28 = 0$

$x = \dfrac{-10 \pm \sqrt{100 - 4(1)(28)}}{2}$

$x = \dfrac{-10 \pm \sqrt{-12}}{2} = -5 \pm i\sqrt{3}$

5) A polynomial function of the form $ax^n + bx^{n-1} + cx^{n-2}$…
has exactly ___n___ roots (keeping in mind that some roots can be double or triple etc. roots and roots may be real and imaginary).

For #6, the "controlling term" i.e the term with the largest degree dictates the end behavior.

6) Assume the coefficient of the term with the largest degree (i.e. with the largest exponent) is **negative**, in a polynomial of degree **n**, where **n** is even:

As $x \to \infty$, $f(x) \to$ ___$-\infty$___ As $x \to -\infty$, $f(x) \to$ ___$-\infty$___ ex $-3x^4$

Explain why this is so: when you raise an input to an even exponent you get a + output, but the neg. k makes both neg.

7) Assume the coefficient of the term with the largest degree (i.e. with the largest exponent) is **negative**, in a polynomial of degree **n**, where n is odd:

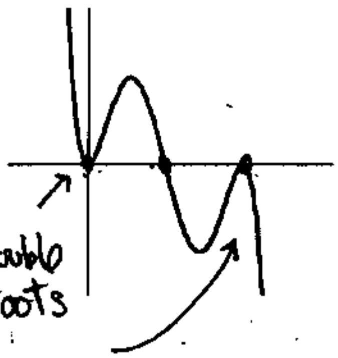
As $x \to \infty$, $f(x) \to$ ___$-\infty$___ As $x \to -\infty$, $f(x) \to$ ___∞___ ex $-3x^3$

Explain why this is so: When you raise a + or − number to an odd exponent, the sign is maintained. But the neg. K switches the signs.

8) For the graphs below, determine the **degree** of the polynomial and if the coefficient of the highest term is positive or negative:

Degree 5

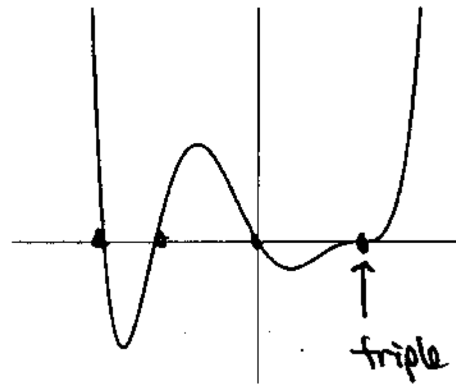
Positive or Negative leading coefficient −



double roots

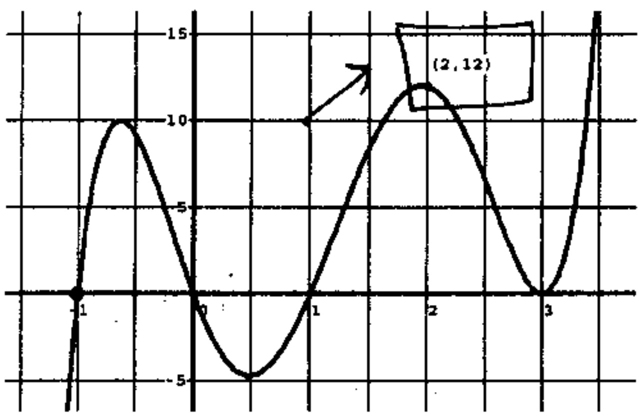
Degree 6

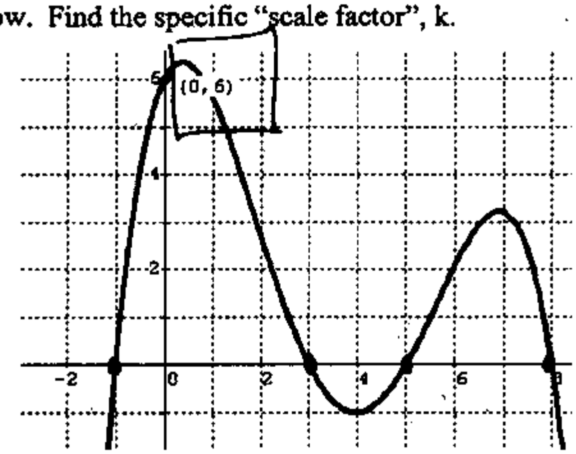
Positive or Negative leading coefficient +



triple root

9) Find the equation for the graphs of the polynomials below. Find the specific "scale factor", k.





$y = k(x+1)x(x-1)(x-3)^2$

$12 = k(2+1)(2)(2-1)(2-3)^2$

$12 = k \cdot 6$

$2 = k$

$\boxed{y = 2x(x+1)(x-1)(x-3)^2}$

$y = k(x+1)(x-3)(x-5)(x-8)$

$6 = k(0+1)(0-3)(0-5)(0-8)$

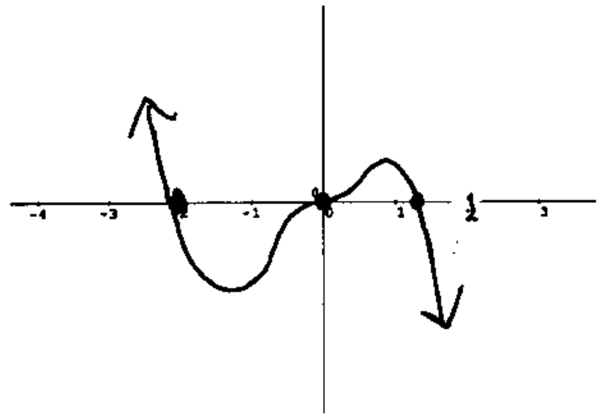
$6 = k(-120)$

$-\frac{6}{120} = k$

$\boxed{y = -\frac{1}{20}(x+1)(x-3)(x-5)(x-8)}$

10) Sketch the following polynomials:

a) $f(x) = -3x^5 - 2x^4 + 8x^3$

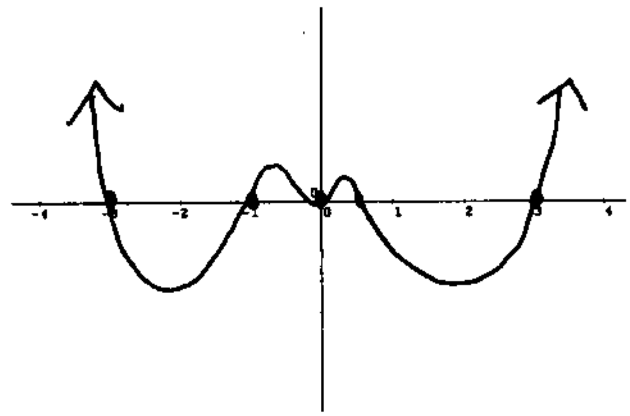


$0 = -x^3(3x^2 + 2x - 8)$

$0 = -x^3(3x - 4)(x + 2)$

$x = 0$ triple root, $x = \frac{4}{3}$, $x = -2$

b) $f(x) = (2x-1)(x^2-9)(x^3+x^2)$



$0 = (2x-1)(x-3)(x+3)x^2(x+1)$

$x = \frac{1}{2}, 3, -3, 0, -1$

double $\quad 2x^6 + \ldots$

11) Find the roots of each polynomial below. (Use scrap paper when necessary!)

a) $x^4 - 5x^2 + 4 = 0$

$(x^2-1)(x^2-4) = 0$

$(x-1)(x+1)(x-2)(x+2) = 0$

$\boxed{x = \pm 1, \pm 2}$

b) $(2x^3 + 3x^2) + (18x - 27) = 0$

$x^2(2x+3) - 9(2x+3) = 0$

$(2x+3)(x^2-9) = 0$

$\boxed{x = -\frac{3}{2},\ x = \pm 3}$

c) $x^3 - 2x + 4 = 0$

check possible roots: $\pm 1, \pm 2, \pm 4$

$(-1)^3 - 2(-1) + 4 \stackrel{?}{=} 0$ No

$(-2)^3 - 2(-2) + 4 \stackrel{?}{=} 0$ yes! $x = -2$

$$\begin{array}{r} x^2 - 2x + 2 \\ x+2 \overline{)\, x^3 + 0x^2 - 2x + 4} \\ -(x^3 + 2x^2) \\ -2x^2 - 2x \\ -(-2x^2 - 4x) \\ 2x + 4 \\ 2x + 4 \\ 0 \end{array}$$

Find roots of $x^2 - 2x + 2 = 0$ by quad. form or compl. the square.

$x^2 - 2x + 1 = -2 + 1$

$(x-1)^2 = -1$

$x - 1 = \pm\sqrt{-1}$

$x = 1 \pm i$

$\boxed{\{-2, 1 \pm i\}}$

d) $2x^3 + x^2 - 5x + 2 = 0$

See scrap paper for (d). →

d) $2x^3 + x^2 - 5x + 2 = 0$

possible roots: $\frac{\pm 2}{1}, \frac{\pm 1}{1}, \frac{\pm 2}{2}, \frac{\pm 1}{2} \rightarrow \pm 2, \pm 1, \pm \frac{1}{2}$

$2(1)^3 + (1)^2 - 5(1) + 2 \stackrel{?}{=} 0$ Yes!

$x = 1$ is a root + $(x-1)$ is a factor

$$
\begin{array}{r|l}
 & \quad 2x^2 + 3x - 2 \\
\hline
x-1 & 2x^3 + x^2 - 5x + 2 \\
 & \underline{-(2x^3 - 2x^2)} \\
 & \qquad 3x^2 - 5x + 2 \\
 & \qquad \underline{-(3x^2 - 3x)} \\
 & \qquad\qquad -2x + 2 \\
 & \qquad\qquad \underline{-(-2x + 2)} \\
 & \qquad\qquad\qquad 0
\end{array}
$$

$(x-1)(2x^2 + 3x - 2) = 0$

$(x-1)(2x-1)(x+2) = 0$

$x = 1, \; x = \frac{1}{2}, \; x = -2$