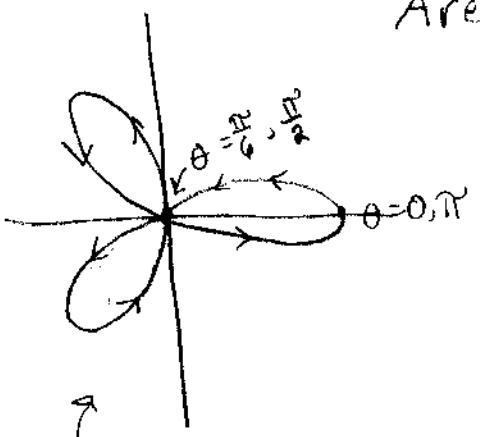
Polar Graphs Finding Area **Name:** Answers
AP Calculus BC

Calculator problems

1) Sketch a graph of $r = 6\cos(3\theta)$ and then find the area enclosed within one "petal" of the rose.

$$\text{Area} = \frac{1}{2}\int_a^b r^2 d\theta$$



$$= \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{\pi}{2}} (6\cos 3\theta)^2 d\theta = 9.425 \text{ sq units}$$

you can choose any petal & find its area, or find the total area & divide by 3.

2) Find the area of the region lying between the inner and outer loops of the limaçon $r = 2\sin\theta - 1$

Area = area of whole limacon − inner loop

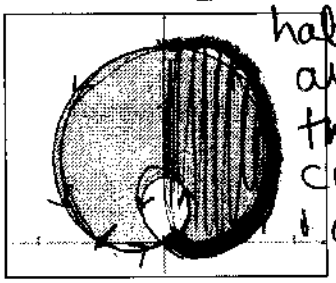
see (*)

$$\text{Area} = 2\cdot\frac{1}{2}\int_{\frac{5\pi}{6}}^{\frac{3\pi}{2}} (2\sin\theta - 1)^2 d\theta - \frac{1}{2}\int_{\frac{\pi}{6}}^{\frac{5\pi}{6}} (2\sin\theta - 1)^2 d\theta$$

$$= 8.88126\ldots - .543516\ldots$$

$$\approx 8.338$$

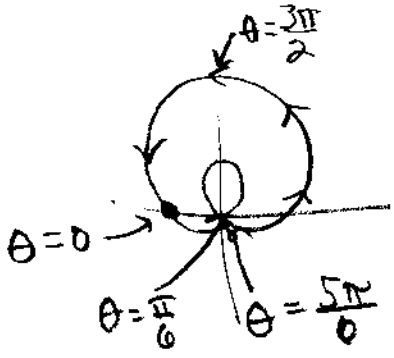
(*) I found half the area of the big curve & doubled it

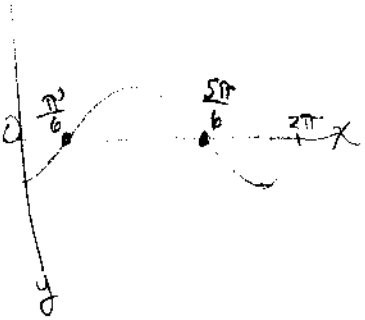


$r(0) = (-1, 0)$

$r(\frac{\pi}{2}) = (0, 0)$

r



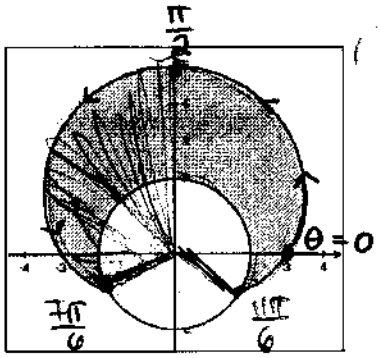


Big area − small area = shaded area

3) Find the area that is inside $r = 3 + 2\sin\theta$ and outside the circle $r = 2$

Find intersections: $2 = 3 + 2\sin\theta$

$-\frac{1}{2} = \sin\theta$ at $\theta = \frac{7\pi}{6}, \frac{11\pi}{6}$



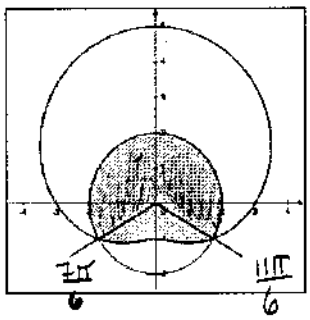
$$A = 2\cdot\frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{7\pi}{6}}(3+2\sin\theta)^2\,d\theta - 2\cdot\frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{7\pi}{6}}(2)^2\,d\theta$$

$$\text{or } A = \int_{\frac{\pi}{2}}^{\frac{7\pi}{6}}\left((3+2\sin\theta)^2 - 4\right)d\theta \approx \boxed{24.187}$$

4) Find the area that ~~is~~ common to ~~$r = 3 + 2\sin\theta$ and~~ the circle $r = 2$

Add up the areas

$$A = \frac{1}{2}\int_{\frac{7\pi}{6}}^{\frac{11\pi}{6}}(3+2\sin\theta)^2\,d\theta + 2\cdot\frac{1}{2}\int_{\frac{\pi}{2}}^{\frac{7\pi}{6}}(2)^2\,d\theta$$



$\approx 1.992893622 + 8.377580411$ Add up areas

$\approx \boxed{10.370}$

Useful trig id

$\cos^2\theta = \frac{1}{2}(1+\cos 2\theta)$