

POE - Maximizing Power- Activity 1.2.6

Speed Depends on Load Torque

When a motor is loaded with no torque,
it spins at no-load speed, ω_0 .
 ω_0 = no-load speed



When a motor is loaded with the stall torque
 τ_{stall} or more, the motor will stop.
 τ_{stall} = stall torque



The lower-case Greek letter omega is the symbol used for angular speed. Angular speed is measured in units of angle measure per time, like rotations per minute, revolutions per second, degrees per second, or radians per second. Radians per second is the SI unit for angular speed.

The lower-case Greek letter tau is the symbol used for torque. Torque is measured in units of distance times a force, like pound-inches, or newton-meters. Newton-meters is the SI unit for torque. Because the distance and force are in perpendicular directions, it cannot be simplified to units of energy; newton-meters is joules, however, when the force and distance are in the same direction.

Power

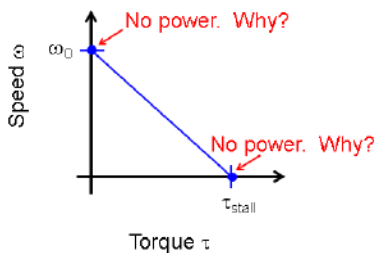
Power = Torque x Angular Speed

$$P = \tau \cdot \omega$$

Power is measured in units of energy per time, like joules per second, or BTU per hour. There are also many common units of power that do not explicitly show an energy per time, like horsepower, but they are still energy per time. The SI unit for power is the watt, equal to a joule per second, which – in agreement with the formula above -- is equal to newton-meters x radians/second.

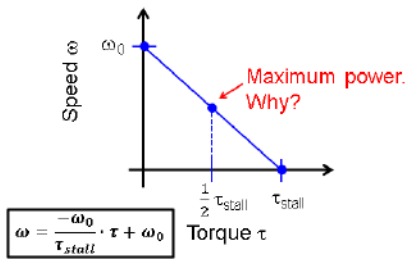
Torque and angular speed are vectors. Power is the dot product or scalar product. This formula, written without vector notation, makes the assumption that the torque and the angular rotation are around the same axes.

Power = Torque x Speed



Since power = torque x speed, power is zero at zero torque (no load speed), and power is zero at zero speed (stall torque).

Maximum Power at ½ Stall Torque



You'll get the most power out of your motor if it has to exert half its stall torque. Why? Again, you can understand the main point of this presentation without the algebra, you just won't know where it came from. If you want to know why...

Speed is linear with torque, with a negative slope.

speed = (- no-load speed/stall torque)(torque) + no-load speed

If you multiply speed by torque, you get a quadratic equation with a negative quadratic coefficient. The next two slides explain why this means that maximum power is at half the stall torque

Why half stall torque?

$$\omega = \frac{-\omega_0}{\tau_{stall}} \cdot \tau + \omega_0$$

Linear equation from last slide.

$$P = \left[\frac{-\omega_0}{\tau_{stall}} \cdot \tau + \omega_0 \right] (\tau)$$

Multiply both sides by torque, product on left is power.

$$P = \frac{-\omega_0}{\tau_{stall}} \cdot \tau^2 + \omega_0 \tau$$

Down-facing parabola.

VEX Application: Pick a Gear Ratio

Step 1. This motor is stalled. What is the stall torque?

$$\tau_{stall} = d \times F_{\perp}$$

$$= (3 \text{ in.})(1.4 \text{ lb})$$

$$= 4.2 \text{ lb}\cdot\text{in.}$$

VEX motor



Step 2. At what torque will the motor deliver maximum power?

$$\tau_{maxPower} = \frac{1}{2} \tau_{stall}$$

$$= \frac{1}{2} (4.2 \text{ lb}\cdot\text{in.}) = 2.1 \text{ lb}\cdot\text{in.}$$

VEX Application: Pick a Gear Ratio



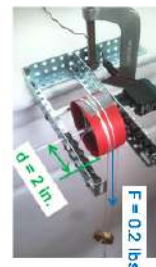
Step 3. The same motor as in the last slide is being used to power a winch with a drum of radius 2 in. lifting 0.2 lb. What torque is the motor applying?

$$\tau_{out} = d \times F_{\perp}$$

$$= (2 \text{ in.})(0.2 \text{ lb})$$

$$= 0.4 \text{ lb}\cdot\text{in.}$$

VEX Application: Pick a Gear Ratio



Step 4. Instead of direct drive, what gear ratio would make the motor deliver maximum power?

$$GR = \frac{\tau_{out}}{\tau_{in}} = \frac{0.4 \text{ in.}\cdot\text{lb}}{2.1 \text{ in.}\cdot\text{lb}}$$

$$= 0.19 \div \frac{12 \text{ teeth}}{60 \text{ teeth}}$$

VEX Application: Pick a Gear Ratio



Now the winch is geared so that the motor is delivering half its stall torque, for maximum power.

$$GR = \frac{\tau_{out}}{\tau_{in}} = \frac{0.4 \text{ in.}\cdot\text{lb}}{2.1 \text{ in.}\cdot\text{lb}}$$

$$= 0.19 \div \frac{12 \text{ teeth}}{60 \text{ teeth}}$$

The motor is connected to a 60-tooth gear, which is driving a 12-tooth gear that shares a drive shaft with the drum. The output torque of 0.4 lb-in. is driven by an input torque of 2.0 lb-in., very close to half of the motor's stall torque. If the winch were lifting a different weight, a different gear ratio would be needed for the motor to deliver its maximum power. The motor will deliver maximum power when it delivers 2.1 lb-in.