Lesson 4.1 Statistics - Overview

Preface

Statistics allow for informative decision-making formulated from theoretical and experimental data analysis. Data analysis has become so integrated into today's modern lifestyles that most individuals are unaware of its role in their daily lives. This can be seen in reality shows that base results upon contestants' national popularity and voting results. Box office movie rankings are based on weekend ticket sales. At an individual level, people use statistics for calculating gas mileage, green calculations such as average energy consumption, insurance rates, and calculating class rank and GPA.

In this lesson students will learn the processes of gathering, organizing, interpreting, and formulating an understanding of data.

Understandings

1. Engineers use statistics to make informed decisions based upon established principles.
2. Visual representations of data analyses allow for easy distribution and understanding of data.
3. Statistics is based upon both theoretical and experimental data analysis.

Knowledge and Skills

It is expected that students will:

1. Calculate the theoretical probability that an event will occur.
2. Calculate the experimental frequency distribution of an event occurring.
3. Apply the Bernoulli process to events that only have two distinct possible outcomes.
4. Apply AND, OR, and NOT logic to probability.
5. Apply Bayes' theorem to calculate the probability of multiple events occurring.
6. Create a histogram to illustrate frequency distribution.
7. Calculate the central tendency of a data array, including mean, median, and mode.
8. Calculate data variation, including range, standard deviation, and variance.

Essential Questions

1. Why is it crucial for designers and engineers to utilize statistics throughout the design process?
2. Why is process control a necessary statistical process for ensuring product success?
3. Why is theory-based data interpretation valuable in decision making?
4. Why is experiment-based data interpretation valuable in decision making?
   5. **Lesson 4.1 Statistics - Key Terms**

Term	**Definition**
Accuracy	Degree of conformity of a measure to a standard value.
Bar Chart	Categorical data graph.
Bayes' Theorem	The probability of an event occurring based upon other event probabilities.
Data	Numbers or information describing some characteristic.
Data Variation	Measure of data scatter.
Deviation	Amount of difference between a value and the mean.

Experiment	An activity with observable results.
Event	A subset of a sample space.
Frequency Distribution	Listing of data values along with their corresponding frequencies.
Frequency Polygons	Frequency distribution graph.
Histogram	Frequency distribution graph.
Mean	Arithmetic average.
Mean Deviation	Measure of variation equal to the sum of the deviations of each value from the mean.
Median	Middle value of a set of values arranged in order of magnitude.
Mode	The value that occurs most frequently.
Normal Distribution	Bell-shaped probability distribution.
Outcome	The result of an experiment.
Pie Chart	Categorical data graph %.
Probability	The calculated likelihood that a given event will occur.
Process Control	To monitor and control a process so that the quality of the output/product improves.
Qualitative Data	Values that possess names or labels.
Quantitative Data	Values that represent a measurable quantity.
Quality Assurance	The use of quality control techniques associated with a process.
Reliability	The probability of satisfactory operation of the product in a given environment over a specified time interval.
Sample Space	A set of all possible outcomes or events in an experiment that cannot be further broken down.
Standard Deviation	The square root of the variance.
Statistics	The collection, evaluation, and interpretation of data.
Statistical Process Control	SPC is a method of monitoring, controlling, and ideally improving a process through statistical analysis. Its four basic steps include measuring the process, eliminating variances in the process to make it consistent, monitoring the process, and improving the process to its best target value.
Tolerance	The difference between the maximum and minimum dimensions allowed within the design of a product.
Variance	The difference between samples.

Methods of Determining Probability

Probability

The calculated likelihood that a given event will occur

Empirical

Experimental observation

Example – Process control

Theoretical

Uses known elements

Example – Coin toss, die rolling

Subjective

Assumptions

Example – I think that . . .

Probability Components

Experiment
An activity with observable results

Sample Space
A set of all possible outcomes

Event
A subset of a sample space

Outcome / Sample Point
The result of an experiment

Probability

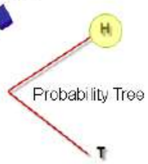
What is the probability of a tossed coin landing heads up?

Experiment

Sample Space

Event H

Outcome



Relative Frequency

The number of times an event will occur divided by the number of opportunities

$$f_x = \frac{n_x}{n}$$

f_x = Relative frequency of outcome x

n_x = Number of events with outcome x

n = Total number of events

Expressed as a number between 0 and 1

fraction, percent, decimal, odds

Total frequency of all possible events totals 1

Probability

What is the probability of tossing a coin three times and it landing heads up exactly two times?

How many desirable outcomes? 3

How many possible outcomes? 8

$$P_x = \frac{f_x}{f_a} \quad P = \frac{3}{8} = .375 = 37.5\%$$

Bernoulli Process

$$P_x = \frac{n!}{x!(n-x)!}(p^x)(q^{n-x})$$

$$= {}_nC_x \; p^x q^{n-x}$$

P = Probability

x = Number of times for a specific outcome within n trials

n = Number of trials

p = Probability of success on a single trial

q = Probability of failure on a single trial

! = factorial – product of all integers less than or equal

Law of Large Numbers

The more trials that are conducted, the closer the results become to the theoretical probability

Trial 1: Toss a single coin 5 times

H,T,H,H,T

P = .600 = 60%

Trial 2: Toss a single coin 500 times

H,H,H,T,T,H,T,T,……T

P = .502 = 50.2%

Theoretical Probability = .5 = 50%

Probability

OR (Addition)

What is the probability of rolling a 4 on a single die?

How many desirable outcomes? 1

How many possible outcomes? 6

$P_4 = \frac{1}{6}$

What is the probability of rolling a 1 on a single die?

How many desirable outcomes? 1

How many possible outcomes? 6

$P_1 = \frac{1}{6}$

What is the probability of rolling a 4 or a 1 on a single die?

$P = P_4 + P_1 = \frac{1}{6} + \frac{1}{6} = \frac{2}{6} = .3333 = 33.33\%$

Probability

What is the probability that the first card is an ace?

$\frac{4}{52} = \frac{1}{13} = .0769 = 7.69\%$

Since the first card was NOT a face, what is the probability that the second card is a face card?

$\frac{12}{51} = \frac{4}{17} = .2353 = 23.53\%$

Since the first card was NOT a ten, what is the probability that the second card is a ten?

$\frac{4}{51} = .0784 = 7.84\%$

Bayes' Theorem

Calculates a conditional probability, based on all the ways the condition might have occurred.

P(A | E) = probability of A, given we already know the condition E

$$= \frac{P(E|A)\cdot P(A)}{P(E|A)\cdot P(A) + P(E|B)\cdot P(B) + P(E|C)\cdot P(C)}$$

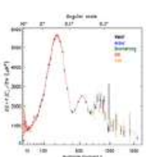
Bayes' Theorem Example

LCD screen components for a large cell phone manufacturing company are outsourced to three different vendors. Vendor **A**, **B**, and **C** supply **60%**, **30%**, and **10%** of the required LCD screen components. Quality control experts have determined that **.7%** of vendor **A**, **1.4%** of vendor **B**, and **1.9%** of vendor **C** components are defective.

If a cell phone was chosen at random and the LCD screen was determined to be defective, what is the probability that the LCD screen was produced by vendor **A**?

Statistics

The collection, evaluation, and interpretation of data



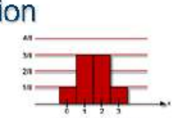
Statistics

Statistics

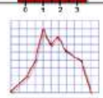
- Descriptive Statistics
  - Describe collected data
- Inferential Statistics
  - Generalize and evaluate a population based on sample data

Graphic Data Representation

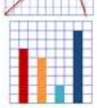
Histogram
  Frequency distribution graph



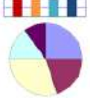
Frequency Polygons
  Frequency distribution graph



Bar Chart
  Categorical data graph



Pie Chart
  Categorical data graph %



Measures of Central Tendency

Mean $\bar{x}$

Arithmetic average

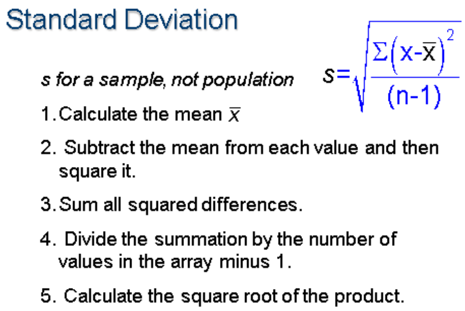
Sum of all data values divided by the number of data values within the array

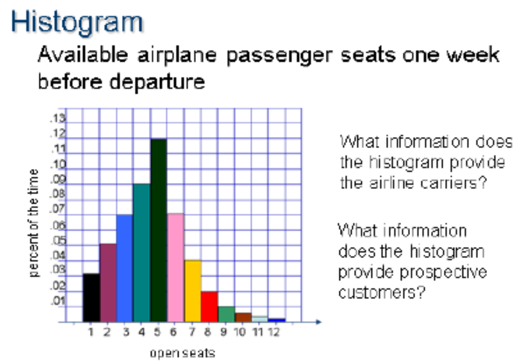
$$\bar{x} = \frac{\Sigma x}{n}$$

Most frequently used measure of central tendency

Strongly influenced by outliers – very large or very small values

Median and Modw





RANGE

Lesson 4.2 Kinematics - Overview

Preface

While statics is concerned with bodies at rest or moving at a constant acceleration, dynamics is concerned with the accelerated motion of bodies. The study of dynamics developed much later than statics because of the need for accurate measurement of time. Galileo Galilee was a major early contributor, performing experiments with pendulums and falling bodies. Newton's development of the three fundamental laws of motion was the springboard for increased understanding and work by other scientists. The two major braches of dynamics are kinematics, which is concerned with the geometric aspects of motion, and kinetics, which is concerned with the forces causing the motion.

In this lesson students will create a vehicle to learn important aspects of motion and freefall. Students will perform an activity that will help them to understand the kinematics concepts involved in projectile motion.

Understandings

1. When working with bodies in motion, engineers must be able to differentiate and calculate distance, displacement, speed, velocity, and acceleration.
2. When air resistance is not taken into account, released objects will experience acceleration due to gravity, also known as freefall.
3. Projectile motion can be predicted and controlled using kinematics equations.
4. When a projectile is launched, velocity in the x direction remains constant; whereas, with time, the velocity in the Y direction in magnitude and direction changes due to gravity.

Knowledge and Skills

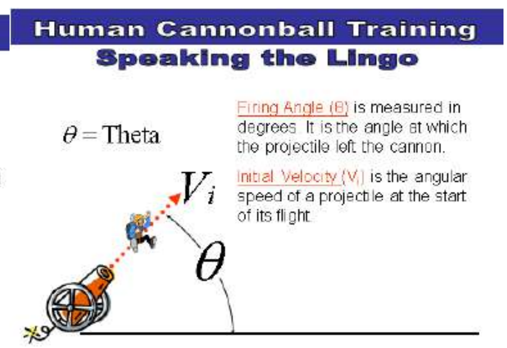
It is expected that students will:

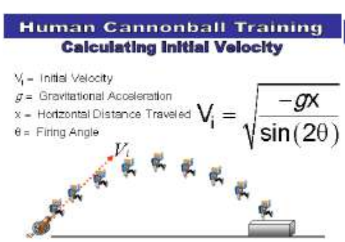
1. Calculate distance, displacement, speed, velocity, and acceleration from data.
2. Design, build, and test a vehicle that stores and releases potential energy for propulsion.
3. Calculate acceleration due to gravity given data from a free fall device.
4. Calculate the X and Y components of a projectile motion.
5. Determine the needed angle to launch a projectile a specific range given the projectile's initial velocity.

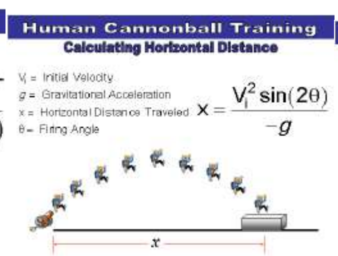
Essential Questions

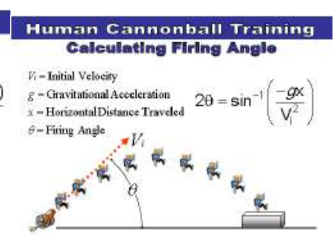
1. What are the relationships between distance, displacement, speed, velocity, and acceleration?
2. Why is it important to understand and be able to control the motion of a projectile?
3. **Lesson 4.2 Kinematics - Key Terms**

Term	Definition
Acceleration	The rate of change of velocity with respect to time.
Free Fall	The condition of unrestrained motion in a gravitational field.
Distance	The total length of path over which the particle travels.
Displacement	A vector quantity giving the straight-line distance and direction from an initial position to a final position.
Velocity	A vector quantity that includes the speed and direction of an object.
Speed	The magnitude of the total distance traveled divided by the time elapsed.









Distance (D) is the total length of the path an object takes.

Displacement (x or y) is the change from initial to final position, including magnitude and direction.

Speed (s) is the change from initial to final position (d) divided by the time (t) elapsed, and only includes magnitude.

$$s = \frac{d}{t}$$

Velocity (V) is the displacement (d) divided by the time (t) elapsed and direction of an object.

$$v = \frac{d}{t}$$