Physics Summary Sheets

AP Physics

International Baccalaureate High Level

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Mechanics *Pg 1*

Force (v) *a push or a pull* *©pfreda@gmail.com 4/4/2023*

Gravitational, ElectroMag,Tension, Compression, Shear, (v) = vector

Buoyant, Friction, Normal, Fluid Resistive (s) = scalar

[in gases due to pressure, in liquids to viscosity]

A Force is required to change the speed or direction of a moving mass

Energy (s) *the capacity to do Work* Units of Joules = N-m

Types; Kinetic [KE], Potential [PE], Thermal/Heat [Q], Nuclear

Sources; Gasoline, Natural Gas, Solar, Chemical

Work (s) *Transfer of Energy between macroscopic bodies*

W = Force dot Distance = F d cos θ [Component of Force in d direction]

Power (s) *Energy [or Work] per unit time* Units Watt = Joule/sec

Displacement (v) Change in position. Net distance travelled.

Gravity natural phenomenon, all physical bodies (mass) attract each other

Universal Law of Gravitation $F_{(ULG)} = G\, m_1\, m_2 / r^2$

[Newton's ULG] $G = 6.67 \times 10^{-11}\ N\ m^2\ kg^{-2}$

Newton's Laws apply only to inertial frames of reference (No Accel)

Accelerating frames create fictitious Forces due to Inertia

Newton's Laws of Motion *"….united the Heavens and Earth"*

1st Law Inertia $F = 0 \rightarrow a = 0 \rightarrow v = constant$

The tendency of mass is to resist a change in motion/velocity.

Nature's natural state is NOT rest, but constant velocity

Inertia is Property of Mass Inertia is NOT a Force

But … it "feels like" a Force in the Acceleration Frame

If the velocity of a body is not constant, then a net Force must be acting upon it. This is how nature works in the absence of External Forces such as Gravity or Friction

2nd Law $\Sigma F = Net\ Force = m\,a \qquad \Sigma F = dp/dt$

$\Sigma F = d/dt\,[m\,v] = m\ dv/dt + v\ dm/dt$

$\sum$ Vertical Forces = $m\,a_v$; $\sum$ Horizontal Forces = $m\,a_H$

The sum of the Forces [net force] equals the rate of change of momentum [dp/dt = d/dt (m v)] = m a (if m is constant)

The acceleration of a body, not velocity, is proportional to the unbalanced / net Force acting on it.

Note; a = 0 if the objects are stationary or at constant velocity

Can use 2nd Law = the ULG to show that a marble and Bowling ball will fall at the same acceleration and speed to Earth

But ….. with air resistance, heavy objects fall faster

3rd Law $F_{AB} = -F_{BA}$ **[action → equal & opposite reaction]**

For every action there is an equal and opposite reaction

Forces are always created in pairs …… No Isolated Forces exist

The Forces act on ***two different bodies AND are = & opposite***

They should NOT be thought to balance or cancel

and they both must be contact or non-contact Forces

This Law explains why rockets work in the vacuum of outer space

Ex; 1/ Gun recoil; bullet & gun. 2/ Walking; Friction Person ←→ ground

Conservation Laws *Energy can neither be created nor destroyed*

1/ Energy $\Delta ME = 0$ *ME = Mechanical Energy* $ME = KE + PE$

Kinetic Energy = $KE = \frac{1}{2}\, m\, v^2$ = *Energy of translational* ***motion***

Potential Energy = $PE = m\,g\,h$ *= Energy doing work against a Conservative Force*

Σ *KE + PE (before) = Σ KE + PE (after)*

2/ Momentum $\Delta p = 0 \qquad p = mv$ (assumes no external Forces)

$\Sigma\, m\, v$ *(before) = $\Sigma\, m\, v$ (after)*

The Sum of the Initial and Final Momentums of the combined 2 bodies must be equal ; assuming an isolated system and ***no external forces***

3/ Angular Momentum L $\Delta L = 0$ (no external Torques)

Point Mass $L = r \times p = m\,v\,r = I\,\omega$ $\Sigma\, mvr$ *(before)* = $\Sigma\, mvr$ *(after)*

Work – Energy Theorem $W = \Delta KE$ **or** $W = \Delta ME$

Work done ON or BY the system = $W = \Delta ME$ **if all Forces are Conservative**

Free Body Force Diagrams Picture description of the Force Vectors

All Force vectors act on ONLY ONE body. Example; A ladder leaning on a wall

5 forces act on it; Weight, 2 friction (wall, floor), 2 normal (wall, floor)

Projectile Motion Parabolic trajectory

Vertical & Horizontal Motion are independent of each other

Vertical motion is *constant acceleration* a = – g

Horizontal motion is *constant velocity* → a = 0

Max Range → θ = 45 deg; Range = $v^2 \sin 2\theta / g$

Position Equation for both vertical and horizontal motion

$s(t) = s_0 + v_0\, t + \frac{1}{2}\, a\, t^2$

s_0 = initial position , v_0 = initial velocity, t = time

Derived by Integrating $(d/dt)^2$ y(t) = a two times

Suvat Equation v = final velocity, u = initial velocity, a = constant

$v^2 - u^2 = 2\,a\,s$; 1/ $s = \frac{1}{2}(u + v)\,t$; 2/ $a = \frac{v - u}{t}$

Solve for "t" in 1 and substitute in 2 or visa versa

Circular Motion / Constant Speed

Circular motion requires a Centripetal Force & acceleration to change the tangential motion to circular motion.

Supplied by either .. 1/ Gravity 2/ Tension 3/ Friction 4/ banked curve

$F(centripetal) = m\,v^2/r$ = F(gravity) or F(tension) or F(friction)*

Examples; Ball swinging on string - String; Orbits – Gravity;

Turning Car – Friction between tires & road ; Satellites – Gravity

Centripetal **Acceleration** $a = v^2/r = \omega^2 r$

Centripetal **Force** $F = m\,a = m\,v^2/r = m\,\omega^2 r$

Tangential **Velocity/Acceleration** $v_T = \omega\, r \qquad a_T = \alpha\, r$

v_T = Circumference / Period $v_T = 2\,\pi\, r / T$

a & F radial vectors toward the center or axis of rotation

v_T & a_T vectors in the tangential direction of motion

ω & α angular vectors [perpendicular to plane of rotation]

T = Period of Rotation/Revolution $f = 1/T$ = frequency

Satellite Physics …… Projectile Motion never reaching Earth

Potential Energy ***PE = Work done against a Conservative Force***

Negative work is done from ∞ to R by the Field/Force [not against it] due to attraction

** **PE** = $\int F\, dr = \int GMm/r^2\, dr = -GMm/r$ (PE = 0 at infinite r) $PE = -GMm/r$

** **F(cent) = F(G)** ; $m\,v^2/r = GMm/r^2 \rightarrow m\,v^2 = GMm/r \rightarrow KE = GMm/2r$

Total Mechanical Energy **TE** = PE + KE = $-GMm/r + GMm/2r = TE = -GMm/2r$

** For Escape ; **KE = PE** → $\frac{1}{2}\, m\, v^2 = GMm/r$ → v[escape] = Sqrt [2GM/R]

** For Orbit ; **F(cent) = F(G)** ; $m\,v^2/r = GMm/r^2$ → v [orbit] = Sqrt [GM/R]

v[orbit] = Sqrt [GM/R] **v[escape] = Sqrt [2GM/R]**

Satellites exist in a negative potential well with KE < PE

Lower orbit satellites have > KE but lower TE ; If TE = 0 or > 0 the object will escape

Work against Gravity is needed to go to a larger radius

Elliptical Motion / Orbits F(Centripetal) = F(Gravitation)

Orbital motion is a form of *Projectile Motion*

Velocity is not constant Conservation of Angular Momentum =>

L = m v r = constant because r x F = τ = 0 at all pts → m1 v1 r1 = m2 v2 r2

$F(centripetal) = m\,v^2/r = m\,\omega^2\, r = F_{(ULG)} = G\,M\,m / r^2$

$v^2 = G\,M / r$; $\omega = 2\pi/T$ $a = \omega^2 r = 4\pi^2 r / T^2 = G\,M / r^2$

Kepler's 1st Law / Orbits Planetary orbits are ellipses

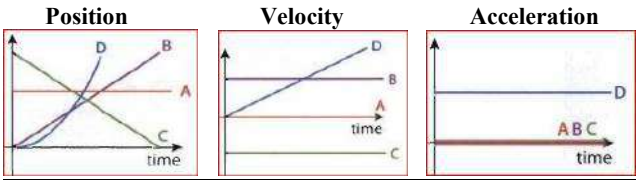
Kepler's 2nd Law / Areas Equal areas swept in equal times

Kepler's 3rd Law/ Periods $T^2 / a^3 = 4\,\pi^2 / G\,M = constant$

"a" = half of Elliptical Major Axis or Radius of Circle

Radius (r) / 1/2 Semi-major axis (a) ← → Period T

Displacement / Velocity / Acceleration Time Graphs



(slope) differentiate → *← integrate (area)*

The area under the y = f(x) graph is "y times x"

To go from one graph to the next one ……

Find ***Slope*** of the tangent line at that point OR ***Area*** under the curve to that point

Area under curve; v → s , a → v ***Slope*** of tangent line; s → v ; v → a

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Linear Momentum $p = m v$ ***Collisions // No potential energy***

***Elastic*; $\Delta p = 0$ & $\Delta KE = 0$** Momentum & Energy are Conserved

Inelastic*; $\Delta p = 0$** Momentum ***ONLY is Conserved

Elastic hard objects ; No energy is lost

Inelastic objects crush or stick together

Energy is lost to heat, sound and vibration

If the objects stick together it is called "perfectly inelastic"

Explosions are always inelastic, the reverse of a perfectly inelastic collision.

The small bits get most of the KE from the chemical energy.

Conservation Laws $\sum m v_{initial} = \sum m v_{final}$ & $\sum KE_{initial} = \sum KE_{final}$

Impulse $J = \Delta p = F \Delta t$ $\Delta p = \Delta mv = m \Delta v = m a \Delta t = F \Delta t$

Newton's 2nd Law $F = dp/dt = m\, dv/dt + v\, dm/dt = ma + v\, dm/dt$

For 1 Dim Elastic Collisions ONLY ***Easier to use than the KE equation***

Sum of initial and final velocities for each mass are equal ***for Elastic Collisions***

$v1 + v1' = v2 + v2' = [(2m1)\, v1 + (2m2)\, v2] / [m1 + m2]$ ***..and ..***

Relative velocities $v1 - v2 = -(v1' - v2')$ *equal & opposite, before & after*

Net momentum change = Impulse is equal and opposite for the masses

$\Delta p = 0$ Total change p = 0 ➔ $m1 [v1' - v1] = -m2 [v2' - v2]$

Classic 1 Dimensional Elastic problem mass 1 collides w mass 2

v ' = v(final) v = v(initial)

$v1' = [(m1 - m2)/(m1 + m2)]\, v1 + [(2m2)/(m1 + m2)]\, v2$

$v2' = [(2m1)/(m1 + m2)]\, v1 + [(m2 - m1)/(m1 + m2)]\, v2$

If v2 = 0 the 2nd mass / object is stationary ……

$v1' = [(m1 - m2)/(m1 + m2)]\, v1$

$v2' = [(2m1)/(m1 + m2)]\, v1$

Friction ➔ ***Friction ALWAYS OPPOSES motion*** ➔

1/ A Force that resists the sliding or rotating motion

2 Kinds; Static/Starting Friction [Stiction] & Kinetic Friction

Both are proportional to the Normal Force $F_{fr} = \mu F_N$ μ = coefficient of friction

2/ A 3rd Law reaction Force that propels a car or person to move forward in reaction to the force backwards by the tire or foot.

Normal Force ➔ ***Always Perpendicular to the surface*** ➔

A Contact Force …… ***But not always = m g !***

Elevator Acceleration = a $F = mg \pm ma$

You weigh more on an elevator accelerating up or decelerating on its way down; Weight = $F = mg + ma$

You weigh less on an elevator accelerating down or decelerating on its way up; Weight = $F = mg - ma$

Elevator in free fall [a = g] ➔ Wt = F = 0 ➔ weightlessness

Einstein's Equivalence Principle of Gravity and Acceleration

1/ Acceleration is equivalent to and thus appears to create a "pseudo" Gravitational Force ***in the opposite direction*** of the acceleration. In the Inertial Frame it is an ***INERTIAL FORCE***.

2/ Free Fall is equivalent to Rest in a Mass/Gravity free region

Both are Inertial Frames of Reference; Newton's Laws hold.

Terminal velocity of a falling object is the velocity of the object when the sum of the upward drag force (prop to v^2) and upward buoyancy force equals the downward force of gravity. Since the net force on the object = 0, the object has no acceleration

Hooke's Law Springs Mass-Spring Simple Harmonic Oscillator

$F[Restoring] = -k x$ [1676 Robert Hooke]

Restoring Force is proportional to spring compression/stretching

PE = Work done compressing/stretching = $\int F\, dx = \int k x\, dx = \frac{1}{2} k x^2$

$PE = \frac{1}{2} k x^2$ spring energy stored = Area under the F vs x curve

Young's Modulus E ***Stress σ to Strain ∈ Ratio***

$E = \sigma / \epsilon = [F/A] / [\Delta L / L]$; for the linear region …

Hooke's Law constant 'k' = $E\, A / L = F / \Delta L$

Equilibriums $\sum F = 0$

1/ Translational Sum of Forces = 0

No Acceleration or unbalanced forces. But the body can still be in motion at constant speed …… Skydiver falling at terminal velocity speed is in TE.

2/ Rotational Sum Torques = 0

3/ Static Sum of forces = 0 & Sum of Torques = 0

Object in SE can be both spinning at constant speed and have its center of mass moving linearly at constant speed.

Rotational Motion Torque = Rotational Force

Position Equation for Rotating motion

$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$

θ_0 = initial position , ω_0 = initial velocity, t = time

Suvat Equation for Rotating Motion

$\omega_f^2 - \omega_0^2 = 2 \alpha \theta$ $v = \omega r$ $x = \theta r$

$KE_{(rotation)} = \frac{1}{2} I \omega^2$ $a = \alpha r$

Linear – Rotational		Parameter Analogs	
x = displacement	$\theta = x / r$	θ	angle
v = dx/dt	$\omega = d\theta/dt$	ω	angular velocity
a = dv/dt	$\alpha = d\omega/dt$	α	angular acceleration
F = ma	$\tau = I \alpha = r \times F$	τ	Torque
F = dp/dt	$\tau = dL/dt$	τ	Rate of Change of Momentum
p = mv	$L = I \omega = r \times p = mvr$	L	Angular momentum
$KE = \frac{1}{2} mv^2$	$KE = \frac{1}{2} I \omega^2$	KE	Mechanical Energy
W = F dot d	$W = \tau$ dot θ	W	Work

Moment of Inertia = Rotational Inertia = Angular Mass

For a point mass M, $I = M R^2$ R = distance from axis

Moment of Inertia $I = \sum m r^2 = \int r^2\, dm = \rho \int r^2\, dV$

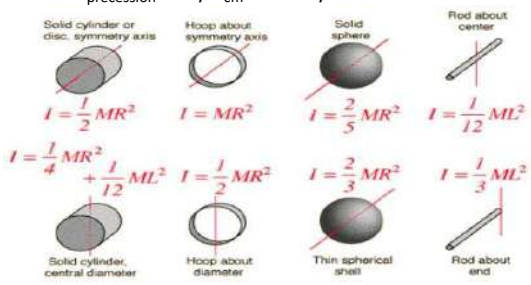
Parallel Axis Theorem $I' = I + m d^2$ d= distance CofM to new axis

Perpendicular Axis Theorem $I(z) = I(x) + I(y)$

Precession ; L vector "chases" the τ (torque) vector

Rotation of the Axis of rotation due to an external Force / Torque

$\omega_{precession} = \tau / L_{cm} = r \times F / I \omega$



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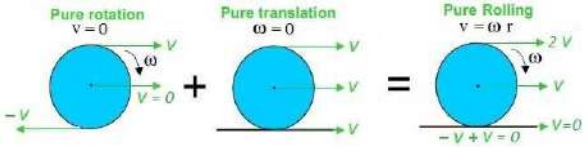
See Wikipedia page .. / Listings_of_Moments_of_Inertia

Rolling Motion ➔ $v = \omega r$ **v = velocity CofM**



Rotation Axis = Center No Axis Rotation Axis = Bottom

$KE = \frac{1}{2} I \omega^2$ $KE = \frac{1}{2} m v^2$ $KE = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$

Pure Rolling …. Rotation axis is bottom of wheel. The top of a car's wheel moves twice as fast, 2x, as the car itself, its CofM ! ! ! $\frac{1}{2} I \omega^2 = \frac{1}{2} I (v/r)^2$

Pure Rotation ***(spinning)*** => ***v[top] = v ; v[CM] = 0 ; v[bottom] = −v***

+ Pure Translation ***(sliding/slipping)*** => ***v[top] = v[CM] = v[bottom] = v***

= Pure Rolling => ***v[top] = 2v ; v[CM] = v ; v[bottom] = 0*** $v = \omega r$

Inclined plane Two objects ***Rolling or Sliding*** down plane of ∠ β

Objects w different mass, radius, density, length OR geometry ….

Rolling	Solid Cylinder	$a = 2/3\, g \sin\beta$	0.67	$I_C = \frac{1}{2} MR^2$
Rolling	Hollow Cylinder	$a = 1/2\, g \sin\beta$	0.5	$I_C = 1\, MR^2$ [same as point mass]
Rolling	Solid Sphere	$a = 5/7\, g \sin\beta$	0.7	$I_C = 1\, MR^2$
Rolling	Hollow Sphere	$a = 2/5\, g \sin\beta$	0.4	$I_C = 1\, MR^2$
Sliding	Solid or Hollow	$a = g \sin\beta$ [No Friction]		$a = g(\sin\beta - u \cos\beta)$ [With Friction]

s(t) v(t) & a(t) depend ONLY on geometry ∠ β , and ***moment of inertia*** I_C

A race down the plane, either rolling or sliding, ***is a tie*** *as long as both cylinders are* ***hollow or solid****. The solid cylinder will always beats the hollow one*

Incline Motion is same as Projectile, Orbital, Pendulum & Free Fall motion

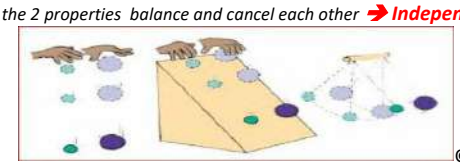
The position, velocity and acceleration are independent of Mass, Radius, Density or Length ! ! Highly non intuitive

Explanation Sliding; ….. Like Dropping ; Diff mass ➔ Same acceleration

Larger Mass ➔ More ***Force****/acceleration, F = mg ….* ***BUT*** *…*

Larger Mass ➔ More ***Inertia*** */ Newton's 1ST Law ➔ Resists acceleration*

So the 2 properties balance and cancel each other ➔ ***Independence of Mass***



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Simple Machines

Mechanical Advantage; Output / Input of Force, Distance, or Speed. Increase/Decrease [or Magnify] Force, Distance or Speed by a tradeoff due to the *Conservation of Energy*

Lever ; Lever Trades distance for reduction in Force; Force or Distance magnifier

Levers Types / Classifications *"what is in the middle ? "*

1/ Pivot [Fulcrum]; crowbar, scissors

2/ Load ; wheelbarrow, nutcracker, bottle opener

3/ Effort ; tweezers, shovel

Inclined Plane ; Trades distance for Effort/Force. Knife/Blade/Axe; are double inclined planes

Wedge ; trades lateral force for longitudinal force

Screw ; is an Inclined Plane wrapped around a shaft Trades/Converts Rotational motion for/to Linear motion

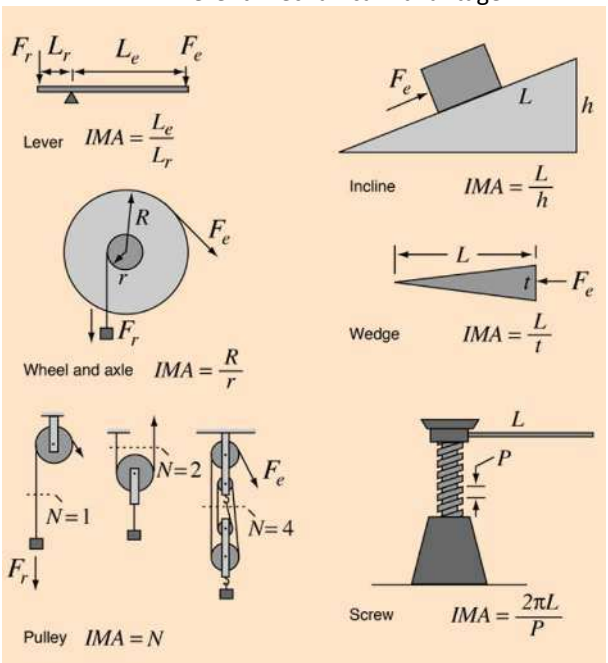
Scissors ; Type 1 Lever, Force magnifier uses torque [force] away from pivot

Gears ; Force Distance Speed magnifiers

Wheel & Axel ; Continuous Rotational Lever using torque [handle to axel] Arm is a distance magnifier Ex; bike handle or pedal, car steering wheel, doorknob

Wheel Barrow ; Type 2 Lever with load in between the pivot and the force

Pulleys; Single fixed allows pulling down rather than up Mechanical advantage = # ropes lifting the load x 100% 3x, 4x, etc reduce force needed by factor of 3, 4, etc Force magnifier ; Smaller force moves the longer distance Friction, rope & pulley mass reduces multiplier < 100% IMA = Inherent Mechanical Advantage



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Gravitational Mass; the property of mass concerning G Field

1/ Active Gravitational Mass ➔ ***creates the Field*** / attractive Force

2/ Passive Gravitational Mass ➔ ***feels the Field*** / attractive Force

Inertial Mass; property of mass that resists a change in velocity resists a change in motion. More Mass ➔ More resistance

***Mass has both properties** ➔ **motion is independent of mass**

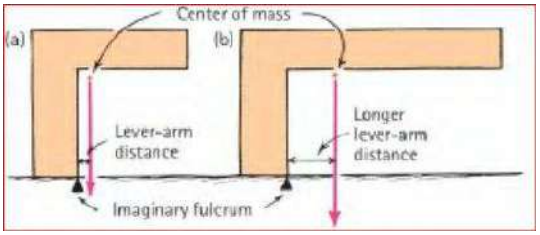
Stability of Objects **Centers of Mass (CM) & Gravity (CG)**.

Instability occurs when the vertical force due to CM is ***outside the base*** of the object. This results in a torque that tends to rotate the object and cause it to fall over.

Objects with ***wide bases*** and ***low CG*** are more stable

If CM is not over the base, the Level Arm length OR distance to the pivot point is a measure of instability

Larger the Lever Arm ➔ Greater the instability



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To find CM, suspend object from a point. CM lies on the vertical line to ground. Repeat for another point and the intersection is CM. You should be able to balance the object from the CM.

** Since 'g' decreases with height and mass does not ➔

➔ Center of Gravity CG is below CM for large objects ….

Empire State Bldg CG is 1 mm below its CM

CM can be located *outside the boundaries* of the mass

This is why the upside down Fosbury Flop high jump works so well. The high jumper can get his body over the bar while his CM is under the bar. So he does less work raising his CM / Weight.

Stability of Ships;

Differs from Stability of Objects due to the additional Buoyant Force.

FB Buoyant Force is **UP** ; FG Gravity Force is **DOWN**

CB = Center of Buoyancy = CM Center of Mass of the displaced H_2O

CG = CM Center of Mass of the Ship

For Stability; CG ***must be under*** the CB ***& as low as possible***

CG Force is down & CB Force is up. When aligned, they cancel

If not vertically aligned ➔ torque

The ***ship will rotate*** until the 2 CM's line up vertically ➔ no torque

CG above CB **causes unstable rotation & ship to *capsize*,**

CB above CG **causes stabilizing restoring torque & stability.**

Standing up in a boat raises CG above CB ➔ instability

Fundamental Forces; Gravity, Electromagnetic, Weak, Strong Nuclear.

Non Fundamental Forces; Normal, Friction, Tension, Compression, Elastic, Buoyant, Bernoulli.

Conservative Forces; Gravity, Electromagnetic, Spring Energy exchanges between Kinetic & Potential Energy. Work is independent of path.

Non Conservative Forces; Friction, Stiction, Tension, Compression, Drag

Ordinary differential equations (particles)

Partial differential equations (fields)

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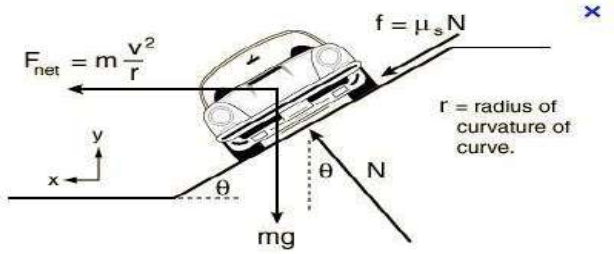
Banked Curve Analysis ….

Two Forces; Gravity and Friction

Decompose each force to X and Y components

Sum the forces in horizontal and vertical directions



Force equations at maximum speed v, at threshold of sliding up incline.

$$\Sigma F_x = m\frac{v^2}{r} = N \sin\theta + \mu_s N \cos\theta$$

$$\Sigma F_y = 0 = N \cos\theta - \mu_s N \sin\theta - mg$$

Solving this pair of equations for the maximum speed v gives:

$$v_{max} = \sqrt{\frac{rg(\sin\theta + \mu_s \cos\theta)}{\cos\theta - \mu_s \sin\theta}}$$

The limiting cases are:

$v_{max} = \sqrt{rg \tan\theta}$ Frictionless case

$v_{max} = \sqrt{rg\,\mu_s}$ Flat roadway

carbank.gif

Weightlessness "feeling" no Force on you; absence of Normal F

1/ Being in a region with no nearby mass/gravity

2/ Having ***only the Force of Gravity*** acting on you

3/ Being in a room in free fall so you feel no normal upward force.

4/ No Normal Force up ➔ You ***DO NOT*** feel your own weight, mg.

Angular Momentum & Torque

Any mass with Linear Momentum about a point (axis of rotation) will also have an Angular Momentum

Angular Momentum = Linear Momentum x Radius

$$L = p \times r = m v \times r$$

r = Radius = Perpendicular distance between object and the point (axis) p = Linear momentum

L = Angular Momentum v = linear velocity of object

$$\text{Magnitude of } L = ||L|| = m v r \sin\theta$$

θ angle between r and v vectors

Conservation of Angular Momentum

For rotation of a mass $|L(\text{center})| = m\,|\,r \times v\,| = m r v$

L = Spin angular momentum

L is conserved ONLY about the center point

$dL/dt = dr/dt \times p + r \times dp/dt$ [dp/dt = F]

$dL/dt = r \times F = \tau$ [Torque] ; **$\tau = 0$ ➔➔ $dL/dt = 0$**

Conservation of Angular Momentum ➔➔ $dL/dt = 0$

If r and F are parallel $\sin\theta = 0$ so no torque

[analogous to d/dt [mv] = m dv/dt = m a = F]

Angular momentum L of a system will not change if no net external torque [r x F] is present

L will be Conserved.

Ice skater spins faster as she brings her arms / mass to the center which reduces the Moment of Inertia

$L = I\,\omega$, Since L is constant, if I goes down ➔ ω goes up

Rigid Bodies & Oscillation

Spin Angular Momentum L is an intrinsic property of Rotating mass ……. just as Inertia is of Linear mass

$\tau = r \times F$ $\alpha = \theta''$ $\omega = \theta' = d\theta/dt$

$\tau = I\,\alpha$ $\tau = r \times F = r \times m a = m(r \times a)$

$L = r \times p$ $L = I\,\omega$ $dL/dt = \tau$ [external]

$L = r \times p = r \times m v = m(r \times v)$

Conservation of Angular Momentum ➔ $dL/dt = 0$

Conservation of Linear Momentum ➔ $dp/dt = 0$

Rod on frictionless surface; spin it about its c of m

If F= 0 then τ (any point) = 0

L = same relative to any point

$L_{cm} = I\,\omega = [\,\tfrac{1}{2} m l^2\,]\,\omega$ [I from Moment tables]

intrinsic spin angular momentum

Ruler [l=2d] lying on a frictionless surface

hit by Impulse I at its end.

C of M moves along line of direction of I vector

$I = F\,\Delta t = M a\,\Delta t = M v$ ➔ $v_{cm} = I / M$

L [before] = 0 t not 0 ➔ L [after] not zero

velocity of c of m not f(where I hits the ruler)

but L is affected

$\tau = 0$ L conserved 0 before and 0 after

Hanging ruler pin at point p ; forces thru pt p

b = distance C of M to p ; l = length

$\tau = r \times F = M g b \sin\theta = -I\,\alpha$

rp vector[center to p] x mg thru pt c (c of m)

$Mg b t \theta + I\,\theta'' = 0$

$\theta'' + [Mgb/I]\,\theta = 0$ ➔ SHM

$\theta(t) = \theta_{rest} \cos\omega t + \phi$ $\omega = \text{Sqrt}[Mgb/I]$

$T = 2\pi\,\text{Sqrt}\,[I / Mgb]$

Hanging Hula Hoop

$\tau = M g R \sin\theta$ $I = MR^2 + \text{same} = 2MR^2$

$T = 2\pi\,\text{Sqrt}[\,2MR^2 / M g R]$

$T = 2\pi\,\text{Sqrt}[2R/g]$

Period for both ruler and Hula is only dependent on geometry. This is highly non intuitive as one would expect mass to affect Period.

Just as with falling mass or a simple pendulum …..

$T = 2\pi\,\text{Sqrt}\,[L/g]$ [L = length of pendulum]

Comparison of Swing Period T for different objects

$T = 2\pi\,\text{Sqrt}\,[X]$ X = …….

Pendulum	L / g	Ring	2 R / g
Rod	2 L / 3 g	Disc	3 R / 2 g

➔ ***The motion for all shapes is independent of mass !*** ➔

Period for Spring-Mass System = T = 2π Sqrt [m/k]

Parallel Axis Theorem $I' = I + m d^2$

Parallel Axis Theorem adjusts Moment of Inertia for new axis

d= distance C of M to new axis

$I_p = 1/12\ M\ l^2 + M b^2$

$T = 2\pi\,\text{Sqrt}\,[(l^2/12 + b^2)/gb]$

Physics // Symbols Units Equations ©P. Freda 11/21/2018

Symbol	Concept	Units	Fundamental Units	Vector Scalar	Equations
(a)	Acceleration	meters / sec^2	meters / sec^2	V	$a = \Delta v / \Delta t$
(θ)	Angular Displacement	radians or degrees		S	$\theta = S / r$ = arc length / radius
(ω)	Angular Velocity	rad / sec		V	$\omega = v / r$; $\omega = 2\Pi f$; $\omega = 2\Pi / T$
(α)	Angular Acceleration	rad / sec^2		V	$\alpha = a / r$; $\alpha = d\omega / dt$
(L)	Angular Momentum	kg – m^2 / sec	kg –meter^2 / sec	V	$L = I \omega$; $L = r x p = r x m v = m(r x v)$
(I)	Moment of Inertia	no units	kg –meter^2	S	Obtained from Tables of Objects $I = M R^2$
(τ)	Torque / Moment	N – m	kg meter^2 / sec^2	V	$\tau = I \alpha$; $\tau = r x F = r x m a = m(r x a)$
(q)	Charge	Coulombs (C)		S	FUNDAMENTAL UNIT
(I)	Current	Amperes = Coulombs / sec		V	$I = q / t$
(s)	Displacement	meters (m)	meter	V	$s(t) = s_0 + v_0 t + \frac{1}{2} a t^2$
(E)	Energy	Joules = Newton – meter	kg meter^2 / sec^2	S	$E = P t$
(KE)	Kinetic energy	Joules (J)	kg meter^2 / sec^2	S	$KE = \frac{1}{2} m v^2$
(PE)	Potential Energy	Joules	kg meter^2 / sec^2	S	$PE = m g h$
(W)	Work	Joules	kg meter^2 / sec^2	S	$W = F d$
(Q)	Heat	Joules	kg meter^2 / sec^2	S	$Q = m c \Delta T + m L$
(L)	Latent Heat	Joules / Kg	meter^2 / sec^2	S	Vaporization or Fusion
(f)	Frequency	Cycles / sec	1 / sec	S	$f = 1 / T$
(F)	Force	Newton (N)	kg meters / sec^2	V	$F = m a$
(P)	Pressure	Pascal (Pa) = N / m^2	kg / meter sec^2	V	$P = F / A$ $P = \rho g h$
(m)	Mass	Kilograms (kg)	kg	S	FUNDAMENTAL UNIT
(p)	Momentum	N – sec	kg meter / sec	V	$p = m v$; $F = dp/dt$; $F = m dv/dt + v dm/dt$
(I)	Impulse	N – sec	kg meter / sec	V	Change in Momentum $I = \Delta p = F \Delta t$
(s)	Position	Meters	meter	V	FUNDAMENTAL UNIT
(P)	Power	Watts = Joules / sec	kg meter^2 / s^3	S	$P = E / t = W / t = F d / t = F v = \tau \omega$
(P)	Electrical Power	Watts = Joules / sec	kg meter^2 / s^3	S	$P = I V$; $P = I^2 R = V^2 / R$
(R)	Resistance	Ohms = Volts / Amps		S	$R = \rho L / A$
(v)	Speed	Distance / time	meter / sec	S	$v = \Delta d / \Delta t$
(T)	Period	Seconds	sec	S	$T = 1 / f$
(t)	Time	Seconds (sec)	sec	S	FUNDAMENTAL UNIT
(T)	Temperature	degrees		S	degK = degC + 273
(v)	Velocity	Displacement / time	meter / sec	V	$v = \Delta s / \Delta t$
(V)	Voltage	Volts = Joules / Coulomb		S	$V = I R$; $V = E / q$

Fundamental Unit Comparisons

$p = m v = F \Delta t$ = kg m / s Momentum

$F = m a$ = kg m / s^2 Force (Newton)

$W = F d$ = kg m^2 / s^2 Work / Energy (Joule)

$P = F d / t$ = kg m^2 / s^3 Power (Watt = Joule/sec)

F / m = m / s^2 Field Strength

= F/m = F / kg = J / m kg = kg [m/s^2] m / kg m ➔ m/s^2

PE / m = m^2 / s^2 Field Potential Energy = F x d

= PE/m = J / kg = kg [m/s^2] m / kg = [m/s^2] m

Thermal Physics

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Work W *Energy transfer between physical micro or macroscopic bodies*

Heat Q *Energy transfer given from a* ***hot body to a cold body***
by Conduction, Convection or Radiation

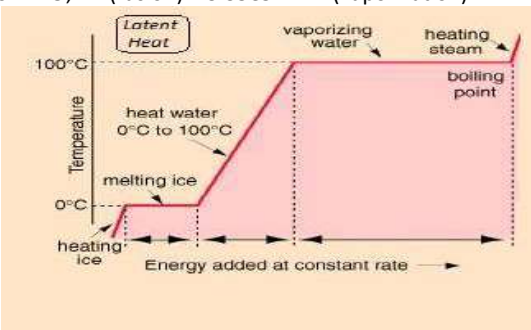
Temperature T *a measure of the* ***average KE*** *of all the molecules*

degKelvin = degCelsius + 273 ; degFahrenheit = 9/5 degC + 32

Internal Energy U *the* ***sum of the KE*** *of all the molecules*

Joule's Law Energy U = f(T) ..... U = 3/2 PV = 3/2 nRT

Latent Heat L *Amount of heat needed to change state without a ΔT*
Latent Heat of Vaporization ; change liquid ➔ gas
Latent Heat of Fusion ; change solid ➔ liquid
For H2O; L (fusion) = 3.35e5 L (vaporization) = 2.27e6 J / Kg



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Q = m c ΔT + m L **C = Q / ΔT**

Q = Heat in **Joules** ΔT = change in temperature
m = mass, kg; c = Specific Heat Capacity; for H2O = 4200 J / kg degC
L = Latent Heat **J / kg** C = Thermal Capacity **J / degC**

N_A = Avogadro's # = # particles/mole = 6.022 x 10^{23} particles/mole
Mass of Hydrogen Atom = 1 / N_A in grams **1 gram H = 1 mole**
Mass of Hydrogen Atom = 1.66 x 10^{-24} grams **12 gram C = 1 mole**
1 mole = mass number of the atom in grams [A = Protons + Neutrons]
Mole has equal # molecules/particles **N_A** but not equal mass
1 mole @ RoomTemp & 1 atm *ALWAYS* has Volume = RT/P = 24 liters
Absolute Zero = – 273 degC = 0 degK ; All motion ceases to exist

Avogadro's law 1/ "Equal volumes of all gases, at the same temperature and pressure, have the same number of molecules."
2/ For a given mass of ideal gas, the volume and amount (moles) of the gas are directly proportional if the temperature and pressure are constant.

Three States of Matter; Solid, Liquid, Gas [Plasma]
Liquid ➔ Gas 1/ ***Boiling***; all of the liquid is at same Temp
2/ ***Evaporation***; only at the surface; can be at any Temp
Rate of evaporation proportional to
1/ surface area 2/ blowing the vapor cloud away
3/ increasing Temp [KE] ***Evaporation is a cooling processes !***

Ideal Gas Identical, perfectly elastic, very small spheres
No forces between ➔ constant velocity motion
Molecules hit the wall of container causing Pressure P = F / A
Triple point [all states in equilibrium] H20 is 0.01 degC at P = 4.6 mm Hg

*****Linear relation Laws which led to the Ideal Gas Law *****
Charles' Law V vs T ; **Boyle's** P vs V ; **Gay-Lussac's** P vs T ➔

Ideal Gas Law **PV = n R T** ***Molar View***
n = # moles R = Molar Gas Constant = 8.31 J mol^{-1} $degK^{-1}$

Boltzmann Gas Law **PV = N k T** ***Molecular View***
N = # molecules k = Boltzman Constant = 1.38 x 10^{-23} J / deg K
N = n N_A N k = n R k = R / N_A R = k N_A
Molecular view vs Molar view, Ideal Gas Law = **Characteristic Gas Law**

Units Pressure **1 atm = 1.01 Bar = 14.7psi = 760 mm Hg = 10 m H2O**
1 atm = 101325 Pa = 1.01x10^5 Pa = 1.03 kg / cm^2 = 10,356 kg / m^2
1 Pascal = 1 N / m^2 ; 1 Torr = 1 mm Hg , 760 Torr = 760 mm Hg = 1 atm

Units Heat Joules OR Calories 4.2 Joules = 1 Calorie ; 4200 J = 1 KiloCalorie which what is found on food labels. 1 Calorie = Energy to raise 1 gm H2O 1degC

Stephan - Boltzmann Radiation Law **Q = e σ T^4 A t**
e = emissivity 0 < e < 1 σ = sigma = 5.67 e -8 J / m^2 s K^4

0th Law of Thermodynamics ***Temperature***
All diathermal walls are equivalent All heat is of the same nature
All thermal equilibriums are equivalent
Allows Temperature to be a valid State variable = Work value of Heat

1st Law of Thermodynamics ..... ***Energy***
ΔU = Q + W ΔU = change in internal energy,
Q = Heat added to system W = work done on the system
➔ *Energy can neither be created or destroyed* ➔

2nd Law of Thermodynamics ***Entropy*** **ΔS > 0** **[Eq1]**
All isolated systems spontaneously evolve towards thermodynamic equilibrium
Nature abhors differences in P, T, U Internal Energy
Natural processes have a preferred direction of progress
It is not possible to convert Heat completely to Work OR Work completely to Energy. **[W ➔ ΔU with Q = 0 not possible]**

Entropy ***Measure of Disorder*** **ΔS = ΔQ / T J / deg K** **[Eq2]**
Entropy is the ***measure of Disorder*** *of a system* **S = k Ln N** **[Eq3]**
Entropy is the ***measure of the # of States*** *of the system*
Total Entropy always increases [for an isolated or closed system]
Energy always spreads out. Systems tend toward equilibrium
Energy always flows naturally from hot to cold; Water flows downhill
Nature tends toward more disorder [chaos] & lower energy states
& takes ***path of least resistance*** */ energy dissipation*
Entropic processes are ***NOT reversible****;*
Disorder more Probable than Order Can not re-collect heat energy
Perpetual motion machines are "not possible"

3rd Law of Thermodynamics ***Chaos***

Heat ..... Energy is Transferred by ....
Conduction **physical contact** of Hot & Cold to one body/material molecular agitation. No material transfer
Convection **intermediate material** between Hot & Cold
Hot fluid (air, water) expands ➔ motion, circulation currents
Natural convection – expansion Forced convection – Fan or Heart/Pump
Convection occurs in all Fluids [liquids & gases] .....
Winds near bodies of water result from *convection currents* due to differences in Temp and density of air. These are called Thermals and cause vertical motion of the air. The differences in Heat Capacity between land and water cause horizontal motion; morning => land to sea, evening => sea to land; California Santa Ana winds.
Radiation **emission of Infrared Electromagnetic waves**

Heat Capacity, C, amount of energy/heat a substance can hold. It depends on the material AND its mass / quantity / size.

Specific Heat Capacity, c , is the heat required to raise Temperature of 1 gram of material by 1 deg. Depends on material only !
Water has much higher Specific Heat than other substances
Liquids have higher C than solids due to degrees of freedom of motion

Thermal Conductivity
What we perceive as hot or cold is NOT its temperature but its ***rate of energy transfer****; Metals will feel hotter or colder than plastic even though both are at the same Temp.*
High rate of energy transfer ➔ Good conductor
Good Conductors; Metals
Poor Conductors = Insulators ; Most Liquids and Gases, Air, Vacuum, Wood, Glass, Most Ceramics, Plastics

Thermal Expansion / Contraction
Matter expands when heated, contracts when cooled
Matter contains Internal Energy, not Heat
Liquids expand more than solids; Gases more than liquids
Water is very unusual; it contracts when heated from 0degC to 4degC ; is less dense than its solid form, ice. It is the
ONLY substance to have its solid less dense than its liquid.
ONLY substance to have its solid float in its liquid.
Newton's Law of Cooling; Rate of cooling is proportional to Temp difference of the object and its surroundings
This means it has an exponential decay
Good absorbers of radiant energy are also good emitters

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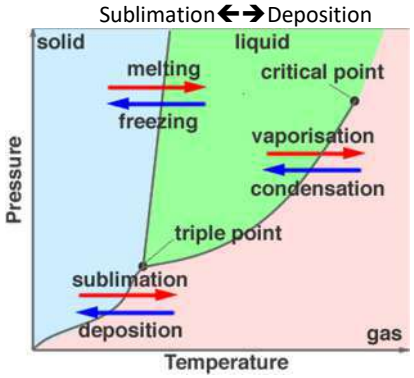
Phase Diagrams P vs T

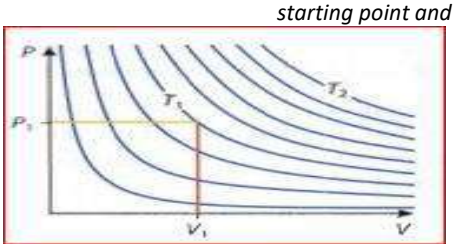
Shows the State/Phase [solid, liquid, gas] vs P T points
Boiling & Melting points change with Pressure & Temp
Triple Point is where all 3 States/Phases can co-exist
Melting ←→Freezing; Vaporization←→Condensation
Sublimation←→Deposition



Change of Phase
Energy is absorbed; Solid ➔ Liquid ➔ Gas/Vapor
Energy is released; Gas/Vapor ➔ Liquid ➔ Solid
Heat of Vaporization H_2O L(vapor) = 540 Cal/gr = 2.27 KJ/gr
Heat of Fusion H_2O L(fusion) = 80 Cal/gr = 335 KJ/gr

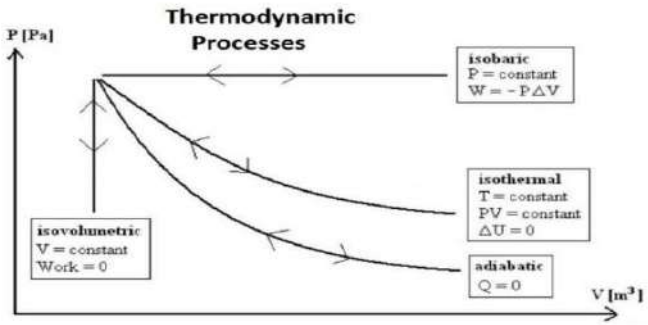
PV Diagrams of State P vs V Pressure vs Volume

Thermodynamic State .... *a point on the PV Diagram =* **Energy**
Thermodynamic Process .... *Transition from one* ***State*** *to another* ***State*** *on the PV Diagram*
Thermodynamic Cycle ..... *4 processes (or more) which end at the starting point and continually repeat*



Isotherms are lines of constant Temperature
Isotherms; the hyperbolic curves of constant PV = n R T

Joule's Law Internal Energy U = f(T) ..... U = 3/2 PV = 3/2 nRT
Point (V , P) ➔ U ➔ T ; Path pt – pt ➔ Work done on or by the gas
**** If Δ V = 0 ➔ No work is being done W = 0 ****
V↑ ➔ W < 0 ➔ ***Work done by the gas chemical energy***
V↓ ➔ W > 0 ➔ ***Work done on the gas***
Decreasing volume V↓ ➔ piston does work on the gas W > 0
Increasing volume V↑ ➔ gas doing work on the piston W < 0
**** Work = Force x Net Distance ➔ *piston must move***
Work W = – P ΔV = Area [PV = (F /A) V = F s = Work]
Gas can be ..... heated or cooled [Q > 0 or Q<0]
..... compressed or expanded [Δ V = V↑or V↓]
Δ V ➔ work done ***by or on*** the gas



Processes on PV Diagram Δ U = Q + W Equation

**** Joule's Law U = f (T) = 3/2 nRT Energy is a function of T alone ****
PV = nRT ➔ T ***up or down*** ➔ U goes ***up or down***
If Δ T = 0 ➔ Δ U = 0 No change in internal energy
Q > 0 ➔ Heat is gained by gas; Q < 0 ➔ Heat is lost from the gas
W > 0 ➔ Work done on the gas; W < 0 ➔ Work done by the gas

1/ Isobaric ; **Constant pressure P** **Δ P = 0** P = constant
Expansion; PV = nRT ➔ V↑ T↑ => ΔU > 0, V↑=>W<0; Q>0
Compression; PV = nRT ➔ V↓ T↓ => ΔU < 0, V↓=>W>0, Q<0

2/ Isochoric; **Constant volume V** **Δ V = 0** V = constant
PV = nRT ➔ P↑ T↑ => ΔU↑, ΔV=0 ➔W=0, Q>0, **Q=ΔU**
PV = nRT ➔ P↓ T↓ => ΔU↓, ΔV=0 ➔W=0, Q<0, **Q=ΔU**

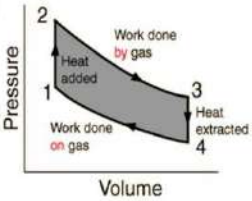
3/ Isothermal; **Constant temperature** **ΔT=0 ΔU=0** PV = constant
Expansion; ΔT=0 => ΔU= 0, **Q= –W**, V↑ => W<0, Q>0
Compression; ΔT=0 => ΔU= 0, **Q= –W,** V↓ => W>0, Q<0

4/ Adiabatic; **No heat is exchanged** **Q = 0 ➔ ΔU = W** PV^z = constant
occurs so quickly ➔ heat has no time to exchange
Expansion; V↑ => W<0 =>ΔU < 0, T↓=> int E ↓
Compression; V↓ => W>0 =>ΔU > 0, T↑=> int E ↑

Thermodynamic Cycle .....

Thermodynamic Cycles

- A cyclic thermodynamics process is a closed path on a PV diagram.
- The most efficient thermodynamic cycle is called the Carnot cycle.
- It consists of two adiabats and two isotherms.



➔ *Examples of Thermodynamic Cycles 1/ Refrigerator*
2/ Internal Combustion Engine [your car] 3/ Air Conditioner
For any Thermodynamic Process ➔ *Work done = Area under*
For any Thermodynamic Cycle ➔ *Net Work done = Area inside the curve of PV Diagram* [done on or by the gas]
*****For a Process ; *Work = the Area under the PV Curve ******
*****For a Cycle ; *Work = the Area inside the PV Loop ******
CW ➔ V↑ net work done ***by*** the gas W < 0 ➔ ***heat engine***
CCW ➔ V↓ net work done ***on*** the gas W > 0 ➔ ***heat pump***
Heat pump is a refrigerator OR air conditioner
Heat Engine is automobile Internal Combustion Engine
Refrigerators and air conditioners; cool through refrigerant phase change
Expansion forced vaporization AND Compression forced condensation.
Heat taken from cold, given to something hot by the work added
Contrary to what Nature would do ..... 2nd Law / Entropy
Isothermal expansions not possible [except at 0 deg K]
It implies heat can be transferred directly thru the gas to the piston without a T increase. This violates the 2nd Law which says it is not possible to convert heat directly in to work
Secret to all heat engines is that the gas is cooled before it is compressed back to original volume. The gas must be made hotter than the environs

Carnot Cycle; Perfect Engine Theoretical Limit
Only use Isothermal, Δ T = 0, or Adiabatic Δ Q = 0 processes

Otto Cycle; Internal Combustion Engine
An Otto cycle consists of 4 processes: 2 isentropic (reversible adiabatic Δ Q = 0)
2 isochoric (constant volume Δ V = 0)

Diesel Cycle; The Diesel Cycle differs from Otto cycle by using an increased compression of the fuel to ignite the fuel rather than using a timed spark
compression ignition" vs "spark ignition

Other Cycles; Stirling, Rankine, Ericsson

Wave Optics & Simple Harmonic Motion Pg 1

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Oscillation a continuing repetitive motion ; periodic motion
Amplitude (x_m) meter SHM maximum displacement
Cycle one complete oscillation (revolution, 360 deg, 2 π radians)
Frequency (f) cycles per sec ; 1 cy/sec = 1 Hertz (Hz) $f = 1/T$
Wavelength (λ) meters / cycle $\lambda = v\,T \rightarrow v = f\,\lambda$ m/sec
Velocity (v) meters/sec = f(λ) is constant but different in every media
Period (T) sec/cycle ; = time to complete one cycle
$T = 2\pi [X]^{1/2}$ Simple Pendulum X = L/g Spring-Mass X = m/k, also = d/g hanging
Physical Pendulums; X = I / m g d Rod X = 2L/3g Ring X = 2L/g
d = dist CM to Pivot I = Moment of Inertia Disc X = 3R/2g
Angular frequency (ω) radians/sec $\omega = 2\pi f = 2\pi / T$
used for circular motion where 2 π is one cycle
Equilibrium position the position where the system is normally at rest; also the point of highest speed and KE
Forced oscillation: an oscillation caused by a repeating external force [compared to a one time disturbance / impulse]
Natural frequency: the oscillation frequency of a disturbed system determined by its natural physical characteristics
Resonance: increase in amplitude of a system because it is being forced to oscillate near its natural frequency
Loud rattles heard while driving a car; Δ speed ➔ rattle stops
Damping: friction causing a loss of energy to a system
Wave : disturbance which travels from 1 location to another
A transfer of energy, NOT mass. Usually thru a medium like water or a gas like air. EM waves need no medium / no **'ether'**

Simple Harmonic Motion [SHM]

Restoring force is proportional to the displacement from the equilibrium position

Differential Equation of Motion … $dx^2/dt^2 + \omega_0^2 x = 0$

➔ ***SHM produces pure sinusoid solutions for position, velocity and acceleration*** ➔

x_m = maximum displacement / amplitude ; x ' = dx/dt

position	x (t) =	$x_m \cos \omega t$
velocity, v(t)	x ' (t) =	$-\omega\, x_m \sin \omega t$
acceleration, a(t)	x '' (t) =	$-\omega^2\, x_m \cos \omega t$
velocity	v(x) =	$\omega (x_m^2 - x^2)^{1/2}$
acceleration	a(x) =	$-\omega^2 x$
For circular motion	a(rad) =	$\omega^2 r = v^2 / r$

KE = 1/2 m v^2 = 1/2 m ω^2 ($x_m^2 - x^2$) ; Total energy = 1/2 m ω^2 x_m^2

Examples of SHM; Circular motion at constant speed,
Mass – Spring set , Pendulum (sin Θ = Θ for small Θ) , Tuning fork
Object floating in a fluid, LC Resonant Oscillations

Waves *propagation of a disturbance / energy*

Sound, Water, Light, Electromagnetic Energy, Gravitational
1 , 2 , or 3 dimensional
No *medium* is needed for EM waves; Any solid, liquid or gas for sound waves

4 Properties of Waves

1/ **Reflection** reversal of direction when meeting a new medium
Law of Reflection ; *Angle in = Angle out*
Specular reflection from a smooth or wet surface
Diffuse reflection from a rough surface

2/ **Refraction** change of speed *entering new medium*.
A *change of direction or bending* also occurs; See Snell's Law

3/ **Interference** waves observe superposition
2 waves can occupy the same space and thus add/subtract amplitudes. Particles cannot do this !
Interference (Superposition) can be Constructive or Destructive

4/ **Diffraction** *spreading out or bending + Interference* of waves which interact with objects; apertures, slits/pupils , corners, lenses

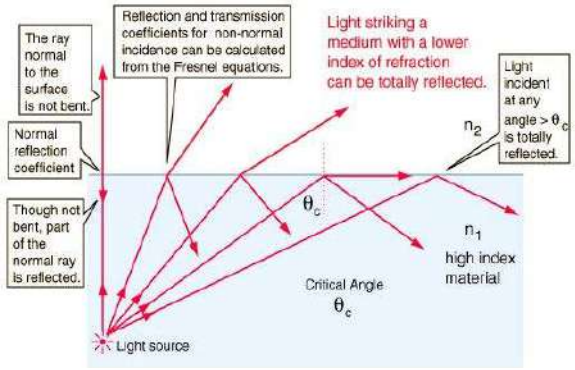
Huygen's Principle: Many synchronized point sources = ***Wavelets***
Distant Optic Source ➔ ***Plane wave*** ➔ ***Many Huygens Wavelets***

Fermat's Principle: Light follows the ***path of least time***
Used to derive Law of Reflection & Snell's Law

Reflection *Reversal of Direction when wave meets a new medium/material*

Law of Reflection ; *Angle in = Angle out*

Total Internal Reflection [TIR] $\Theta_{CRIT} = \text{Arcsin } n_2/n_1$ $n_1 > n_2$
Wave travelling from refractive index medium; high n_1 to low n_2
TIR Occurs when refraction angle > or = 90 degrees
Θ > or = Θ_{CRIT} ➔ **TIR ;** Fiber optic cables, Diamonds
Θ_{CRIT} = Water 49deg , Glass 42deg



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Note; Any energy going up at Θ_{CRIT} or greater is totally reflected

Refraction *1 / Change of speed of a wave entering a new media.*

2 / Change of direction [bending of the wave] due to Huygen's Principle
3 / Frequency does NOT change when wave enters a new medium
Refractive Index n is a measure of the change of speed.
The Index of Refraction is defined as the speed of the wave in *vacuum* divided by the speed of the wave in the *medium*

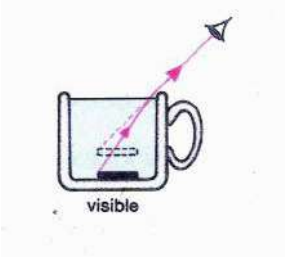
$n = c / v$ ***n > 1 ;*** $n_1 v_1 = n_2 v_2$ $v = f \lambda$ ➔ $n_1 \lambda_1 = n_2 \lambda_2$

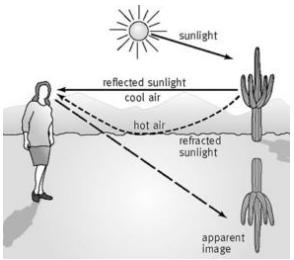
and from Snell's Law $n_1 \sin \Theta_1 = n_2 \sin \Theta_2$

➔ ***Bending of the wave does not occur if incidence angle is 0 degrees, but the change of speed of the wave does occur.*** ➔

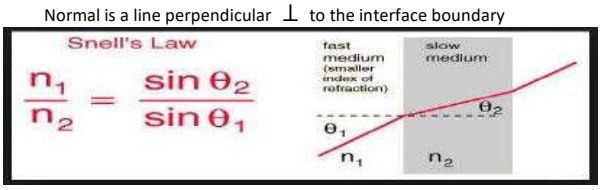
Sound waves travel faster in physically dense media like water
EM waves travel slower in optically dense media
Optical Density is NOT same as Physical Density
Refracted wave bends ***toward / away*** from the ***Normal***
FST = Fast to Slow ➔ ***Toward*** ; **SFA** = Slow to Fast ➔ ***Away***
Fast Media = Low index refraction = Optically low density
Slow media = High index refraction = Optically high density

Consequences of Refraction ……..
Objects appear closer or farther , distorted or bent , larger or smaller when looking thru a translucent media.
Refractive Index n = Real Depth / Apparent Depth
Apparent magnification is x1.33 for Water, x1.5 for Glass





Snell's Law **quantifies relation of bending angle to refractive index due to *the change in velocity***
➔ **Angles measured with respect to the *Normal*** ➔
Normal is a line perpendicular ⊥ to the interface boundary



Interference

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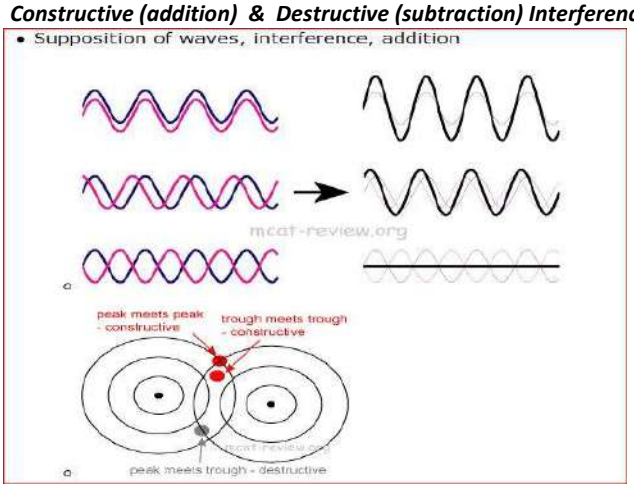
Superposition of Waves …..

Constructive (addition) & Destructive (subtraction) Interference



2 Circular Wave Sources; No obstacle ; Interference calculations

Maxima d = 1x 2x 3x 4x λ Minima d = 0.5x 1.5x 2.5x λ

d = path length difference Phase angle **$\phi = 2\pi d / \lambda$**

If d = whole # wavelengths m λ ➔ in phase ➔ **Constructive** /add

If d = odd # ½ wavelengths (m + ½) λ ➔ out of phase ➔ **Destructive**/cancel

Diffraction ~ = Interference

Wavelets spread out

Many Huygens wavelets + Interference ➔ Max & Minima

Single Slit Diffraction; ***Narrow slits diffract more than wider slits***

because larger angle is needed to obtain the same path length difference

Longer wavelength red light diffracts more than shorter blue

Narrow slit acts like a point source ➔ circular wave fronts

Slit diffraction….. (at) small angles ➔ tan Θ = sin Θ = Θ

Calculating half width ' y ' of a maximum on a screen or retina or film

From screen geometry Tan Θ = Θ = y/D From slit ΔL = a sin Θ = m λ

sin Θ = m λ / a and Θ = y / D ➔ **y = m λ D / a**

y = half width of maximum D = screen distance

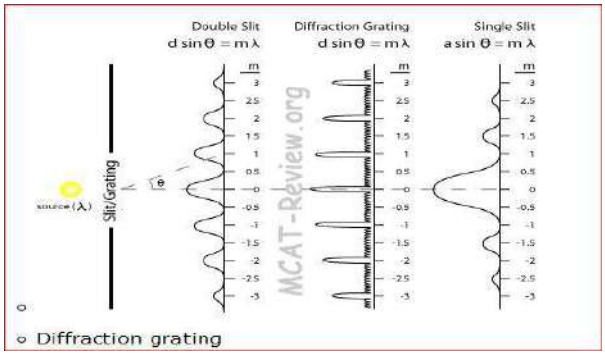
λ = wavelength a = slit width m = mode #

Central Maximum Width 2y ; For Circular Aperture the central maximum width is 1.22 times wider than for a slit.

Slit Interference Max / Min Plane wave incident

	Maximums (λ)	Minimums (λ)	Equation for a Maximum
1 slit	0 ±1.5 2.5	± 1 2 3 4	m λ = a sin θ
2 slits	± 0 1 2 3	± .5 1.5 2.5	m λ = d sin θ
Diff Grating	± 0 1 2 3	± .5 1.5 2.5	m λ = d sin θ

λ = wavelength d = slit separation a = slit width m = node order



Fraunhofer Diffraction … Optic Source and screen are very far from the aperature

Fresnel Diffraction … Optic Source & screen not far from aperature. Much more complex analysis

Rayleigh Criterion & Diffraction Limited Eye Resolution

Non Resolvability of 2 sources due to the 2 maxima converging **$\theta_R \geq 1.22\ \lambda / a$**

Two point sources are resolved when the principal diffraction maximum of one image coincides with the first minimum of the other. 2 Objects seen thru a lens, mirror, pupil or aperture making an angle smaller than the angular resolution **θ_R** cannot be resolved. As distance object to slit/pupil ↑ so Θ ↓ converging the 2 maxima ……This equation also gives the angular spreading of a source of light having a beam diameter 'a'. Resolving Power = 1 / Resolution

Diffraction Limit of resolution of any imaging system is about equal to the imaging wavelength

Optical microscope resolution limit 200 nm

Electron microscope limit 0.1 nm (size of an atom)

Effective λ = 0.02 nm

Polarization Brewster's Angle Ω

If angle between reflected and refracted rays is 90 deg, ***polarization of reflected wave is 100%***

Ω = the angle of incidence necessary for this condition

$\Omega = \text{Arctan } n2 / n1$ n2 > n1

wave travels from n1 medium => n2 medium

Ω = 53 deg air : water ; Ω = 56 deg air : glass

Note; These angles will vary with wavelength λ of light

Malus' Law ➔ $I = I_0 \cos^2 \Theta$

I = Intensity of light passing thru 2 polarizers

Θ = angle between the polarizers

Polarizers reduce intensity by 50% at Θ = 45 deg

For Θ = 90, 60, 45, 30, 0 deg ; $I_0 \cos^2 \Theta$ = 0, 0.25, 0.5, 0.75, 1.0

Types of Waves

Progressive / Travelling Waves ; plane or circular wave fronts

Transverse Wave; disturbance is perpendicular to direction of energy flow; Ex; Water, EM Radiation

Longitudinal Wave; disturbance is in direction of energy flow. Ex; Sound. Can not be polarized

Standing Waves / Stationary Waves SW

Incident and Reflected waves Interfere (add/subtract) to form a SW

Reflection + Interference at the right frequency ➔ ***Resonance***

Many "harmonic /right" frequencies ➔ ***Resonant Modes***

Can exist in Solids, Liquids or Gases

Node (Not Moving); place on SW where amplitude stays at zero

Antinode; place amplitude changes from + max to – max

Speed of wave on a string $V \text{ (string)} = [\ T / u\ ]^{1/2}$

T = tension , u = mass per unit length = grams / meter

$f = v / \lambda$ v = velocity λ = wavelength

Wavelength (λ) for Acoustic Resonance vs (L) Physical Length

Harmonic	1st	2nd	3rd	4th	5th
String	2L	L	2/3 L	½ L	2/5 L
Closed Pipe	4L	-	4/3 L	-	4/5 L
Open Pipe	2L	L	2/3 L	½ L	2/5 L

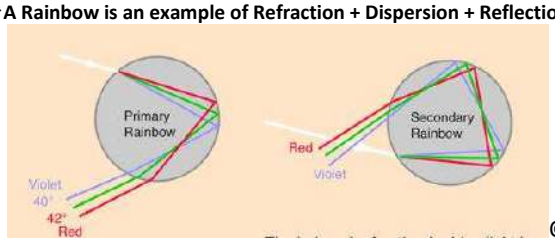
String ➔ Node both sides ; Open ➔ antinodes both sides ; Closed ➔ node at 1 end

Dispersion; v = f(λ) & n = f(λ)

Change in wave speed & refractive index n with frequency.

Dispersive devices; Water , Prism , Diffraction Grating

➔A Rainbow is an example of Refraction + Dispersion + Reflection



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Doppler Effect *change in apparent frequency due to motion*

1/ Sound $c = 330$ m/s

f_{OB} = observed freq f_S = source frequency

c = speed of sound/light v = velocity of the wave

Source [transmitter] **Observer [receiver]**

$$f_{ob} = f_s \frac{c \pm v[ob]}{c \pm v[s]}$$

Moving Source ➔ v[ob] = 0 ; Moving Observer ➔ v[s] = 0

2/ Light & EM Radiation $c = 3 \times 10^8$ m/s

$f_{0B} = f_s + \Delta f$ $\Delta f = [\ v / c\ ]\ f_s$

$f_{0B} = f_s \pm [\ v / c\ ]\ f_s$ $v << c$

Electric Fields & Forces

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Electric Field & Electric Force [+ & – charge]

Field …. A place where you find something [potato, soccer, rice, football]
Electric Force Vector Field is a region where a + charge experiences a Force Vector and moves in the direction of the Force Vector

Charge +q creates radial Electric Force Field that accelerates a + charge away
Electric Field vectors begin on a + charge and end on a - minus charge

Like charges repel ***Unlike charges attract***

Static Charge ➔ Electric Field
Moving charge / current ➔ Magnetic Field
Accelerating Charge ➔ EM Radiation

Maxwell's Equations, displacement current dE/dt ….. **predicted radio waves** and showed that ***light is an electromagnetic wave***
Hertz confirmed 8 yrs later–Tesla wireless xmission–Marconi TransAtlantic
Gauss ➔ there are ***no magnetic monopoles – point chrges***
Ampere's Law ➔ B = f (current) Oersted ➔ Current produces magnetic field
Biot-Savart Law ➔ relates magnetic field **B** at position **r** due to a *steady* current I
Kirchoff's Laws are a special case of Faraday's Law omitting the magnetic induction of electric field

Our economy runs on these principles & Faraday's Law to make Electricity.
Need Energy to move the magnet or the coil
Coal, Gas ➔ Steam, Water/Hydro Gravity , Wind, Nuclear

Electrostatics ➔ *Force* F_E

Triboelectric Effect is charge transfer due to contact/friction

+ Glass rod	**	– Silk cloth		+ CatFur	**	– Rubber rod
+ Hair	**	– Comb		+ Wool	**	– Plastic Balloon

Attraction by **Induction**; charged comb with paper bits
Note some bits fall back down after hitting the comb and some stick to it. ***Gravitational Force vs Static charge Force***
Electrical force ≫ Gravitational force ; Ratio of F_E to F_G = 10^{36} ! !
Conservation of Charge *charge is neither created nor destroyed*
Fields and Forces add as vectors PE and Voltage add as scalars

For point sources, the following Equations apply …….

F_E Electric Force Coulomb's Law

$F_E = k\ Q_1 Q_2 / r^2$ ➔ *Force* units **Newtons N**
$k = 1/4\pi\varepsilon_0 = 9 \times 10^9\ N\ m^2 / C^2$ ***Force = Gradient of PE***
$\varepsilon_0 = 8.85 \times 10^{-12}\ C^2 / N\ m^2$ = vacuum permittivity

E Electric Field Strength ➔ *Force per unit charge*

$E = k\ Q_1 / r^2 = F_E / q$ units **N / C = V / m**
$E = dV_E / dh$ ***Field Strength = Gradient of Potential***
➔ *Field lines show direction a + charge will move*
Gravitational Analog = $g = GM/r^2 = F / m =$ N / Kg

PE Potential Energy ➔ *Force x Distance* units **Joules J**

$PE = k\ Q_1 Q_2 / r$ Work done $W = k\ Q_1\ q / r$
Work done taking a point charge + q in from infinity [zero potential]
$PE = \int F\ dr$ ***PE= work done against the Field*** ***PE = Integral of Force***

V_E Electric Potential ➔ *Work done per unit charge*

$V_E = k\ Q_1 / r = PE / q = E\ r$ units **Volts = J / C**
$q = 1.6 \times 10^{-19}$ J / ev [CV/e^-] [$kg\ m/s^2$ x m /kg]

$V_E = \int E\ dr = \Delta$ EPot ***Potential = Integral Field Strength***

V = Work done ***by*** charge moving thru E Field across Equipotential lines
V = Work/q = F d/q = (F/q)d = E d W = q ΔEP = q V = q E Δh Joules
In a uniform field between 2 plates, the Force & Field are the constant everywhere for a given charge q [Joule = 6.24 e18 ev]

Electric Dipole E(on axis) = $2kp/r^3$ E(⊥ axis) = kp/r^3 p = q d

➔ **Maxwell's Laws of Electromagnetism** ➔

1st Law	Gauss Law Electrostatics	2nd Law	Gauss Law Magnetostatics
3rd Law	Faraday's Law	4th Law	Ampere's Law

Gauss Law Electrostatics **Maxwell's 1st Law** ***E = Flux Density = ϕ / A***

$\phi = \int E \cdot dA = Q / \varepsilon_0$ Σ **Flux Out = Charge enclosed in the volume**
$\phi = E\ A \cos\theta = Q / \varepsilon_0$ *Flux* $\phi = E\ A$ **[if E ⊥ to A's Normal vector]**
ϕ Electric Flux = Volt–meter = $N\text{-}m^2$/Coulomb [Magnetostatics $\int B \cdot dA = 0$]
E Flux lines begin on a + Charge and end on a – Charge

Electric Circuits

Kirkhoff's Laws

KVL ; *The sum of the voltages around a loop = 0*
KCL ; *The sum of the currents in to any node = 0*

Parallel vs Series Circuit

A *parallel* circuit; the *voltage across* the components is the same
Note; parallel does not mean to be geometrically parallel
A *series* circuit; the *current thru* the components is the same

Ohm's Law $V = I\ R$ Voltage = Current x Resistance
Power Laws $P = I\ V = I^2 R = V^2 / R$ Watts = Joules / sec

Resistor Combinations

Series; RT = Total R = R1 + R2 + R3 …. + Rn
Total Series Resistance is the sum of the resistors

Parallel;

a/ Reciprocal Rule; $1 / RT = 1/R1 + 1/R2 + 1/R3 \ldots + 1/R$
This works for any number of R's but is cumbersome Math

The following 2 Rules work ONLY for 2 Resistors ……
But …. Any combination of 'n' resistors can be calculated With multiple steps, 2 R's at a time.

b/ Product Over Sum Rule $RT = [R1\ R2] / [R1 + R2]$

c/ N + 1 Rule N = R(large) / R(small) $RT = R(large) / N + 1$
This works even if 'N' is a fraction, not an integer

Capacitor Combinations are opposite of Resistor Combination rules
R Series laws = C Parallel Laws AND R Parallel Laws = C Series Laws
Series; Charge same, Voltage adds --- Parallel; Charge adds, Voltage same

Capacitor Equations $q = C\ V$ $C = \epsilon_0 A / d$
$E = V / d = q / \epsilon_0 A$ $V = q / C = q\ d / \epsilon_0 A = E\ d$

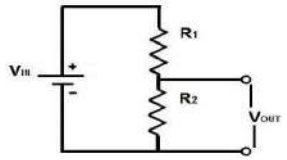
Conservation of Energy

Capacitor $E = ½\ C\ V^2 = ½\ Q^2 / C = ½\ Q\ V$; $I = C\ dV/dt$
Inductor $E = ½\ L\ I^2$ $\phi = L\ I$ $(q = CV)$ $V = L\ di/dt$
Since energy can not be created or destroyed
Energy can not move/change instantaneously, thus …..
The voltage across a capacitor can not change instantaneously
The current in an inductor can not change instantaneously
DC current seeks path of least Resistance; AC current least Inductance

Dielectric ➔ C ↑ Efield ↓ Energy density $n_E = ½\ k\ \epsilon_0\ E^2$ ↓
Polarization of the dielectric molecules reduces the total E Field

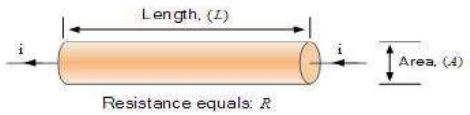
Drift Speed …. F = ma = qE & a ~ g ~ E = F/q & v = a t = a = F /m = qE/m … v = a t = qEt/m v = J / nq

Voltage Divider Rule



For Resistors; Vout = Vin [R2 / R1 + R2]
For Capacitors; Vout = Vin [C1 / C1 + C2]
Replacing the R1, R2 with C1, C2

Resistivity ρ Resistance $R = \rho\ L / A$
Resistivity ρ has units of Ohm - meter cc



1/ Lines of Electric Flux begin at + charge and terminate on – charge.
2/ The direction of the electric field and electric force is tangent to the field/flux line.
3/ Electric Field Vector (lines of force) always *normal to the surface* of charged body.
4/ Two electric lines of force cannot intersect each other.
5/ Electric lines of force in the same/opposite direction repel/attract each other
6/ Electric Potential exists due to charge. Voltage = EP difference between 2 pts
7/ Electric Field inside a conductor = 0 (if I = 0). E Potential = constant – dV/dr =E
8/ Free charges on a conductor reside only on the surface due to mutual repulsion
9/ Inside a surface charged hollow spherical volume, E Field = 0 due to symmetry

Magnetic Fields & Forces

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Magnetostatics

B Magnetic Field ➔ *Force per unit current* units **Tesla**

Magnetic Field Strength B = Flux density = ϕ / A

➔ ***Flux lines are the Field / Force vector*** ←

F_M Magnetic Force & **B** Magnetic Field

Units for B Tesla = Newton - sec / Coulomb - meter

Magnets exist as dipoles with a North and South pole

Like poles repel and unlike poles attract ➔ Force F_M

Opposing fields from magnets or currents ➔ Force F_M

Static charge ➔ Electric field

Moving charge [current] ➔ Magnetic field

Accelerating Charge ➔ Radiation

Earth's geographic North Pole is a Magnetic South Pole ☺

Pauli Exclusion Principle explains magnetism with idea of e- spin

Ferromagnetism; can be **permanent magnets**, Iron, Nickel, Cobalt, Neodymium Gadolinium and Alloys of Rare-Earth metals

Paramagnetism; materials **attracted** to a magnetic field

paramagnetic atoms from Z = 1 to Z = 20 are:

H, Li, B, C, N, O, F, Na, Al, Si, P, S, Cl, K.

unpaired electrons => paramagnetic; paired, => diamagnetic

Diamagnetism; materials **repulsed** by magnetic field; ***most materials***

Wood, Paper, Plastic, Glass, Heavy Metals, Copper, Marble, Water, Salt

Ferrimagnetism **Strongly attracted** to B Fields Examples .. Magnetite $MgFe_2O_4$ (magnesioferrite), $Y_3Fe_5O_{12}$ (yttrium iron garnet), $NiFe_2O_4$ (trevorite),

Antiferromagnetism; no net magnetic field possible

Substances that are *non-magnetic* include copper, aluminum, gases, and plastic. A material may exhibit more than one form of magnetism depending on its temperature.

Magnetodynamics

Lorentz Force Law ; Interaction of 2 Magnetic Fields ➔ *Force*

F_M (due to current) = I L x B **L = length vector, I = current**

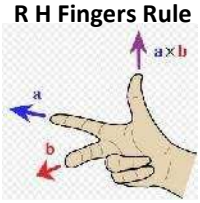
F_M (due to motion) = q v x B **v = charge velocity vector**

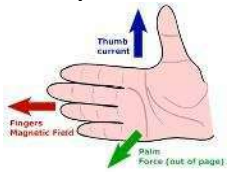
➔To go from q v B to I L B ; q v = q (L / t) = (q/t) L = I L

Cross Product Direction Rules ….

Positive charge => Right Hand Rule; Negative charge => Left Hand Rule

R H Fist/Grip Rule **R H Fingers Rule** **R H Slap/Palm Rule**





Right Hand Grip/Fist Rule ➔ Current (thumb) vs Magnetic Field (fingers)

Solenoid Rule ➔ Magnetic Field (thumb) vs Current (fingers)

RH Rule; v (index finger…a) X B (middle finger…b) => F is (thumb…a x b)

RH Slap Rule v (thumb) X B (index finger) => F is (middle finger)

F vector = qvB sin θ (v-B) is always Perpendicular to plane of v and B vectors

F vector = ILB sin θ (L-B) is always Perpendicular to plane of L and B vectors

1/ Magnetic forces are perpendicular to both the velocity of charges or current, and to the magnetic field. Since magnetic forces are perpendicular to the velocity, they do no work ! ! ! (W=F · r)

2 / Speed of particles moving in a magnetic field remains constant in magnitude, the direction changes. Kinetic energy is constant! (no work).

3 / F_B= q v x B r = mv / qB for circular / helical motion of q in uniform magnetic field.

4 / Magnetic dipoles align with the magnetic field same as electric dipoles

2 Parallel Wires; current in ***same*** direction ***attract*** each other in ***opposite*** directions, ***repel*** each other. Why ? Current in wire 1 creates B field that wire 2 sees. The Lorentz Force on wire 2 points to wire 1 ; F_M = q v x B. Similarly, Force on wire 1 points to wire 2 The Force decreases with distance d

$B_1 = u_0\ I_1 / (2\pi d)$ & $F = I_2 L B_1$ ➔ $F = u_0 I_1 I_2 L / (2\pi d)$

Note how this Force is different from Gravitational or Electric Forces between a Field and a Mass / Charge. Since there is no Magnetic monopole, the Force arises from the interaction of the two Dipoles created by the B Fields.

Electric Dipole Torque = **p x E** **p = q d** *d =dipole separation* **p** *direction + charge*

Magnetic Dipole Torque = **N I A x B** *on wire loop of Electric Motor*

Gauss Law Magnetostatics Maxwell's 2nd Law

$\phi = \int B \cdot dA = 0$ Net Flux = Σ Flux Out = 0 ➔ Flux In = Flux Out

There are ***No Magnetic Monopoles*** like there are Electric Charges

B Flux lines begin on a North Pole and end on a South Pole

ϕ = Magnetic Flux = Lines of **B** Field Webers OR Maxwells

Flux; An idea for understanding Mag or Elec field. Flux lines never cross each other. Make closed loops thru the magnetic material

Flux lines proportional to Current [Magnetic] OR Charge [Electric]

B = Flux density = ϕ / A ➔ $\phi = B \cdot A = B\ A \cos\theta$

For 'N' turns of coil $N\phi = N B A \cos\theta = N B A \cos \omega t$

Units Flux Density B = ϕ / A in Tesla OR Gauss …. Flux ϕ in Webers OR Maxwells

Tesla = 1 Weber / m^2 = 10,000 Gauss

Gauss = 1 Maxwell / cm^2 Weber = 10^8 Maxwell

Tesla/Gauss units = Newton-sec/Coulomb-meter

Electromagnetic Induction dB/dt ➔ emf & current is induced

Faraday's Law / Maxwell's 3rd Law $V_{EMF} = - d\phi/dt = \oint E \cdot dl$

$V_{EMF} = - d\phi / dt = - d/dt \int B \cdot dA = - d (A B \cos \omega t) / dt = A B \omega \sin \omega t$

Rate of change of Magnetic Flux ➔ Induces E Field ➔ Volt/Emf ➔ Current

…. If a Conductor is moving in a Magnetic Field OR a Magnetic field is moving/changing near a conductor

No matter how the ***change*** is produced, voltage/EField will be generated

The Flux change could be produced by ……

1 /changing the magnetic field strength or frequency

2/ moving a magnet toward or away from the coil,

3/ moving the coil into or out of the magnetic field,

4/ rotating coil or magnet relative to each other

Lenz's Law *The direction of the induced current creates a field that opposes the field that produced it*

➔ *It is the minus sign in Faraday's Law* ←

Ampere's Law / Maxwell's 4th Law

$\oint B \cdot dl = \mu_0 I$ $u_0 = 4\pi\ e{-}7$ T–m / A = N/ A^2

For Specific Geometries …..

B (near a wire r > R) = u_0 I / 2πr **B (center of a I loop) = u_0 I / 2 r**

B (inside wire r < R) = u_0 I r / 2πR² B (dipole on axis) = u_0 I / 2 π z^3

B (center solenoid) = u_0 N/L I [r << L] B (toroid) = k u_0 N I / 2 π r

B (Ring / Current loop on z axis) = u_0 I r (2 π r) / 4 π ($z^2 + R^2)^{3/2}$

Biot-Savart Law; *Relates B Field (ϕ/A) to the current I*

magnitude, direction, length and proximity (r) of ***current*** producing it.

It is the ***Magnetostatic equivalent to Coulomb's Law for Electrostatics***

For a constant current I ….. **$B = u_0 I / (4 \pi r^2)$ Integral dl x r**

Fleming Rules; Motor-Generators 3 Fingers at right angles

	Fingers; ***Index***	***–*** ***Middle***	***– Thumb***
Left Hand Rule; Motors;	Field	Current	Force
Right Hand Rule; Generators;	Field	Current	Motion

Similarities Electric / Magnetic Force Field

1) Both Force Fields are entities of Electromagnetism ➔ electricity and magnetism are not separate subjects but are intimately related.

2) A time varying electric field gives rise to a magnetic field and a time varying magnetic field generates an electric voltage / current in a conductor. If one of them varies with time or space, the other one is induced.

3) Both are vector fields and exert forces on electric charges

4) Superposition, attraction & repulsion, E and B Flux penetrates matter

Major Differences Electric / Magnetic Force Field & Miscellaneous Points

1) Electric field has sources and sinks (+ and – charge). Begin on + and terminate on – . But, Magnetic fields have no sources or sinks (are solenoidal) ***No Magnetic monopole exists.*** North & South Poles come in pairs called a Dipole. *The Field begins on N Pole and ends on S Pole* by convention.

2) For static charges Electric fields have non-zero Divergence and zero Curl; For steady currents, Magnetic fields have zero Divergence and non zero Curl.

3) Electric field force on charges are independent of velocity while magnetic field forces vary with charge velocity; $F_M = q (v \times B)$

4) A static charge will move in an Electric Field BUT not move in a Magnetic Field

5) Charge motion is parallel to E field lines but moves perpendicular to B field lines

6) Electric fields exert force parallel to charge motion and thus perform work on them while Magnetic fields exert forces perpendicular to motion so they ***do no work*** on them. W = F d cos θ = 0 (cos 90 deg = 0)

7) No work ➔ No change in KE or speed. BUT, direction & velocity both change.

8) A flowing electric current gives rise to a Magnetic Field that circles the current

9) B Fields Interact only with certain materials. Have circular, closed field lines

10) A time-changing Electric Field gives rise to a Magnetic Field that circles the **E** field

11) A time-changing Magnetic Field gives rise to a Electric Field that circles the **B** field

Fluid Statics *Pg 1* © *pfreda@gmail.com* *4/4/2023*

Pascal's Law ***(Blaise Pascal)*** **P** [pressure]

P = Force / Area = F / A = Energy/Volume ➔ E = PV

$P_1 - P_2 = \rho\ g\ [\ h_2 - h_1\ ]$ $P = \rho\ g\ \Delta h$ ➔ $P = \rho\ g\ h$

ρ = density fluid **g =** 9.8 m/s^2 **h =** height in fluid = depth

➔ ***Pressure increases linearly with depth*** ➔

➔ ***Pressure at any depth is the same*** ➔

Absolute pressure = Atmospheric pressure + Gauge pressure

$P = p_0 + \rho\ g\ h$ p_0 = P (depth = 0)

P is independent of Direction, Volume, Weight or Shape of the vessel and depends ONLY on the ***weight of the fluid*** above the given point

Atmospheric pressure *is due to gravity. We live under the weight of the fluid that is the atmosphere.*

Fluids are Liquids, Gases, Plasmas [Shape of their container ➔ Volume]

Air pressure at sea level is ~ 1 kg / cm^2 = 10,000 kg/m^2

100 kg pressure on your hand both top and bottom

Mercury barometer is 0.76 m high = 760 mm high

Water barometer is 10.33 m high ; 13.6x higher than Hg

10 meters of water produce an overpressure of 1 atm

Units Standard Atmospheric Pressure (1 atm = 6 different units)

1 atm = 14.696 psi = 10.33 m H_2O = 760 mm Hg = 1.01325 Bar

1 atm = 101325 Pa = 1.01x10^5 Pa = 101.325 kPa = 1.01325 Bar

1 atm = 760 mm Hg = 10,356 kg / m^2= 1.01325 Bar

1 Torr = 1 mm Hg = 133.3 Pa, 760 Torr = 760 mm Hg = 10.3 m H_2O

1 Pa = 1 Pascal = 1 N / m^2 ρ [mercury] = 13.6 x 10^3 kg / m^3

1 Bar = 10^5 Pa ; ρ [water] = 1000 kg / m^3 = 1.0 gram / cm^3

Pressure vs Height in the Atmosphere

$P(h) = P_0\ e^{(-h / H_0)}$ $H_0 = kT/mg = 8000m$

h= 2400m ➔ P= 0.75 atm H_2O boils at 92C

h = 8.9km ➔P = 0.33 atm H_2O boils 70C [Mt Everest]

h= 30km ➔ P = 1/45 atm = 17 mm Hg H_2O boils 20C

e ^ - (2400/8000) = 1.349 ➔ 1/1.349 = 0.74 … checks ok

Snorkel at depth of 1 m is impossible given hydrostatic pressure

To inhale, you must expand your chest & overcome H_2O and Atmospheric Pressure. You can not inhale (suck in/expand your chest) even at a depth of only 1 meter ➔ need for pressurized air tank

Letting air out underwater is easy. H_2O pressure helps.

Siphon; *Atmospheric pressure* down on the liquid in upper tank pushes fluid up into the top of the siphon [as it does in a barometer] where the pressure is lower. So it flows up and over the top.

The "Chain-pulling + tensile strength" explanation is *incorrect.*

By equating the PE at the top tank to the KE at the bottom tank yields outflow velocity to be $v = [\ 2\ g\ h\ ]^{(1/2)}$ [Note no Volume or Mass]

This is the same equation for fluid leaving the bottom of any tank.

Straw sucking height limit calculation

Sucking up thru a straw from a height ...how high can one go ?

The **work done** to raise the column to h is $W = F\ dot\ s = P\ A\ h/2$

The **potential energy** PE of the column = $m\ g\ h/2\ = (\rho A h)\ g\ (h/2)$

PE of the column = The work done moving the fluid ➔

W= $P\ A\ (h/2) = PE = \rho g h^2\ A/2$ [h/2 = avg distance] ➔ $h = P_{1atm} / \rho\ g$

h = 1.01 e5 [N/m^2] / [1000 kg/m^3 10 m/s^2] = **10.1 m** at sea level

This is the height of a water H_2O barometer at sea level

Pascal's Barrel Experiment; He attached a narrow vertical tube 10 m long to a barrel of H_2O, then filled it with water. It caused the barrel to explode. Why ? Internal P now 2 atm >> External P of 1 atm … ***see Pascal's Vases***

Torricelli's Law solves Bernoulli Equation for the speed of water leaving a tank ➔ speed of emptying liquid = Sqrt [2 g h] h = height of tank

Laplace's Law; Wall Tension T = PR Vessel radius ↑ ➔ Surface area A ↑.

If P = F / A is constant & A ↑ ➔ F, wall tension force ↑ (must go up)

Poiseuille's Law;. Laminar Flow rate in cylindrical pipe ***drops*** *inversely with length and viscosity* and with *linearly with 4th power of the radius and Pressure drop*.

For blood flow, a 19% decrease in radius due to cholesterol ***will cut flow rate in half*** !

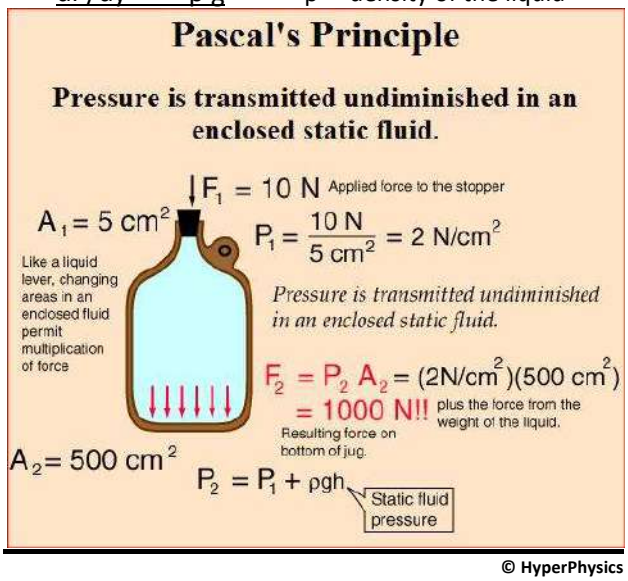
Pressure in the pipe drops linearly with length just as voltage drops along a wire.

Pascal's Principle; ***External pressure on an 'enclosed' incompressible fluid at rest is transmitted to all points in the fluid & acts in all directions. [perpendicular to the walls]***

P (scalar) = Force / Area = F / A ➔ F d / A d = Work / V

P = Work / Volume = Energy Density

dP/dy = – ρ g ρ = density of the liquid



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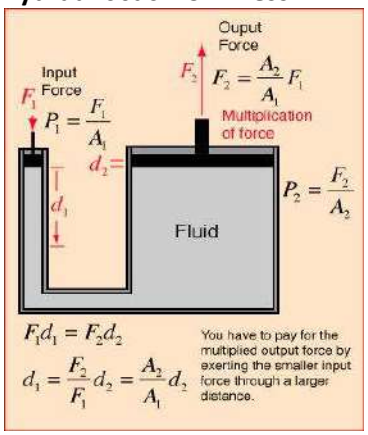
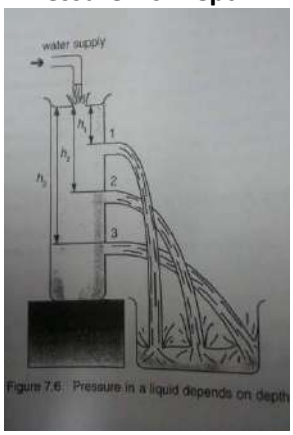
Hydraulic Jack or Press **Pressure vs Depth**





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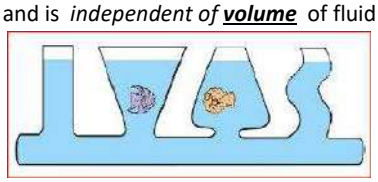
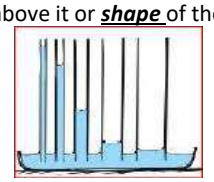
Note that if the vessel at right is dropped and thus is in free fall, the Equivalence Principle adds an upward pseudo gravity equal to the downward gravity, so there is no longer a differential pressure on H_2O Thus water will fall as one solid body**. No net Gravity ➔ No Pressure differential with depth ➔** Not only is there no differenc e in stream outflow velocity, BUT **All 3 streams will stop flowing out**.! !

Applications; Auto braking system, Siphon, Hydraulic Jack/Press

Pascal's Vases demonstrate fluid pressure depends only on ***height***; and is *independent of* ***volume*** of fluid above it or ***shape*** of the vessel.





Capillary Action / Capillarity ***the rise of a liquid in a fine tube or narrow spaces. Caused by cohesive/adhesive forces. Height determined by Gravity.*** Examples 1/ Towel or hair dipped in water 2/ The wick of an oil lamp 3/ Paint brush 4/ Transport of water from a plant root to its leaves

Vacuum levels vs Method Mechanical Pump 1 Pa; Vapor Diffusion / Jet 10^{-8} Pa Sublimation Pumps 10^{-12} Pa

U Tube with 2 fluids. The difference in heights ➔ relative densities

Fluid Statics Pg 2

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Archimedes Principle & Buoyant Force 3rd century BC

➔ ***Buoyant Force = weight of displaced fluid*** ➔

Wt [displaced fluid] = mg = [ρ (fluid) V (displaced fluid)] g

ρ & g are constants ➔ ***Displaced Fluid Volume*** ➔ ***Buoyant Force***

WHY ??? Buoyancy arises ***due to Gravity*** causing fluid pressure to increase with depth (Pascal's Law) *[liquid or gas]* ….. thus ….

The bottom of a submerged object has more pressure than the top.

So there is a ***Net Upward Force*** on any submerged object called the Buoyant Force. ***Downward Force*** = Wt of object

If the object floats ➔ Object Wt down < Buoyant Force Up

*** Floating is an Equilibrium ; F[buoyant] = F[gravity] ****

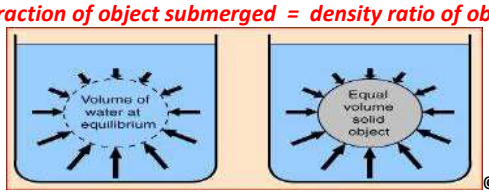
*** Equal submerged volumes feel the same Buoyant Force ****

*** The Buoyant Force is the same at all depths for given Volume ****

*** Submerged objects displace their volume BUT ….***

Floating objects displace their weight *

*** Fraction of object submerged = density ratio of object / fluid***



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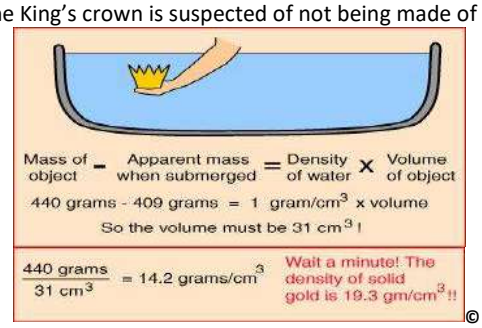
Archimedes Solution of King's Crown Density Problem

The King's crown is suspected of not being made of Pure Gold



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Problem: to find the density of the material in the King's crown

Weight crown in water = Wt of crown – Buoyant Force Upward

Buoyant Force Upward = Wt of crown – Weight crown in water

Buoyant Force = Weight of displaced fluid

Wt of displaced fluid = Wt of crown – Weight crown in water

$\rho_W V_{cr} g = \rho_{cr} V_{cr} g$ – [known by measurement]

V_{cr} = volume of displaced water [measured] ➔ ρ_{cr}

Iceberg in water; ρ[ice] = 0.92 g/cm³ ρ[w] = 1 g/cm³

Floating ➔ No motion ➔ Wt of object = Wt of displaced water, Fb

$M_O g = M_{UW} g$ ➔ $\rho_O V_O g = \rho_W V_{UW} g$

V_{UW} = V [ice under water] $\rho_O V_O = \rho_W V_{UW}$

$V_{UW} / V_O = \rho_O / \rho_W = 0.92 / 1 = 0.92$

so … 92% of an iceberg's volume is underwater

Vertical Cylinder A x L in water; h = length under water

F_B = ***Buoyant Force = Weight of displaced fluid*** = ρ_w A h g

If floating / stable ➔ F_B = Mg + Air Pressure

ρ[fluid] A h g = ρ[object] A L g + Air Pressure

independent of the shape or volume ! !

To float ➔ *ρ[object] < ρ [fluid = H_2O] & h < L*

Classic Problem; Boat in a pool. A Rock in a is thrown overboard

Does the water line… of the pool …go up or down ? Of the boat ? m = mass of rock

In *boat* rock displaces its *Weight* In the *pool* rock displaces its *Volume*

Buoyant Force = F_B = Wt of displaced H_2O = ρ[H_2O] V[displaced] g

Rock In water; Volume [H_2O displaced] = ***V of rock***

Rock In boat; Floating ➔ $F_B = F_G$ => ρ[H_2O] V [H_2O displaced] g = m g = Wt rock

ρ[H_2O] V[H_2O displaced] g = ρ[rock] V[rock] g

Volume [H_2O displaced] = ***(ρ[rock] / ρ[H_2O]) V[rock]*** Comparing Volumes ;

If ρ[rock] >> ρ[H_2O] ➔ more H_2O displaced with rock in the boat so BOTH pool & boat water lines goes **down**

Fluid Dynamics

Bernoulli's Principle

An *increase in the speed* of a fluid occurs simultaneously with a *decrease in pressure*

OR a decrease in the fluid's potential energy.

Venturi Effect is the ***reduction in fluid pressure*** that results when a fluid flow speeds up through a constricted section of pipe

HIGH pressure area is where speed is LOW, not high

LOW pressure is where speed is HIGH **** highly counter intuitive ! ! ****

➔ From Bernoulli Eq at same height h, If v ↑ then P ↓ and visa versa

Law of Mass Conservation / Continuity ➔ $A_1 v_1 = A_2 v_2$

If ***Area ↓ then fluid velocity 'v' must increase*** [Finger on a garden hose]

And to satisfy the ***Law of Conservation of Energy,*** since

Energy = Work = *P V must not change*, ➔ if V ↑ , then P must ↓

$P1 - P2 = 1/2\ \rho\ (v2^2 - v1^2)$ from Bernoulli Equation, h=0

Bernoulli's Equation Conservation of Energy`

$1/2\ m v^2 + m g h + P V = Constant$ V = Volume

OR $1/2\ \rho v^2 + \rho g h + P = Constant$ v = velocity

Trade speed for height, h, or pressure, P, in a tube of fluid

$P = ½\ \rho v^2$ = KE / V $P = \rho g h$ = PE / V

Examples; 1/ Airplane wing / **air foil**; air on top travels farther and thus faster due to mass continuity creating lower pressure on top. 2/ NASCAR **spoiler** stabilizer bar pushes the car down with inverted air foil. 3/ Blow up on a funnel with a ping pong ball inside and you can NOT get it up or out due to the speed causing a low pressure in the narrow region between wall and ball. Turn it upside down and you can keep it in the funnel against gravity; hard to believe until seen 4/ Vacuum hose blow ping pong ball vertically will be very ***stable horizontally*** due to the Bernoulli Effect as long as LP region is below C of Mass. Even at angle of 30 degrees 5/ Spray can **Atomizer**; air flow over a tube going down to the perfume creates LP and the perfume rises and flows out with the air flow.

Balloons *Air is a fluid ➔ Buoyancy forces, as with water*

Think of a water tank holding an ice cube & rock halfway down and let go. The ice rises and the rock sinks.

Same with He balloon & apple in air. One rises one falls

Specific Gravity = Ratio of densities of the object to the fluid

**** If density of the object < density of the fluid ➔ object rises and visa versa, sinks ****

Hot Air Balloon

M[total] g = Mass M of gas + M[rest of the materials]

Fb = weight of displaced air = V ρ[air] g

To rise ; Fb > Mg ρ is density

V ρ[air] g > V ρ[gas] g + M[rest]

density of air > density of gas

necessary but not a sufficient condition

This is why hot air balloons are so large; Need big V

Since air ρ decreases with height, a He balloon rises only to the height where ρ [air] = ρ [He] = approx. 20 miles. But a hot air balloon can go higher by further reducing balloon air density with more heat.

Acceleration Buoyancy

****Acceleration of box in horizontal [or any] direction simulates gravity****

Einstein's Equivalence Principle, Gravity – Acceleration

Acceleration creates an inertial/pseudo force due to Inertia

➔ ***Imagine a hanging Apple & He balloon on strings in a box in outer space***

1/ Accelerating the box in outer space ➔ no gravity or air. The apple & balloon ***both stand still due to inertia***. But *in the box* they both seem to move in a direction, opposite the box acceleration vector.

2/ Same accelerated box , but with Air added inside

Air inertia creates higher density in back of Apple/ He balloon & thus a differential air[fluid] pressure ➔ buoyancy

Apple moves opposite acceleration vector [sinks in the fluid/ gravity]

He balloon moves with acceleration vector [rises] due to buoyancy made possible by inertia & acceleration

Box + Air + acceleration ➔ differential pressure ➔ buoyancy

No air [fluid] ➔ no buoyancy ; No accel ➔ no buoyancy

No box ➔ no buoyancy ; No gravity ➔ no buoyancy

3/ Back on Earth, in a car with gravity or acceleration & air

Hanging apple & He balloon will go in opposite directions if you hit the gas OR the brakes ➔ ± Acceleration

4/ Driving around a bend, you & apple seem to move to the outside but He balloon moves to the inside due to Centripetal Acceleration and the inertia of the air

Atomic & Nuclear Physics

History of Atomic Models

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Billiard Ball Model 1803 Dalton **Plum pudding Model** 1897 JJ Thompson

Rutherford Model 1911 Experiment firing alpha particles at a thin Au sheet; most went right thru, but some reflected straight back. ➔ small positive nucleus.

Bohr Model 1913 of a mini Solar system flawed; can not explain why electrons do not fall in to the nucleus OR nuclear protons do not fly out of the nucleus

Electron Cloud Model 1926 E Schrodinger Quantum model / Wave Function model

Three Theories of Light / Electromagnetic Energy

λ = wavelength

1/ λ << object size ➔ Ray Optics ; Geometric Optics

2/ λ approx. = object size ➔ Wave Theory Optics

3/ λ >> object size (atomic dimensions) ➔ Quantum Mechanics

Light λ = 5e –7 m Atom d = 1e –10 m Nucleus d = 1e –15 m

Photoelectric Effect / Experiment

Heinrich Hertz 1887

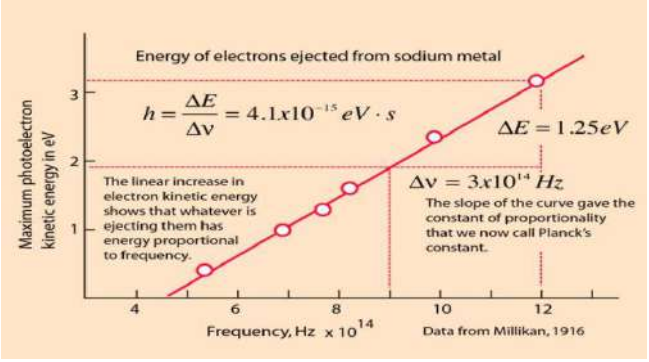
Light shining on metal => electrons arc more easily with UV light

Predictions; 1/ change in the ***intensity*** of light would induce changes in the kinetic energy & velocity of the electrons emitted 2/ there is a delay in electron emission at low light intensities/energies

Results; 1/ electron energy depended on light ***frequency***, not intensity 2/ below a threshold frequency, no electrons are emitted regardless of light intensity 3/ weak violet light creates higher energy electrons than intense red or yellow light 4/ There was no delay in electron emission at low light intensities/energies ➔ ***Light Waves have particle like properties thus a new model is needed. The Quantum Model***

KE (electron) = h f – W W = Work function = Y intercept

KE = Energy of emitted electron f = freq incoming light , h = Planck's constant



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Max Planck's Quantum Model ➔ **E = h f , c = f λ ➔ E = h c / λ**

h = Planck's Constant = 6.63 e –34 J-sec = 4.1 e –15 eV-sec

Photon is a packet of energy; a particle as well as a Wave

Louis DeBroglie 1924 ***Electron/particle is like a wave***

Related electron momentum p to wavelength λ_e

Wavelength $\lambda_e = h / p$ $p = h / \lambda_e = h f / c = h \nu / c$ [f = ν]

Electrons/Particles have a wavelike nature; superposition & interference

➔ **Wave – Particle Duality** - *Einstein believed light is a particle (photon) and the flow of photons is a wave much like a water wave is a flow of atoms*

Erwin Schrodinger Wave Equation ; Quantum Wave Function

$$h/2m\ \partial^2\Psi/\partial x^2 + V\Psi = i\ h\ \partial\Psi/\partial t$$

Max Born Psi Ψ^2 = Probability of finding the electron at point x in space

Electron in Potential Well $KE = ½\ m\ [\ h / m\ \lambda]^2$

Standing Wave metaphor for the electron trapped in a box model is too simple

Edwin Schrodinger replaced Standing Wave with Probability Wave Function ψ^2

Electrons/ Protons as Mass & Charge Probability waves

Werner Heisenberg Uncertainty Principle *Limit to precision*

Pairs of physical properties of a particle such as position *x* and momentum *p*, can NOT be known simultaneously.

$\Delta p\ \Delta x = \Delta E\ \Delta t > h / 4\pi$ $\hbar = h / 2\pi$ [h bar]

Albert Einstein $E = m c^2$ ***Energy and Mass are equivalent***

1 kg = 9e12 Joules 1 u = 935.1 MeV u = 1/12 M[C^{12}]

$E = m^2 c^4 + p^2 c^2$ $p = mv$ not valid near c

Wave Properties Reflection, Refraction, Interference/Superposition, Diffraction, Polarization. **Particle Properties** *can not explain* the last three

Compton Effect demonstrated that inelastic scattering of light by collision with a charged particle, produces light of longer wavelength [less energy] than that of the incident radiation. ... The effect is significant because it showed that light cannot be explained purely as a wave phenomenon

Other Important Early Quantum Theory Experiments;

1/ Davisson – Germer 2/ Franck - Hertz 3/ Stern – Gerlach

Atomic Emission & Absorption Spectra ➔

Electron Orbit Radii ➔ Quantized Electron Energy Levels

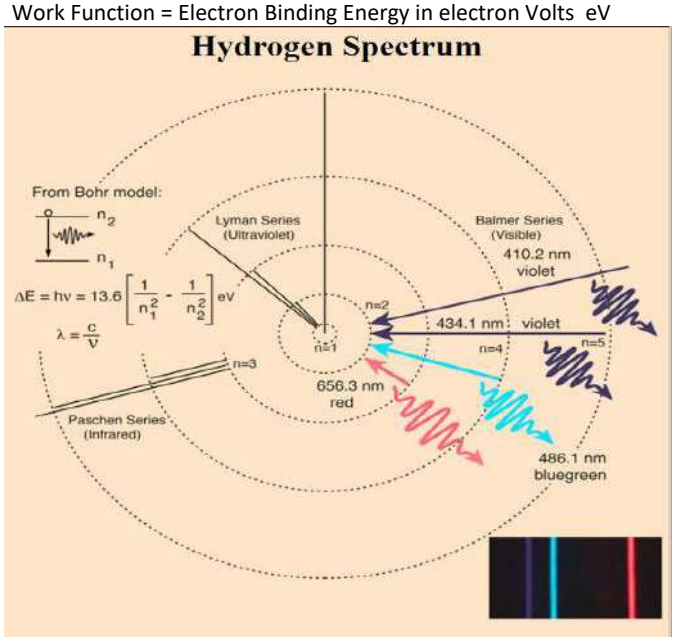
Atoms have discrete electron energy levels / states

Spectral Absorption & Emission lines ➔ Quantized electron energy levels

Size of the change in electron energy levels in the atom ➔

➔ frequency of light/ electromagnetic energy emitted or absorbed

Work Function = Electron Binding Energy in electron Volts eV



Balmer Series

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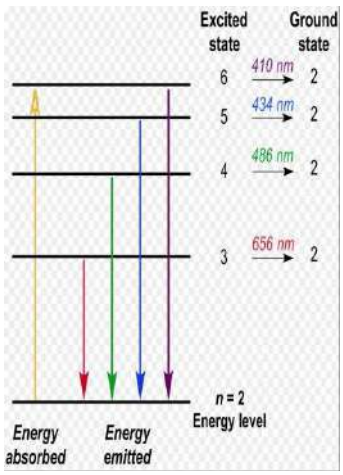

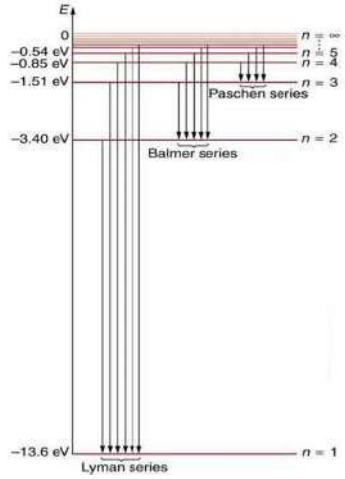

Negative energy levels in electron levels => it is bound to the atom

Work is done => PE, but done by the charge, not on/to the charge

Analogous to satellite negative Potential Energy PE

Energy level comparisons …

Per Atom	Energy	Per Atom	Energy
KE gas @RT	0.02 ev	Chemical	50 ev
Red Light	1.75 ev	Nuclear	200 Mev
Blue Light	3.1 ev		

Ex; Δ of 6 eV ➔ f = E / h = 6 (1.6e –19 J/ev)/6.63e –34 = 1.45 e15 Hz => UV 207 nm

Visible Light Photon 1.5 – 3.5eV , Medical Xrays 200KeV , Gamma/Beta 0-3MeV

Alphas 2-10MeV, Cosmic Ray 10MeV – 1kTeV, Kinetic Energy mosquito = 1 TeV

Energy electron ***1eV = 1.6e –19 Joule ; # electrons / Coul = 6.24 e18 q / C***

Unified Atomic Mass Unit 1 u = 931.5 MeV/c^2 = 1.66054e–27 kg

Mass Electron 9.1 e –31 kg = 0.00054858 u = 0.511 MeV/c^2

Mass Proton 1.672e–27 kg = 1.00727647 u = 938.28 MeV/c^2

Mass Neutron 1.675 e–27 kg = 1.00866490 u = 939.57 MeV/c^2

Photons interact with the atom, alphas bounce off

Ionization; electron(s) removed leaving a positive + charged ion

Absorption / Emmision Spectra => additional evidence of ***quantum*** energy levels

Phosphorescence Electron gun and Zinc Sulfide detector screen emits light if electrons excite the ZnS electrons

Electrons thru a thin graphite film give a pattern that looks like a wave diffraction pattern. Particles appear to behave as a diffracting wave

Atomic & Nuclear Physics

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Atomic Symbols and Terms

A = Z + N **A = Mass Number = # Protons Z + # Neutrons N**

Z = Atomic Number = # Protons = # Electrons

Atomic Mass/Weight = the average mass [grams] of all of the naturally-occurring isotopes of an element; e.g Carbon C 12.01 Lithium Li 6.941 Magnesium Mg 24.31

Experiments show all atoms have mass and charge that were multiples of the same number ➔ nucleus was made of unit masses and unit charges.

Nucleon Neutron or Proton **Nuclide** Unique Nucleus species

Isotope Nuclides with same Z, but different N & A

Symbol X^A_Z or $_AX_Z$ Ex Carbon14 C^{14}_6 or $_{14}C_6$

Z is often omitted; element => Z , e.g. U_{235} U => Z=92 A=235

Neutrino Properties 1/ Zero electrical charge

2/ Mass much smaller than the electron, recent experiments indicate definitely some mass 3/ Spin of ½ 4/ Very weak interaction with matter

Radioactive Decay

Emission of α β or γ Rays (Alpha, Beta or Gamma rays) from a nuclide that decreases Mass & increases Binding Energy per nucleon => making it more stable

Ex; Alpha Decay Δm = 0.0304 u BE = 28.3 Mev

M[alpha]= 4.00153 u, M[2 protons + 2 neutrons] = 4.03188

Fe56 (Iron) has 3rd highest BE/A ➔ so stable ➔ abundant

** **Generally, stable nuclei have more neutrons than protons in order to compensate for the repulsion of protons** **

No nuclei are stable for A > 83 [Lead is 82, Bi is 83]

Light nuclei most stable if N = Z ; Heavy nuclei if N > Z

Note; Nuclear BE >> Electron BE by factor of 10^6

Conservation of Energy & Momentum ➔ the decay particle will get the highest speed [KE] in the decay. ***The sum of both the Mass Numbers and Atomic Numbers do not change***

1/ Alpha Decay α ***2 protons + 2 neutrons*** **4He2** ***[Helium nucleus]***

226Ra88 => 222Rn86 + 4He2 [Transmutation]

238U92 => 234Th90 + 4He2

Typical energy of 5 Mev Highly ionizing

Range of 2-3 cm of air Can not penetrate paper

2/ Beta Minus Decay β(−) ***electron*** [Transmutation]

Neutron ➔ Proton + electron β(−) + Antineutrino

14C6 => 14N7 + $^0_{-1}e$ − + ν_e bar

209Pb82 => 209Bi83 + $^0_{-1}e$ − + ν_e bar

Range of 30cm air, 1 mm in Al Not highly ionizing

3/ Beta Plus Decay β(+) ***positron*** [Transmutation]

Proton ➔ Neutron + positron β(+) + neutrino

22Na11 => 22Ne10 + $^0_1e+$ + ν_e

19Ne10 => 19F9 + $^0_1e+$ + ν_e

Range of 30cm air, 1 mm in Al Not highly ionizing

4/ Gamma Ray Decay = No Change in Element

Gamma Rays are photons of very high Energy

Range centimeters of Pb (Lead) Not ionizing

5/ Neutrino No charge, little mass, unreactive,

Range 1000's km of Pb (Lead) **!!** Not ionizing

Antimatter = positive electrons + negative positrons

Changes in Z, N & A numbers with Decay Radiation Type

#	Z	N	A
Alpha α	− 2	− 2	− 4
Beta β −	+ 1	− 1	=
Beta β +	− 1	+ 1	=
Gamma γ	0	0	=

Radiation protection; Distance, Lead or Concrete Shielding

Radiation The unit of activity, R, is the *Curie, Ci*

1Ci = 3.7 x 10^{10} decays/second The SI unit of activity is the *Becquerel, Bq*

1Bq = 1 decay / second ➔ 1 Ci = 3.7 x 10^{10} Bq

The most commonly used units of activity are the mCi and the μCi

Average Nucleus radius is $r = r_0 A^{(1/3)}$ r_o = 1.2 x 10^{-15} m

Average Atomic radius = 1.0 x 10^{-12} m

Half Life *The time it takes for the mass to halve by Decay*

Rate of decay ∝ Mass M [# of Nuclei] ➔ dM/dt ∝ λ M

Exponential Growth or Decay ➔ Rate of change is proportional to size

Decay follows an Exponential decay curve

λ = Decay constant = Rate of decay in percent per unit time [years]

$dM/dt = -\lambda M$ ..... Solution is M[final] = M[initial] $e^{-\lambda t}$

Exponential Decay Equation $M(t) = M_0 e^{-\lambda t}$

Decay Constant λ s^{-1}

The Log of both sides yields $\ln[M] = \ln[M_0] - \lambda t$

$\ln[M / M_0] = -\lambda t$

$\ln[1/2] = -\lambda t$

$t_{[half\ life]} = \ln(1/2) / -\lambda = 0.693 / \lambda$

Plot of Log of M vs time is linear with slope = − λ

Carbon Dating measures ratio of radioactive C14 to C12 ratio.

C14 Half Life is 5730 years; Organic materials only

Potassium-40 ➔ Argon Dating t [half] = 1.25e9 yrs

Nuclear Binding Energy = Δm c^2 = Energy to split the nucleus

Binding Energy BE = Mass Defect ➔ ***M[separate] = M[bound] + BE***

***1/** Energy not present in the separate masses is what holds the nuclei together. **2/** Energy given to KE of decay products **3/** Nuclear binding energy is used to determine whether fission or fusion will be a favorable process*

*** In Decay, Fission or Fusion processes, **BE/nucleon ↑ goes up** ***

Nuclear Reactions Fission & Fusion

14N7 + 1n0 => 14C6 + 1p1 Nitrogen to Carbon

To find Energy released, subtract masses A before and after the reaction in units of "u" and x by 931.5 MeV/u

Energy came from Binding Energy of nucleus, NOT from Mass

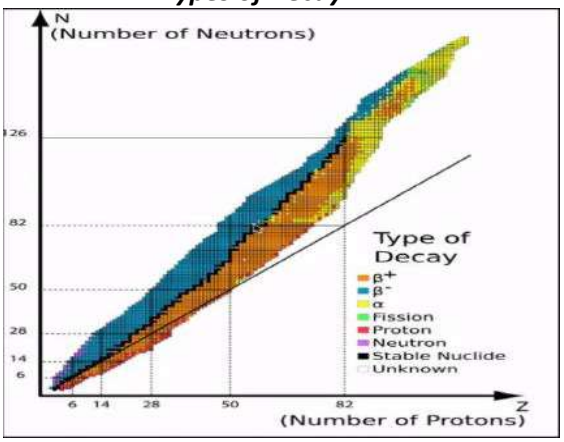
Fusion and Fission increase Binding Energy per Nucleon

Binding Energy Nucleus = 10^6 x Binding Energy of an Atom

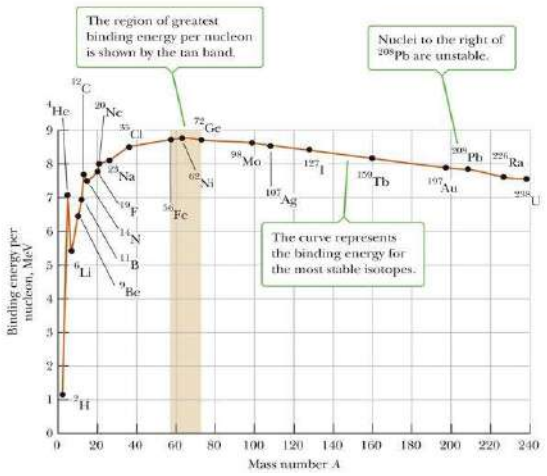
Nuclear Strong Force holds nuclei together only over short range

Neutrons are stable in nucleus but unstable outside; half life = 15 min

Types of Decay



Binding Energy per Nucleon



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Electron Configurations Shells, Subshells & Orbitals

Shell 1st number of a subshell like 2 in $2p^4$
The Energy Level with a known Work function,
Principle quantum number 'N'
Mean radial distance of the electron from the nucleus

Subshell The letter after the Shell # p in $2p^4$
There are 4 subshells at present s , p , d ,f
Angular quantum number 'L' *the shape of the orbit or standing wave order*
*S*ome *P*hysicists *D*on't *F*ind *G*iraffes *H*iding *I*n *K*itchens [G,H,I K not needed yet.]

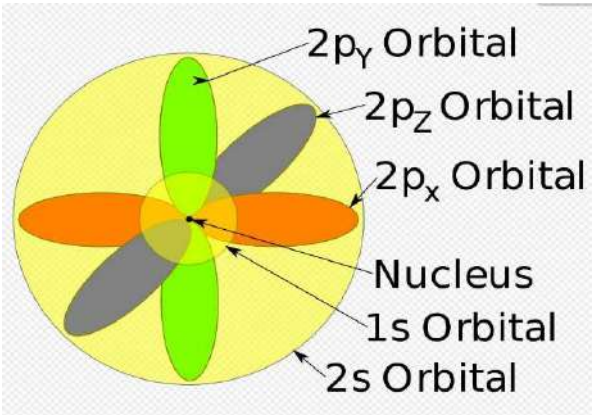
Orbital Divisions of the Subshell each of which can contain 2 electrons of opposite spin
There is 1 s orbital, 3 p orbitals, 5 d orbitals, and 7 f orbitals
*Regions within an atom that the **electron** will most likely occupy.*
Each orbital can hold **two electrons**. One spin-up and one spin-down

Electron Configuration A single string of orbital names and superscripts $1s^2\ 2s^2\ 2p^6$
The superscript = # electrons in that subshell // the sum of exponents = Atomic Number
An electron configuration for an atom with every orbital completely filled would be written: $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6\ 5s^2\ 4d^{10}\ 5p^6\ 6s^2\ 4f^{14}\ 5d^{10}\ 6p^6\ 7s^2\ 5f^{14}\ 6d^{10}\ 7p^6$ sum of exponents = 118 No 8s yet

Note that the above list would be the electron configuration for (Oganesson), 118, Og, the highest-numbered atom on the periodic table –
So this electron configuration contains every currently known electron shell for a neutrally charged atom

Each shell can contain only a fixed number of electrons
The 1st shell can hold up to **2** electrons,
The 2nd shell can hold up to **8** (2 + 6) electrons,
The 3rd shell can hold up to **18** (2 + 6 + 10) and so on.
*The nth shell can in principle hold up to **$2(n^2)$** electrons*

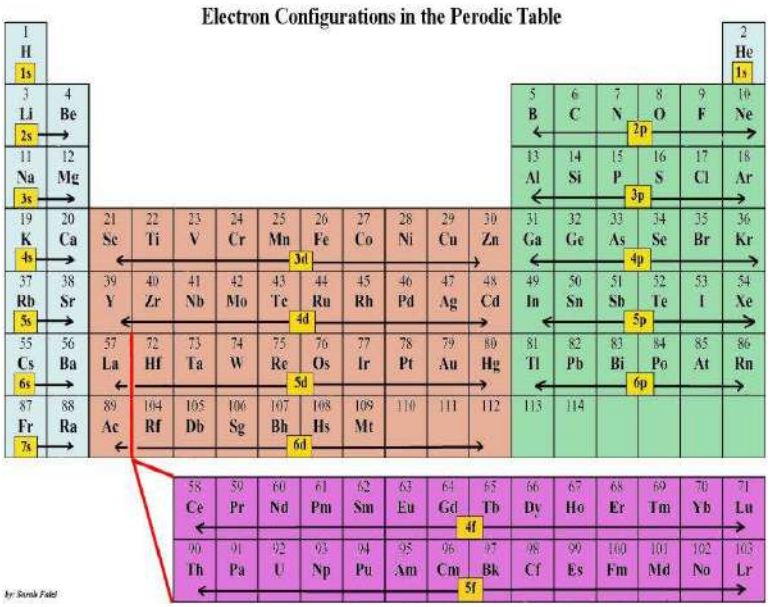
Each subshell has 2 electrons in each orbital.
s subshell has **1 orbital** that can hold up to **2 electrons**,
p subshell has **3 orbitals** that can hold up to **6 electrons**,
d subshell has **5 orbitals** that hold up to **10 electrons**,
f subshell has **7 orbitals** that can hold **14 electrons**.



Four quantum numbers can describe an electron in an atom completely.

Principal quantum number	(*n*)	Shell
Azimuthal quantum number	(*ℓ*)	Subshell 0,1,2,3 for s p d f
Magnetic quantum number	(*m*)	Energy shift (orientation of the subshell's shape)
Spin quantum number	(*s*)	Spin of the electron +1/2 or -1/2

The **electrons do** not **orbit** the nucleus in the manner of a planet **orbiting** the sun, but instead exist as standing waves. Thus the lowest possible energy an **electron** can take is similar to the fundamental frequency of a wave on a string
The **4th shell** of any atom can hold a maximum of **32 electrons = $2n^2$** n=4
There are four subshells --- 4s, 4p, 4d, and 4f - that can hold a maximum of 2 times 1,3,5,7 = 2, 6, 10, and 14 electrons, respectively, for a total of 32
1st = max of 2 **2nd** = max of 8 **3rd** = max of 18 **4th** = max of 32 **5th** = max of 50
The subshells s , p , d , f **s**harp, **p**rincipal, **d**iffuse and **f**undamental, respectively.
The letters and words refer to the visual impression left by the fine structure of the spectral lines
Quantum numbers L 0 , 1 , 2 , 3 correspond to s , p ,d , f
Octet Rule ; Atoms tend to gain or lose electrons to achieve an outer shell of 8 electrons [$s^2\ p^6$ orbitals] ***which is highly stable***.
Atoms in Group 1 & 2 tend to mate with atoms in Groups 16 – 17.
Group 18 are the Noble Gases which are also ***highly stable***



The shape of the periodic table corresponds to the order of orbital sets in electron configurations

When writing an electron configuration for Chlorine,…. think:
"This atom is in third row (or "period") of the periodic table.
It's also in the fifth column of the periodic table's p orbital block.
Thus, its electron configuration will end with ...3p^5 "

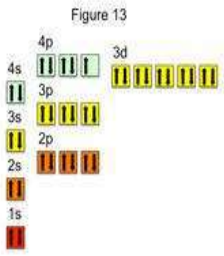
Examples

Na11 or $Al13^{+2}$ ..... $1s^2\ 2s^2\ 2p^6\ 3s^1$
O8 or $N7^{-1}$ or $F9^{+1}$ ..... $1s^2\ 2s^2\ 2p^4$
Sc21 or $Ti22^{+1}$ or $Ca20^{-1}$ ..... $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^1$
Cd48 or $Sn50^{+2}$ ..... $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6\ 5s^2\ 4d^{10}$
Cs55 ..... $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2\ 3d^{10}\ 4p^6\ 5s^2\ 4d^{10}\ 5p^6\ 6s^1$
Lr103 or $No102^{-1}$ ..... $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^6\ 4s^2 3d^{10}\ 4p^6\ 5s^2\ 4d^{10}\ 5p^6\ 6s^2\ 4f^{14}\ 5d^{10}\ 6p^6\ 7s^2\ 5f^{14}$

Shorthand Notation Ca20 = [Ar] **$4s^2$** O8 = [He] **$2s^2\ 2p^4$**
Fe26 = [Ar] **$4s^2\ 3d^6$** Cd48 = [Kr] **$5s^2\ 4d^{10}$** Nd60 = [Xe] **$6s^2\ 4f^4$**
Pb82 = [Xe] **$6s^2\ 4f^{14}\ 5d^{10}\ 6p^2$** Bk97 = [Rn] **$7s^2\ 5f^8$**

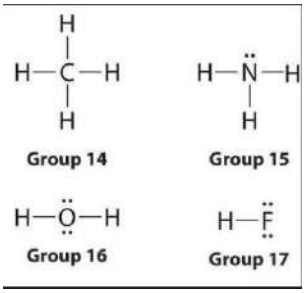
Electron / Gilbert Lewis Dot Diagrams
a shorthand ways to represent *how atoms form covalent or ionic bonds to form molecules*. Lewis dot diagrams use dots arranged around the atomic symbol to represent the electrons in the outermost energy level of an atom.
Single bonds are represented by a **pair of dots or one line** between atoms.
Double bonds are represented by a **pair of lines** between atoms
Electron Filling Diagrams representation of the Electronic Configuration using boxes with 2 electrons of opposite sign per box.
One box = one of the subshell orbitals

Bromine Electron Filling Diagram



bromine

Electron Dot Diagrams



Geometric / Ray Optics

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Optics History

Before 1700 λ << f Geometric / Ray Optics f = focal length
Around 1800 λ ~= f Wave Theory or slit width
Around 1900's λ >> f Quantum Theory

Law of Reflection ; Angle In = Angle Out

Law of Refraction ; **$n1 \sin \Theta_1 = n2 \sin \Theta_2$**

Θ = angle theta = angle from the Normal to the surface
Both Laws can be derived from Fermat's Principle of Least Time
Frequency f does not change in a new medium, but λ does
Index of Refraction **$n = c / v$** ➔ n v = constant = speed of light 3e8 m/s
Since $v = f \lambda$ and $n = c / v = c / f \lambda$ ➔ $f = c / n \lambda$ ➔ $n \lambda$ = constant

$n1\ v1 = n2\ v2$ & **$n1\ \lambda1 = n2\ \lambda2$** & **$n1 \sin \Theta_1 = n2 \sin \Theta_2$**

Geometrical optics does not account for optical effects such as diffraction and interference

Definitions

R = Real V = Virtual Real & Inverted OR Virtual & Upright
I = Inverted U = Upright f = focal length
L = Larger S = Smaller do = object distance from center
If do > f ==> di Real Inverted Image, Opposite side
If do < f ==> di Virtual Upright Image, Same side
If do = f ==> No image / Image is at Infinity

Object anything being viewed by an optical system or device

Image the likeness of an object from using an optical device where light rays cross or focus [mirror or lens]

Real Object ..... rays physically emanate from the Object

Virtual Object .... rays appear to physically emanate from the Object but do NOT

Real Image ..... where the light is, in front of mirror, behind a lens, can be seen on a screen / retina

Virtual Image .... where the object "seems to be" coming from
1/ diverging mirror 2/ object inside focal length of converging lens

A real image occurs where rays converge, whereas a virtual image occurs where rays only appear to converge
Virtual images are formed by diverging lenses or by placing an object inside the focal length of a converging lens
Plane/Flat mirrors, convex mirrors, and diverging lenses can never produce a Real Image

Convex lens is **thicker in the center** than the ends
Concave lens is **thicker at the ends** than in the center

A **concave** mirror converges light to a focal point on the **same side** as the object.
A **convex** lens converges light to the focal point on the **other side** of the object

For a Thin Lens, the power is approx. the sum of the surface powers
Surface Power = (n2 – n1) / R
Lenses have two focal points, one on either side of the lens.
Mirrors have one focal point;
A concave mirror OR convex lens ==> converges light to a focal pt => Real Image.
A convex mirror OR concave lens ==> diverges light from a focal pt => Virtual Image
Concave or Flat Mirror f > 0 positive Convex Mirror f < 0 negative

***Spherical lens* or mirror** is an approximation to a Parabolic Geometry for small angles. The Focal Point is one half the Radius of the Spherical Curvature; **f = R / 2**
Thin Lens => refracts but NO dispersion/chromatic aberration OR spherical aberration

Differences between lenses and mirrors

1/ Light reflects from a mirror
2/ Light goes through, and is refracted by, a lens.
3/ Lenses have two focal points, one on either side of the lens.

Similarities between lenses and mirrors

1/ The equations we used for mirrors all work for lenses.
2/ A convex lens acts a lot like a concave mirror. Both converge parallel rays to a focal point, have positive focal lengths, and form images with similar characteristics.

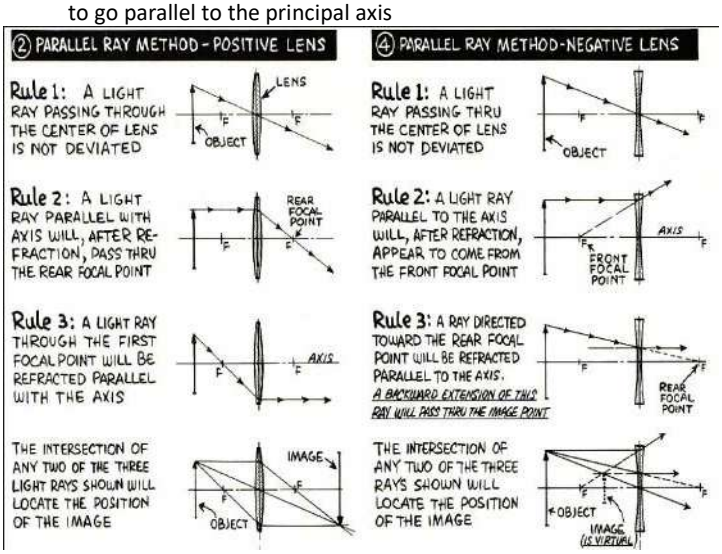
Geometric Analysis

Convex Lens Ray Tracing Rules

The PARALLEL RAY goes through the lens and passes through the focal point on the far side
The CHIEF RAY passes through the center of the lens [for thin lens]
The FOCAL RAY goes through the focal point on the object side and emerges from the lens parallel to the principal axis. It is a mirror image of the parallel ray.

Concave Lens Ray Tracing Rules

The PARALLEL RAY goes through the lens and diverges away from the principal axis going directly away from the focal point on the object side of the lens.
The CHIEF RAY passes through the center of the lens [for thin lens] SAME AS FOR CONVEX LENS
The FOCAL RAY goes through the lens heading toward the focal point on the far side of the lens. It is re-directed by the lens to go parallel to the principal axis



Algebraic Analysis

LENSMAKER'S FORMULA
Spherical surface

$$1 / f = (n - 1) (1 / R1 + 1 / R2)$$

LENS EQUATION
Parabolic surface

$$1 / f = 1 / d_o + 1 / d_i$$

$$f = d_o d_i / d_o + d_i$$

R radius of curvature f Focal Length d_o object distance d_i image distance
R1 = front surface R1 > 0 R2 = back surface R2 < 0 n = n2 / n1 n1 = 1 for air
m = Magnification P = Lens Power in diopters h = height

$f = d_o d_i / d_o + d_i$ $d_i = d_o f / (d_o - f)$ f = R/2 for Spherical suface

$m = - d_i / d_o = h_i / h_o$ **P = 1 / f in Diopters**

MEANING OF NEGATIVE QUANTITIES .....

do distance, di distance , f focal length, m magnification
Negative image distance di ==> virtual image
Negative Image distance di ==> object and image are on same side of lens
Negative focal length or Power => diverging lens OR mirror
Negative magnification m ==> image is inverted compared to the object.
A positive lens focal point is on the other side of the lens from where the object is placed.
Negative lens focal point is on the same side of the lens from where the object is placed.
Converging lens or mirror ..... f > 0 always
Diverging lens/mirror ..... f < 0 always

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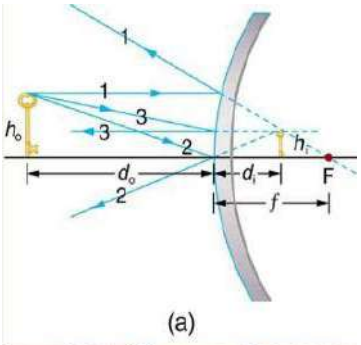
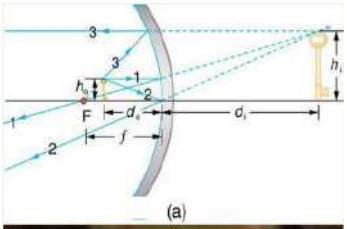
USES FOR LENSES AND MIRRORS

Concave Lens corrects for Nearsightedness Myopia
Convex Lens corrects for Farsightedness Hyperopia
Concave Lens d0 > f glasses/contacts/ nearsighted, myopia
Concave Lens d0 < f
Convex Lens d0 > f glasses/contact/ farsighted, hyperopia
Convex Lens d0 < f Magnifying glass
Correction w converging lens; flashlights, binoculars, telescopes, photography
Correction w diverging lens; cameras, microscopes
Concave Mirror; Focusing Shaving mirrors, Head mirrors, Ophthalmoscope, Satellite dish, Astronomical telescopes, Headlights, Solar furnaces & collectors
Convex Mirror; Fish eye/Diverging Mirror; Inside buildings, Sunglasses, Vehicle mirrors, Magnifying glass, Security
Flat Mirror; Personal hygiene
Reflecting telescopes make use of a concave mirror, a plane mirror, and convex lens
Refracting telescopes use two convex lenses.
Microscopes make use of a concave mirror, a plane mirror, and a convex lens
https://byjus.com/physics/uses-of-convex-mirror/

Convex Lens



Concave Lens

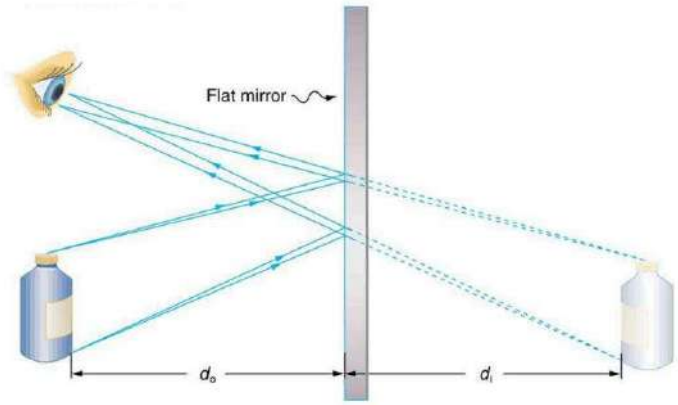


Cartesian Sign Convention

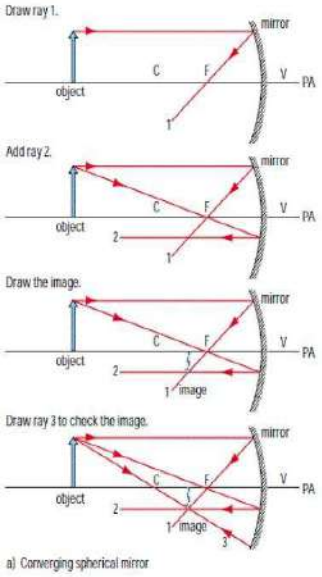
All figures are drawn with light traveling from left to right.
All distances are measured from a reference surface, such as a wavefront or a refracting surface.
Distances to the left of the surface are negative
Angles measured clockwise from the optic axis are negative.
The refractive power of a surface that makes light rays more convergent is positive.
The focal length of such a surface is positive.
The distance of a real object is negative.
The distance of a real image is positive.
Heights above the optic axis are positive.

Hyperphysics Mirrors
http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/mirror.html#c1
Hyperphysics Lenses
http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/lenscon.html#c1

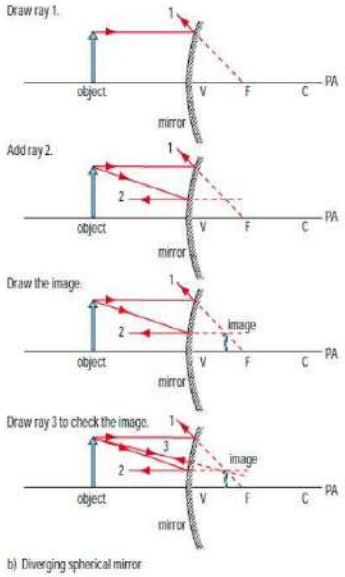
Flat Mirror showing Virtual / Apparent Image



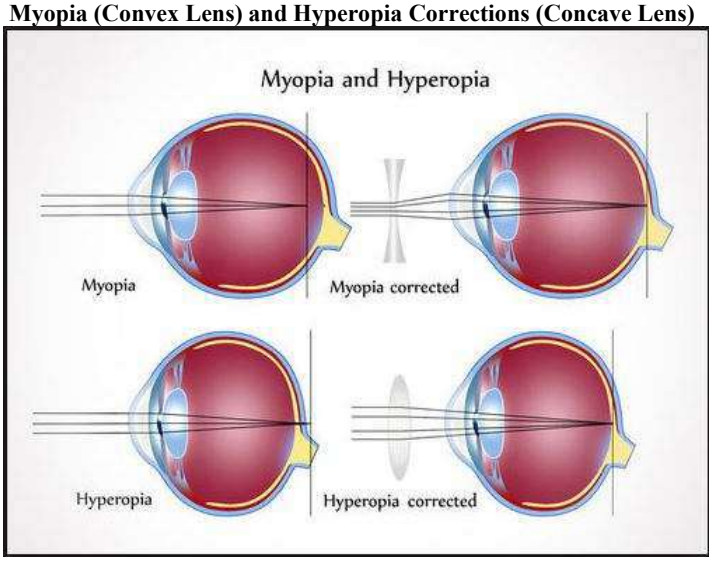
Converging Spherical Mirror



Diverging Spherical Mirror



Myopia (Convex Lens) and Hyperopia Corrections (Concave Lens)



Chromatic aberration … inaccuracies due to dispersion variation of index of refraction with wavelength
Spherical aberration ….. inaccuracies due to outer parts of the lens not focusing well such as with a Spherical lens vs Parabolic lens

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Astonomical Unit AU is Distance *Earth–Sun = 1.5e11 m*

Light Year 1 lyr is Distance λ travels in 1 yr *1 lyr = 9.46e15 m*

Parsec "parallax = 1 arc – second" *1pc = 3.26 lyr = 3.1e16 m*

1 arcsec distant object in triangle with Sun & Earth

1 arcsec = 1/3600 deg ! ! (See "Stellar Distances" below)

Time for light to travel to Earth from ……..

Moon 1 sec, Sun 8 min, Nearest star 5 yrs,

Nearest Galaxy 2e6 yrs

Light travels 10e16 meters/yr, 3×10^8 m/s

Looking back in time

Stars are observed in constant relative position, but

Planets move about ; Planet in Greek = wanderer

Planet orbit is an ellipse with Sun at one focus

40 dwarf Planets one of which is Pluto

Energy Conservation ➔ PE + KE = constant

plus ellipse ➔ radius and thus PE changes

so as R ↓, PE ↓ ➔ KE ↑, v ↑ and visa versa

Earth period 23 hr 56 minutes; Moon period = 27.3 days

Star is a massive plasma

Asteroid R < 100km

Comet; loose particles of ice and rock with a tail

Distances; Star-star 1 lyr; Diameter of galaxy 10^5 lyr;

galaxy to galaxy 10^6 lyr

Fusion inside the sun balances the gravitation forces

Proton proton chain fusion of H to He

Apparent Brightness (b) ***Energy / Area*** $b = L / 4 \pi d^2$

Luminosity (L) total ***Power*** in Watts

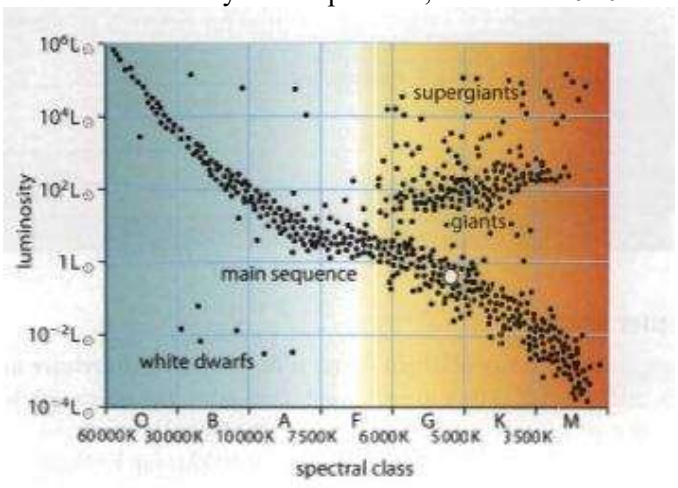
[energy radiated per sec] L[Sun] 3.839×10^{26} W

Stephan-Boltzman Law $P / m^2 = \sigma T^4$

$\sigma = 5.6e-8$ W / m^2 K^{-4} Power emitted = $L = \sigma A T^4$

Wein Displacement Law $\lambda\ [max] = 2.9 \times 10^{-3}$ km / T

Hertzsprung-Russell Diagram ……relates a star's Luminosity to Temperature, created in 1910



Main Sequence HR Diag approx linear L vs T

Stars …. 90% of Stars seen on this line

Stars off the Main Sequence

Red Giant ; cool, large, L = 100x L[Sun]

R = 10x R[Sun]

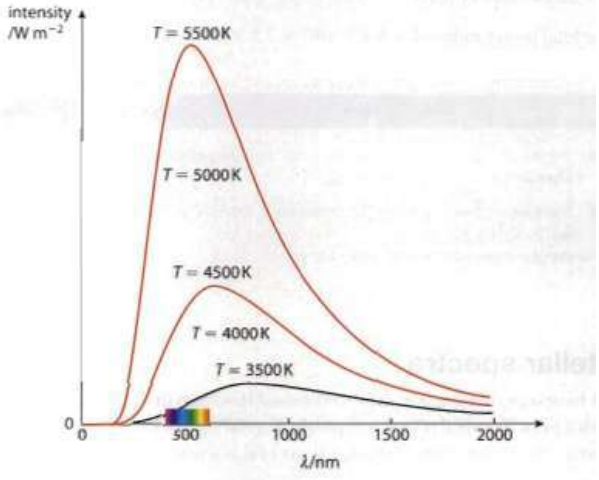
SuperGiants ; cool, very large, L = 10^6 x L[Sun]

R = 1000x R[Sun] , very rare, Betelgeuce

White Dwarf ; hot, small, R ≅ R[Earth], Low L

Neutron Star; Result of SG collapse, hot R≅ 12km

Black Body Radiation Intensity Distribution ….



Area under Intensity-Wavelength curve = Power

Black Body Radiation absorbs all, reflects none

Binary Stars Accretor – Donor pair

For an orbiting body about a mass M ……

Gravitational Force = Centripetal Force

$G M m / r^2 = m \omega^2 r$

so if we measure ω and r ➔ M

Binary Stars almost half the stars out there

$T^2 = 4 \pi^2 d^3 / G (M1 + M2)$ d = separation

T & d ➔ Mass of the 2 stars

Types of Binaries; Visual, Eclipsing, Spectroscopic

Donor gives up mass to the Accretor

Magnitude : Brightness Classifications

Harvard Classification; OBAFGKM

2000 degK to 60000degK

$(2)^5 = 32$ $(2.512)^5 = 100$

Apparent Magnitude m [viewed from Earth]

Greek scale was x2 each step for a range of x32

1 (brightest) to 6 (dimmest) $(2.512)^5 = 100$

Modern scale uses x2.512 each step, range = x100

Dimmest Star seen with; eye m=6, binoculars 10,

large telescope 20, photographic telescope 25

Brightness ratio $b1 / b2 = 2^{(m2-m1)}$

Absolute Magnitude M [viewed from dist 10 Pc]

Distance from Earth $d = 10 \times 10^{(m - M)/5}$ pc

If p= parallax in arcsec, $M = m + 5 (1 + \text{Log } p)$

Distance modulus $u = m - M$ $d = 10^{(1 + u/5)}$

Apparent (m) & Absolute (M) Magnitudes

	m	**M**
Sun	**–26.8**	**4.8**
Full Moon	**–15.6**	
Venus	**–4.4**	
Sirius	**–1.47**	**1.4**
Vega	**0.04**	**0.5**
Betelgeuse	**0.41**	**–5.14**
Polaris	**1.99**	**–3.6**
Pluto	**15.1**	

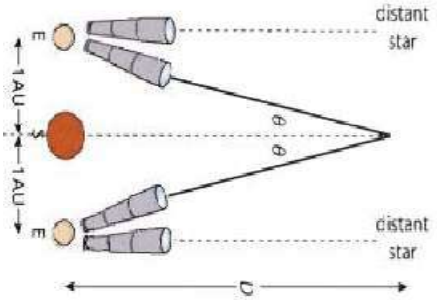
Stellar Distance Measurements

1/ Stellar Parallax Method

For small angles Tan θ = Sin θ = θ in radians

Tan θ = θ = AU / D ➔ **D = AU / θ**

AU is distance Sun-Earth for measurements 6 months apart to create the Parallax



1 Parsec = Distance ….. if θ = 1 arcsecond

so then Distance (pc) = 1 / Angle in arcseconds

smallest angle measurable from Earth = 0.01 arcsec due to atmosphere distortion. Satellites 0.001 arcsec

360 arcsec per arc deg; 1 arcdegree = 2π /360 rad

2/ HR Diagram; Given b & λ [max] ➔ Find L and d

measure λ [max] ➔ T from Wein's Law ➔

L from HR; with L ➔ d from Apparent brightness

$b = L / 4\pi d^2$ ➔ d = Sqrt [L/4πb]

3/ Cepheid Variabes; ***Important Standard Candles***

expand => bright [fast] & contract => dim [slowly]

L vs T is linear on a log-log plot

T ➔ L , + b ➔ d

Stellar Evolution & Processes

Birth dust particle gravitation not enough; a force like a Supernova needed for compression. 10^5 yr process

1/ Protostar; contracts,T↑, λ emitted but not visible due to dust

2/ Pre main sequence; More contraction,T↑, fusion

3/ Main sequence; contraction stops, fusion forces now balance gravitational forces

Star life process moves along & across HR Diagram

Small Star Heating/Fusion stops when Hydrogen runs out He then fuses to Carbon, until Fe iron is left in the core and fusion stops; Fe has max Binding Energy per nucleon,so it can not provide more energy for reaction. Smaller stars never make it to Fe and stop at Carbon

Chandrasekhar Limit**;** White Dwarfs can form only from stars w Mass < 4 M{Sun]

Big Star Layers of fusing elements; H, He, C, Ne, O, Si, Mg, Fe SuperNova to Neutron Star will be stable if Mass < 3 M[Sun]. the ***Oppenheimer-Volkoff Limit***

Black Holes ; If the Neutron star has M > 3 M[Sun] it will continue to collapse [increasing density] until not even light can escape and becomes a Black Hole.

Pulsars; Stars rotate (1 cycle/month)

As they collapse they gain speed due to Conservation of Angular Momentum. Since the magnetic field also then intensifies, exciting EM radiation near the poles that give a flashing of light with period 0.3 to 1.5 sec.

Discovered in 1967 by Jocelyn Bell

Cosmology

Models of The Universe

Newton; Infinte in size & age. Static. Uniform

Olber's paradox; ∞ # stars => why is sky black ?

Galaxies found in clusters, not randomly

Red Shift killed Newton's Static Universe

Big Bang explosion model

universe is expanding vs stars moving thru the universe; Balloon surface analogy for separation

Red shift = $\Delta\lambda / \lambda = v / c$ v = relative velocity

Hubble's Law Recession Speed vs Distance

He measured Recession speed & Distance of many Galaxies and found ***they are linearly related***.

Hubble's Constant H_o is the slope of this line

H_o = 72 km/sec / Mpc Divide by 3.09e19 km/Mpc

yields $H_o = 2.33e{-}18\ sec^{-1}$

Age of Universe = $1 / H_o = \frac{separation\ distance}{recession\ velocity}$

$1 / H_o$ = 4.292e17 sec = 1.36e10 yrs

Age of Universe is ≅ 13.6 Billion Years

Calculation assumes velocity is constant

Atoms did not form until 10^9 yrs when T=4000K which is ≅ 0.4ev < ionization energy of Hydrogen.

Cosmic Microwave Background Radiation CMB

Radiation from the Big Bang

Discovered/Confirmed in 1960's Penzias & Wilson

COBE satellite showed CMB not uniform ➔ galaxies can form

Future Possibilities of the Universe's Expansion

Open; keeps expanding

Flat; rate of expansion tends toward zero

Closed; expansion stops, contraction begins

Critical density is that which will cause Closed Universe

ρ [critical] = $3 H_0^2 / 8 \pi G =$ **10^{-26} kg/m^3**

this is just 6 H atoms per cubic meter ! !

Hubble's Law => $v = H_0 r$

Dark Matter does not emit or interact with light

MACHO Massive Astro Compact Halo Objects

WIMPS Weakly Interacting Massive Particles

Dark Energy is what can explain the accelerating recession of the galaxies.

	R km	M M[sun]	rho gm/m^3
White Dwarf;	e4	0.5	e6
Neutron Star	e1	1.5	e19
Black Hole	0	3.0	∞

Special Relativity

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Speed of Light is an Absolute Maximum

A constant of Nature

STR = Special Theory of Relativity

➔ ***Position, velocity, energy, momentum all relative***

…….. Galileo's Principle of Relativity

All Inertial motion [non accelerated, uniform] is relative

No absolute and well-defined state of rest

No privileged or absolute reference frames

Einstein extended this principle so that it included the constant speed of light [in inertial frames]

STR is defined in the absence of Gravity or acceleration

STR is based on two postulates:

1/ Relativity Principle: The laws of nature are the same in all non accelerated (inertial) or non gravitational reference frames

2/ *The speed of light in a vacuum is the same in **all inertial frames**. It is an **absolute** of Nature and NOT relative*.

STR is about both relative and high velocities

Mass–Energy equivalence $E = m c^2$

Mass–Energy Equivalence => Light bending

Not about Gravity or accelerated frames of reference

At speeds near speed of light 'c' other frames see

Time Dilation $t' = \gamma\, t$ Length Contraction $L' = L/\gamma$

Non Simultaneity [NS]

Gamma γ is the factor $[\, 1 / \text{sqrt}\,(\, 1 - [v/c]^2\,)\,]$

v	0	0.5 c	0.8 c	0.9 c	0.99c	1.0 c
γ	1	1.15	1.66	2.3	7	∞

Lorentz Transformation gives the parametric value in Special Relativity. One multiplies or divides by Gamma

STR Consequences ……

Mass–energy equivalence E = mc^2 => Light bending

Time dilation [in the moving frame seen by rest frame]

Length contraction [in moving frame seen by rest frame]

Relativistic mass increasing with speed

Different reference frames disagree about Simultaneity

Relativity of simultaneity disappears near c

Relativistic Doppler effect Thomas precession

Twin Paradox - travelling twin ages more slowly; which one travelled ? Ans; Neglect accel and decell [thus not inertial frames] is what causes the paradox

More STR Consequences and Paradoxes ……

Bell's spaceship paradox - 2 rockets w string between them going in same direction snapping due to LC

Ehrenfest paradox - Rotating disc; radius 'r' does not contract being perpendicular to motion, but circumference = 2 pi r does

Ladder - Garage Paradox - which contracts ? Incorrectly assumes absolute simultaneity

Mass–energy equivalence is a consequence of special relativity's speed of light limitation on mass

=> Equivalence of mass and energy,

$E = mc^2$ => photon energy behaves as mass; can bend in a G field.

Galileo Galilei had already postulated that all uniform motion is relative AND already showed gravitational motion is independent of mass ! !

Speed of Light 'c' is not just the velocity of a certain phenomenon— namely the propagation of electromagnetic radiation (light) — but rather a fundamental feature of the way space and time are unified as Spacetime.

Newton's other great achievement, the Universal Law of Gravitation, is not compatible with Special Relativity

General Relativity

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Curvature of Spacetime is the Absolute of Nature

GTR = General Theory of Relativity

➔ ***Acceleration & Force are relative concepts too***

Matter tells spacetime how to curve … and …

Spacetime geometry tells matter how to move.

Einsteinian Equivalence Principles

1/ Gravitational Field on Earth = Accelerated Frame in Space

You feel a Force …. *in either situation = [Normal force]*

It is a pseudo Force in that it is a result of Inertia

2/ Free Fall in GField = Inertial Frame in Space [Δ v = 0]

You feel NO Force … *in either situation*

General relativity is a geometric theory of gravitation

STR = Relativity for velocity [inertial frames]

GTR = Relativity for acceleration [non-inertial frames]

GTR removes an asymmetry from Newtonian Gravity, namely the distinction between inertial and non inertial [accelerated] frames of reference.

General relativity is the generalization of special relativity to include gravitation and/or acceleration (non inertial reference frames)

Special relativity is restricted to flat spacetime and constant velocity inertial frames.

Curvature itself => change in velocity direction

=> acceleration => Force = m a

GTR Predictions/Implications/Consequences ……

differ significantly from those of classical physics

geometry of space is curved; Riemannian

gravitational time dilation, - , processes close to a massive body run more slowly

gravitational time delay light signals take longer to move through a gravitational field

gravitational lensing ; light bends thru a G Field

gravitational redshift of light

gravitational waves analogous to EM Waves

orbital decay caused by emission of gravitational waves

predicts existence of black holes

Increases in Velocity or Gravity slow down time

Moving clocks run slower than clocks at rest STR

Clocks on Earth run slower than clocks in space GTR

Light slows down in gravitational fields GTR

$E^2 = (mc^2)^2 + (pc)^2$

E^2 = rest energy squared + momentum squared

v $E / c = pc$

Einstein deduced that free-fall is actually inertial motion

Free fall hides G force; You do not feel any Force

Acceleration creates an opposite pseudo Force

Gravity is a geometric property. Curvature is what creates the force called gravity. If you're driving along the highway and you hit a steep hill, or a curve in the road, that geometric curvature is going to change your velocity ➔ acceleration ➔ Force

The curvature of spacetime is directly related to the energy and momentum of whatever matter and radiation are present; spacetime is a relativistic manifestation of the existence of mass

Evolution of Ideas in Physics History …..

Earth flat => Earth curved;

Space Euclidean flat => SpaceTime Riemannian curved