

Physics Summary Sheets

AP Physics

International Baccalaureate High Level

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Mechanics

Pg 1

Force (v) a push or a pull @pfreda@gmail.com 1/14/2021

Tension, Compression, Shear, Gravitational, EMag, (v) = vector
Buoyant, Friction, Stiction, Normal, Fluid Resistive (s) = scalar
[in gases due to pressure, in liquids to viscosity]

A Force is what is required to move or stop an object/mass

A Force is required to change the speed or direction of a moving mass

Energy (s) the capacity to do Work Units of Joules = N-m

Types; Kinetic [KE], Potential [PE], Thermal/Heat [Q], Nuclear

Sources; Gasoline, Natural Gas, Solar, Chemical

Work (s) Transfer of Energy between macroscopic bodies

$W = \text{Force} \cdot \text{Distance} = F d \cos \theta$ [Component of Force in d direction]

Power (s) Energy [or Work] per unit time Units Watt = Joule/sec

Displacement (v) Change in position. Net distance travelled.

Gravity the natural phenomenon by which all physical bodies (mass) attract each other

Universal Law of Gravitation $F(\text{ULG}) = G m_1 m_2 / r^2$

[Newton's ULG] $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$

Newton's Laws apply only to inertial frames of reference (No Accel)

Accelerating frames create fictitious Forces due to Inertia

Newton's Laws of Motion "...united the Heavens and Earth"

1st Law Inertia $F=0 \Rightarrow a=0$ & $v=\text{constant}$

Inertia is a property of Mass

The tendency of mass is to resist a change in motion.

Nature's natural state is NOT rest, but constant velocity

If the velocity of a body is not constant, then a net Force

must be acting upon it. This is how nature works in the absence of Forces such as Gravity or Friction [Outer space]

2nd Law $\Sigma F = \text{Net Force} = m a$ $\Sigma F = dp/dt$

$\Sigma F = d/dt [m v] = m dv/dt + v dm/dt$

The sum of the Forces [net force] equals the rate of change

of momentum [$dp/dt = d/dt (m v)$] = $m a$ (if m is constant)

The acceleration of a body, not velocity, is proportional to the unbalanced / net Force acting on it.

$\Sigma \text{Vertical Forces} = m a_v$; $\Sigma \text{Horizontal Forces} = m a_h$

Note; $a=0$ if the objects are stationary or at constant velocity

Can use 2nd Law & the ULG to show that a pebble and boulder

will fall at the same acceleration and speed to Earth

But with air resistance, heavy objects fall faster

3rd Law $F_{AB} = -F_{BA}$ [action $\Rightarrow$ equal & opposite reaction]

For every action there is an equal and opposite reaction

Conservation of Momentum/Energy is the basic idea

The Forces act on two different bodies AND are = & opposite

but they should NOT be thought to balance or cancel

This Law explains why rockets work in the vacuum of outer space

Ex; Gun recoil; bullet & gun. Ex; Walking; Person & ground

Conservation Laws Energy can neither be created nor destroyed

1/ **Energy** $\Delta ME = 0$ $ME = \text{Mechanical Energy}$ $ME = KE + PE$

Kinetic Energy $= KE = \frac{1}{2} m v^2 = \text{Energy of translational motion}$

Potential Energy $= PE = m g h = \text{Energy doing work against a Force}$

$\Sigma KE + PE (\text{before}) = \Sigma KE + PE (\text{after})$

2/ **Momentum** $\Delta p = 0$ $p = m v$ (assumes no external Forces)

$\Sigma m v (\text{before}) = \Sigma m v (\text{after})$

The Sum of the Initial and Final Momentums of the combined 2 bodies

must be equal ; assuming an isolated system and no external forces

3/ **Angular Momentum** L $\Delta L = 0$ (no external Torques)

$L = m v r = I \omega = r \times p$ $\Sigma m v r (\text{before}) = \Sigma m v r (\text{after})$

Work – Energy Theorem $W = \Delta ME$

Work = Change in ME = Work done ON or BY the system

Free Body Force Diagrams Vector picture description of the Physics

All Forces acting on ONLY 1 body. Ex: A ladder leaning against wall has

5 forces acting on it; Weight, 2 friction (wall, floor), 2 normal (wall, floor)

Projectile Motion

Parabolic trajectory

Vertical motion is constant acceleration $a = -g$

Horizontal motion is constant velocity $\Rightarrow a = 0$

Max Range $\Rightarrow \theta = 45 \text{ deg}$; Range = v^2 / g

Position Equation for both vertical and horizontal motion

$$s(t) = s_0 + v_0 t + \frac{1}{2} a t^2$$

s_0 = initial position , v_0 = initial velocity, t = time

Derived by Integrating $(d/dt)^2 y(t) = a$ two times

Suvat Equation v = final velocity, u = initial velocity, a = constant

$$v^2 - u^2 = 2 a s ; 1/ s = \frac{1}{2} (u + v) t ; 2/ a = \frac{v - u}{t}$$

Solve for "t" in 1 and substitute in 2 or visa versa

Circular Motion / Constant Speed

Circular motion requires a Centripetal Force & acceleration to change the tangential motion to circular motion.

Supplied by either .. 1/ Gravity 2/ Tension 3/ Friction 4/ banked curve

****F(centripetal) = $m v^2 / r$ = F(gravity) or F(tension) or F(friction)****

Examples; Ball swinging on string - String; Orbits – Gravity;

Turning Car – Friction between tires & road ; Satellites – Gravity

Centripetal **Acceleration** $a = v^2 / r = \omega^2 r$

Centripetal **Force** $F = m a = m v^2 / r = m \omega^2 r$

Tangential **Velocity/Acceleration** $v_T = \omega r$ $a_T = \alpha r$

v_T = Circumference / Period $v_T = 2 \pi r / T$

a & F radial vectors toward the center or axis of rotation

v_T & a_T vectors in the tangential direction of motion

ω & α angular vectors [perpendicular to plane of rotation]

T = Period of Rotation/Revolution $f = 1/T$ = frequency

Satellite Physics Projectile Motion never reaching Earth

Potential Energy **PE = Work done against a Force**. For Orbits it is negative b/c

work is done from ∞ to R by the Field/Force [not against it] due to attraction

**** PE = $\int F dr = \int GMm/r^2 dr = -GMm/r$ (PE = 0 at infinite r) $PE = -GMm/r$**

**** F(cent) = F(G)**; $m v^2 / r = GMm/r^2 \Rightarrow m v^2 = GMm/r \Rightarrow KE = GMm/2r$

$PE = -GMm/r$ $KE = GMm/2r = \frac{1}{2} m v^2$ $TE = -GMm/2r$ Height $h \Rightarrow r = R_E + h$

$|PE| = 2 |KE|$, $TE = KE + PE = KE - 2KE = -KE$ $KE = -\frac{1}{2} PE$, $2 KE + PE = 0$

Total Mechanical Energy $TE = PE + KE = -GMm/r + GMm/2r = -GMm/2r$

At Earth's surface $PE = mgh = PE(R_E + h) - PE(R_E) = GmMh/(R_E(R_E + h)) \sim GmMh/R_E^2 = mgh$

**** For Escape**; $KE = PE \Rightarrow \frac{1}{2} m v^2 = -GMm/r \Rightarrow v[\text{escape}] = \text{Sqrt} [2GM/R]$

**** For Orbit** ; $F(\text{cent}) = F(G)$; $m v^2 / r = GMm/r^2 \Rightarrow v[\text{orbit}] = \text{Sqrt} [GM/R]$

$v[\text{orbit}] = \text{Sqrt} [GM/R]$

$v[\text{escape}] = \text{Sqrt} [2GM/R]$

Satellites exist in a negative potential well with $KE < PE$

Lower orbit satellites have $> KE$ but lower TE ; If $TE = 0$ or > 0 the object will escape

Work against Gravity is needed to go to a larger radius

Elliptical Motion / Orbits $F(\text{Centripetal}) = F(\text{Gravitation})$

Orbital motion is a form of **Projectile Motion**

Velocity is not constant Conservation of Angular Momentum $\Rightarrow$

$L = m v r = \text{constant because } r \times F = \tau = 0 \text{ at all pts} \Rightarrow m_1 v_1 r_1 = m_2 v_2 r_2$

$F(\text{centripetal}) = m v^2 / r = m \omega^2 r = F(\text{ULG}) = G M m / r^2$

$v^2 = G M / r$; $\omega = 2\pi/T$ $a = \omega^2 r = 4\pi^2 r / T^2 = G M / r^2$

Kepler's 1st Law / Orbits Planetary orbits are ellipses

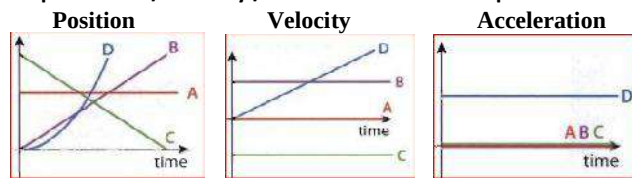
Kepler's 2nd Law / Areas Equal areas swept in equal times

Kepler's 3rd Law / Periods $T^2 / a^3 = 4 \pi^2 / G M = \text{constant}$

"a" = half of Elliptical Major Axis or Radius of Circle

Radius (r) / 1/2 Semi-major axis (a) $\leftrightarrow$ Period T

Displacement / Velocity / Acceleration Time Graphs



(slope) differentiate $\Rightarrow$ $\leftarrow$ integrate (area)

The area under the $y = f(x)$ graph is "y times x"

To go from one graph to the next one

Find Slope of the tangent line at that point OR Area under the curve to that

point Slope of $s \Rightarrow v$ [$v = ds/dt$], Slope of $v \Rightarrow a$ [$a = dv/dt$]

Area under curve; $v \Rightarrow s$ [$s = vt$]; $a \Rightarrow v$ [$v = at$]

Linear Momentum & Collisions

No potential energy

Elastic; $\Delta p = 0$ $\Delta KE = 0$ Energy & Momentum are Conserved

Inelastic; $\Delta p = 0$ Momentum ONLY is Conserved

Elastic hard objects ; No energy is lost

Inelastic objects crush or stick together

Energy is lost to heat, sound and vibration

If the objects stick together it is called "perfectly inelastic"

Explosions are always inelastic, the reverse of a perfectly inelastic collision.

The small bits get most of the KE from the chemical energy.

Conservation Laws $\sum m v_{initial} = \sum m v_{final}$ & $\sum KE_{initial} = \sum KE_{final}$

Impulse $J = \Delta p = F \Delta t$ $\Delta p = \Delta mv = m \Delta v = ma \Delta t = F \Delta t$

Newton's 2nd Law $F = dp/dt = m dv/dt + v dm/dt = ma + v dm/dt$

Classic 1 Dimensional Elastic problem; mass 1 collides w mass 2

$v' = v_{(final)}$ $v = v_{(initial)}$

$$v_1' = [(m_1 - m_2)/(m_1 + m_2)] v_1 + [(2m_2)/(m_1 + m_2)] v_2$$

$$v_2' = [(2m_1)/(m_1 + m_2)] v_1 + [(m_2 - m_1)/(m_1 + m_2)] v_2$$

If $v_2 = 0$ $v_1' = [(m_1 - m_2)/(m_1 + m_2)] v_1$

$$v_2' = [(2m_1)/(m_1 + m_2)] v_1$$

$v_1 + v_1' = v_2 + v_2'$ $= [(2m_1) v_1 + (2m_2) v_2] / [m_1 + m_2]$

For 1 Dim Elastic Collisions ONLY Easier to use than the KE equation

Sum of initial and final velocities for each mass are equal

Relative velocities $v_1 - v_2 = -(v_1' - v_2')$ equal & opposite, before & after

$m_1 [v_1' - v_1] = -m_2 [v_2' - v_2]$ $\Delta p = 0$ total change p = 0

Net momentum change is equal and opposite for the masses

Hooke's Law Springs Mass-Spring Simple Harmonic Oscillator

$F [Restoring] = -k x$ [1676 Robert Hooke]

Restoring Force is proportional to spring compression/stretching

PE = work done compressing/stretching = $\int F dx = \int k x dx = \frac{1}{2} k x^2$

$PE = \frac{1}{2} k x^2$ spring energy stored = Area under the F vs x curve

Elevator Acceleration = a $F = mg \pm ma$

You weigh more on an elevator accelerating up or

decelerating on its way down; Weight = $F = mg + ma$

You weigh less on an elevator accelerating down or

decelerating on its way up; Weight = $F = mg - ma$

Elevator in free fall [a = g] $\rightarrow Wt = F = 0 \rightarrow$ weightlessness

Einstein's Equivalence Principle of Gravity and Acceleration

1/ Acceleration is equivalent to and thus appears to create a "pseudo" Gravitational Force in the opposite direction of the acceleration. In the Inertial Frame it is an **INERTIAL FORCE**.

2/ Free Fall is equivalent to Rest in a Mass/Gravity free region

Both are Inertial Frames of Reference; Newton's Laws hold.

Terminal velocity of a falling object is the velocity of the

object when the sum of the upward drag force (prop to v^2) and upward buoyancy force equals the downward force of gravity.

Since the net force on the object = 0, the object has no acceleration.

Young's Modulus E Stress σ to Strain ϵ Ratio

$E = \sigma / \epsilon = [F/A] / [\Delta L / L]$; for the linear region ...

Hooke's Law constant 'k' = $E A / L = F / \Delta L$

Equilibriums $\sum F = 0$

1/ Translational Sum of Forces = 0

No Acceleration or unbalanced forces. But the body can still be in motion at constant speed Skydiver falling at terminal velocity speed is in TE.

2/ Rotational Sum Torques = 0

3/ Static Sum of forces = 0 & Sum of Torques = 0

Object in SE can be both spinning at constant speed and have its center of mass moving linearly at constant speed.

Friction $\rightarrow$ Friction ALWAYS OPPOSES motion $\leftarrow$

1/ A Force that resists the sliding or rotating motion

2 Kinds; Static/Starting Friction [Stiction] & Kinetic Friction

Both are proportional to the Normal Force $F_{fr} = \mu F_N$ μ = coefficient of friction

2/ A 3rd Law reaction Force that propels a car or person to move forward in reaction to the force backwards by the tire or foot.

Normal Force $\rightarrow$ Always Perpendicular to the surface $\leftarrow$

A Contact Force But not always = m g !

Rotational Motion

Torque = Rotational Force

Position Equation for Rotating motion

$$\theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} \alpha t^2$$

θ_0 = initial position, ω_0 = initial velocity, t = time

Suvat Equation for Rotating Motion

$$\omega_f^2 - \omega_0^2 = 2 \alpha \theta$$

$$v = \omega r \quad x = \theta r$$

$$KE (rotation) = \frac{1}{2} I \omega^2$$

$$a = \alpha r$$

Linear – Rotational Parameter Analogs

x = displacement $x = \theta r$

θ angle

$v = dx/dt$ $\omega = d\theta/dt$

ω angular velocity

$a = dv/dt$ $\alpha = d\omega/dt$

α angular acceleration

$F = ma$ $\tau = I \alpha = r \times F$

τ Torque

$F = dp/dt$ $\tau = dL/dt$

Rate of Change of Momentum

$p = mv$ $L = I \omega = m v r$

L Angular momentum

$KE = \frac{1}{2} mv^2$ $KE = \frac{1}{2} I \omega^2$

KE Mechanical Energy

$W = F \text{ dot } d$ $W = \tau \text{ dot } \theta$

W Work

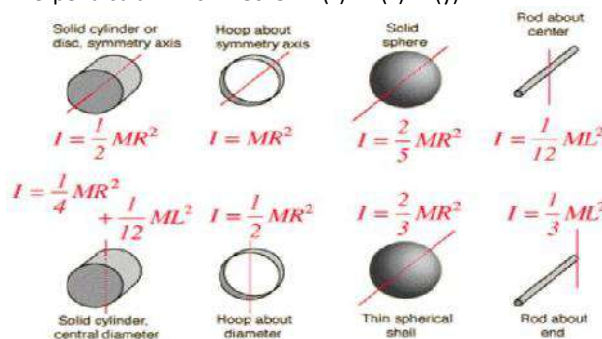
Moment of Inertia = Rotational Inertia = Angular Mass

Moment of Inertia $I = \sum m r^2 = \int r^2 dm = \rho \int r^2 dV$

For a point mass M, $I = M R^2$ R = distance from axis

Parallel Axis Theorem $I' = I + m d^2$ d = distance CofM to new axis

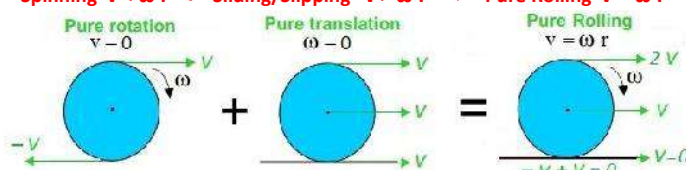
Perpendicular Axis Theorem $I(z) = I(x) + I(y)$



See Wikipedia page .. / Listing_of_Moments_of_Inertia

Rolling w/o Slipping Motion $\rightarrow v = \omega r$ v = velocity CofM

Spinning $v < \omega r$ + Sliding/Slipping $v > \omega r \Rightarrow$ Pure Rolling $v = \omega r$



Rotation Axis = Center

No Axis

Rotation Axis = Bottom

$$KE = \frac{1}{2} I \omega^2 = \frac{1}{2} I (v/r)$$

$$KE = \frac{1}{2} m v^2$$

$$KE = \frac{1}{2} m v^2 + \frac{1}{2} I \omega^2$$

Pure Rolling ... The top of a car's wheel moves twice as fast, 2x, as the car itself !!!

Pure Rotation (spinning) $\Rightarrow v [top] = v$; $v [CM] = 0$; $v [bottom] = -v$

+ Pure Translation (sliding/slipping) $\Rightarrow v [top] = v [CM] = v$; $v [bottom] = v$

= Pure Rolling $\Rightarrow v [top] = 2v$; $v [CM] = v$; $v [bottom] = 0$ $v = \omega r$

Inclined plane Two cylinders **Rolling or Sliding** down at $\angle \beta$

Objects w different mass, radius, density, length OR geometry have

Rolling Solid cylinder $a = \frac{2}{3} g \sin \beta$ $I_c = \frac{1}{2} MR^2$

Rolling Hollow cylinder $a = \frac{1}{2} g \sin \beta$ $I_c = 1 MR^2$ [same as point mass]

Sliding Solid or Hollow $a = g \sin \beta$ [No Friction] $a = g (\sin \beta - \mu \cos \beta)$ [Friction]

$v(t)$ & $a(t)$ depend ONLY on geometry $\angle \beta$, and moment of inertia I_c

A race down the plane, either rolling or sliding, is a tie as long as both cylinders are hollow or solid. The solid cylinder will always beats the hollow one

$\rightarrow$ Sliding has same result as projectile, orbital, pendulum & free fall motion $\leftarrow$

The position, velocity and acceleration are independent of Mass, Radius, Density or Length !! Highly non intuitive

Explanation Sliding: Larger Mass $\rightarrow$ More Force/acceleration, $F = mg$

BUT ... Larger Mass $\rightarrow$ More Inertia $\rightarrow$ Resists acceleration

So the 2 properties balance and cancel each other $\rightarrow$ Independence of Mass



Simple Machines

Mechanical Advantage; Output / Input of Force, Distance, or Speed. Increase/Decrease [or Magnify] Force, Distance or Speed by a tradeoff due to the Conservation of Energy

Lever ; Lever Trades distance for reduction in Force;

Force or Distance magnifier

Levers Classifications "what is in the middle ? "

1/ Pivot [Fulcrum]; crowbar, scissors

2/ Load ; wheelbarrow, nutcracker, bottle opener

3/ Effort ; tweezers, shovel

Inclined Plane ; Trades distance for Effort/Force.

Knife/Blade/Axe; are double inclined planes

Wedge ; trades lateral force for longitudinal force

Screw ; is an Inclined Plane wrapped around a shaft

Trades/Converts Rotational motion for/to Linear motion

Scissors ; Type 1 Lever, Force magnifier uses torque [force] away from pivot

Gears ; Force Distance Speed magnifiers

Wheel & Axel ; Continuous Rotational Lever using

torque [handle to axel] Arm is a distance magnifier

Ex; bike handle or pedal, car steering wheel, doorknob

Wheel Barrow ; Type 2 Lever with load in between the pivot and the force

Pulleys; Single fixed allows pulling down rather than up

Mechanical advantage = # ropes lifting the load x 100%

3x, 4x, etc reduce force needed by factor of 3, 4, etc

Force magnifier ; Smaller force moves the longer distance

Friction, rope & pulley mass reduces multiplier < 100%

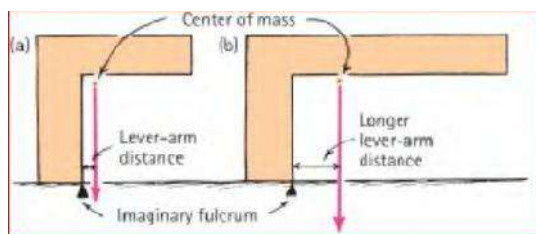
Stability of Objects Centers of Mass (CM) & Gravity (CG).

Instability occurs when the vertical force due to CM is **outside the base** of the object. This results in a torque that tends to rotate the object and cause it to fall over.

Objects with wide bases and low CG are more stable

If CM is not over the base, the Level Arm length OR distance to the pivot point is a measure of instability

Larger the Lever Arm → Greater the instability



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To find CM, suspend object from a point. CM lies on the vertical line to ground. Repeat for another point and the intersection is CM. You should be able to balance the object from the CM.

** Since 'g' decreases with height and mass does not →

→ Center of Gravity CG is below CM for large objects

Empire State Bldg CG is 1 mm below its CM

CM can be located outside the boundaries of the mass

This is why the upside down Fosbury Flop high jump works so well. The high jumper can get his body over the bar while his CM is under the bar. So he does less work raising his CM / Weight.

Stability of Ships;

Differs from Stability of Objects due to the additional Buoyant Force.

FB Buoyant Force is UP ; FG Gravity Force is DOWN

CB = Center of Buoyancy = CM Center of Mass of the displaced H₂O

CG = CM Center of Mass of the Ship

For Stability; CG must be under the CB & as low as possible

CG Force is down & CB Force is up. When aligned, they cancel

If not vertically aligned → torque

The ship will rotate until the 2 CM's line up vertically → no torque

CG above CB causes unstable rotation & ship to capsize,

CB above CG causes stabilizing restoring torque & stability.

Standing up in a boat raises CG above CB → instability

Fundamental Forces; Gravity, Electromagnetic, Weak, Strong Nuclear.

Non Fundamental Forces; Normal, Friction, Tension, Compression, Elastic, Buoyant, Bernoulli.

Conservative Forces; Energy exchanges between Kinetic & Potential Energy. Gravity, Electromagnetic, Spring

Non Conservative Forces; Friction, Stiction, Tension, Compression, Drag

Ordinary differential equations (particles)

Partial differential equations (fields)

Gravitational Mass; the property of mass concerning G Field

1/ Active Gravitational Mass → creates the Field / attractive Force

2/ Passive Gravitational Mass → feels the Field / attractive Force

Inertial Mass; property of mass that resists a change in velocity resists a change in motion. More Mass → More resistance

*Mass has both properties → **motion is independent of mass**

Inertial Force [Pseudo, Fictitious]

An apparent force in an accelerating frame, [non inertial frame]

1/ an accelerating, decelerating or turning car 2/ an elevator

3/ a turning or rotating frame that explains the observation or "feeling of a Force".

These "apparent forces" are considered fictitious because they **do not exist in inertial frames** [that are not accelerating].

Einstein's Equivalence Principle of Gravity and Acceleration

→ States that acceleration is equivalent to a force due to Inertia ←

If $F = m a$, then not only does Force create acceleration, but acceleration creates "pseudo" Force due to Inertia.

Ex 's; Centrifugal, Coreolis, or Simulated Gravity 'forces'.

Newton's Laws do not hold in non inertial frames. So there is no 3rd Law reaction force seen in this frame

***Gravitational Force itself is a pseudo force in Einstein's General Theory of Relativity. Acceleration and Attraction of mass is caused by Mass creating a Curvature of Space-Time

***In a curved spacetime, all frames are non-inertial ***

Centrifugal Force is NOT a Force of Nature.

It is really an Inertial Force Inertia Newton's 1st Law CF exists [is seen & felt] only in the motion [non inertial] frame of reference, not the stationary frame.

It is an Inertial, pseudo or fictitious force.

Ex; A ball will appear to move toward the outside radius

[passenger door] of a car turning left w driver on the left.

But it is really moving straight due to its Inertia. It is the door that is moving to the ball due to friction acting on the tires.

More Examples; Conical Pendulum, Orbits, Banked curve,

Ball swinging on string in an accelerating car

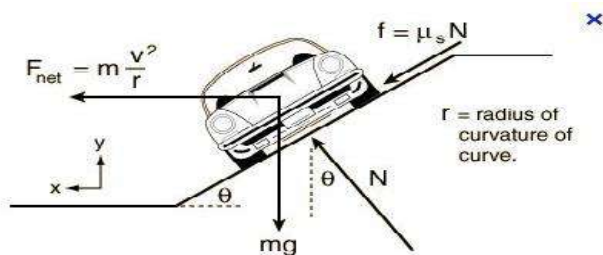
Coreolis Force the Fictitious "Force" that explains curved motion in a rotating frame. Hurricanes rotate CCW in NorHemisp

Banked Curve Analysis

Two Forces; Gravity and Friction

Decompose each force to X and Y components

Sum the forces in horizontal and vertical directions



Force equations at maximum speed v , at threshold of eliding up incline.

$$\Sigma F_x = m \frac{v^2}{r} = N \sin \theta + \mu_s N \cos \theta$$

$$\Sigma F_y = 0 = N \cos \theta - \mu_s N \sin \theta - mg$$

Solving this pair of equations for the maximum speed v gives:

$$v_{\max} = \sqrt{\frac{rg(\sin \theta + \mu_s \cos \theta)}{\cos \theta - \mu_s \sin \theta}}$$

The limiting cases are:

$$v_{\max} = \sqrt{rg \tan \theta}$$

Frictionless case

$$v_{\max} = \sqrt{rg \mu_s}$$

Flat roadway

[carbank.gif](#)

Weightlessness "feeling" no Force on you; absence of Normal F

- 1/ Being in a region with no nearby mass/gravity
- 2/ Having **only the Force of Gravity** acting on you
- 3/ Being in a room in free fall so you feel no normal upward force.
- 4/ No Normal Force up $\rightarrow$ You **DO NOT** feel your own weight, mg .

Angular Momentum & Torque

Any mass with Linear Momentum about a point (axis of rotation) will also have an Angular Momentum

Angular Momentum = Linear Momentum $\times$ Radius

$$L = p \times r = m v \times r$$

r = Radius = Perpendicular distance between object and the point (axis) p = Linear momentum

L = Angular Momentum v = linear velocity of object

$$\text{Magnitude of } L = |L| = m v r \sin \theta$$

θ angle between r and v vectors

Conservation of Angular Momentum

For rotation of a mass $|L(\text{center})| = m |r \times v| = m r v$

L = Spin angular momentum

L is conserved ONLY about the center point

$$dL/dt = dr/dt \times p + r \times dp/dt \quad [dp/dt = F]$$

$$dL/dt = r \times F = \tau \quad [\text{Torque}]; \quad \tau = 0 \iff dL/dt = 0$$

Conservation of Angular Momentum $\iff dL/dt = 0$

If r and F are parallel $\sin \theta = 0$ so no torque

[analogous to $d/dt [mv] = m dv/dt = m a = F$]

Angular momentum L of a system will not change if no net external torque $[r \times F]$ is present

L will be Conserved.

Ice skater spins faster as she brings her arms / mass to the center which reduces the Moment of Inertia

$L = I \omega$, if I goes down $\rightarrow \omega$ goes up

Rigid Bodies & Oscillation

Spin Angular Momentum L is an intrinsic property of Rotating mass just as Inertia is of Linear mass

$$\tau = r \times F \quad \alpha = \theta'' \quad \omega = \theta' = d\theta/dt$$

$$\tau = I \alpha \quad \tau = r \times F = r \times m a = m (r \times a)$$

$$L = r \times p \quad L = I \omega \quad dL/dt = \tau \quad [\text{external}]$$

$$L = r \times p = r \times m v = m (r \times v)$$

Conservation of Angular Momentum $\rightarrow dL/dt = 0$

Conservation of Linear Momentum $\rightarrow dp/dt = 0$

Rod on frictionless surface; spin it about its c of m

If $F = 0$ then τ (any point) = 0

L = same relative to any point

$$L_{cm} = I \omega = \left[\frac{1}{2} m l^2 \right] \omega \quad [I \text{ from Moment tables}]$$

intrinsic spin angular momentum

Ruler $[l=2d]$ lying on a frictionless surface

hit by Impulse I at its end.

C of M moves along line of direction of I vector

$$I = F \Delta t = M a \Delta t = M v \rightarrow v_{cm} = I / M$$

L [before] = 0 t not 0 $\rightarrow L$ [after] not zero

velocity of c of m not 0 (where I hits the ruler)

but L is affected

$\tau = 0$ L conserved 0 before and 0 after

Hanging ruler pin at point p ; forces thru pt p

b = distance C of M to p ; l = length

$$\tau = r \times F = M g b \sin \theta = -l \alpha$$

$r p$ vector [center to p] $\times$ mg thru pt c (c of m)

$$M g b \sin \theta + l \theta'' = 0$$

$$\theta'' + [M g b / l] \theta = 0 \rightarrow \text{SHM}$$

$$\theta(t) = \theta_{\text{rest}} \cos \omega t + \phi \quad \omega = \text{Sqrt}[M g b / l]$$

$$T = 2 \pi \text{Sqrt}[l / M g b]$$

Hanging Hula Hoop

$$\tau = M g R \sin \theta \quad I = M R^2 + \text{same} = 2 M R^2$$

$$T = 2 \pi \text{Sqrt}[2 M R^2 / M g R]$$

$$T = 2 \pi \text{Sqrt}[2 R / g]$$

Period for both ruler and Hula is only dependent on geometry. This is highly non intuitive as one would expect mass to affect Period.

Just as with falling mass or a simple pendulum

$$T = 2 \pi \text{Sqrt}[L / g] \quad [L = \text{length of pendulum}]$$

Comparison of Swing Period T for different objects

$$T = 2 \pi \text{Sqrt}[X] \quad X = \dots\dots$$

Pendulum	L / g	Ring	$2 R / g$
Rod	$2 L / 3 g$	Disc	$3 R / 2 g$

$\rightarrow$ The motion for all shapes is independent of mass ! $\leftarrow$

Period for Spring-Mass System = $T = 2 \pi \text{Sqrt}[m/k]$

Parallel Axis Theorem $I' = I + m d^2$

Parallel Axis Theorem adjusts Moment of Inertia for new axis

d = distance C of M to new axis

$$I_p = \frac{1}{12} M l^2 + M b^2$$

$$T = 2 \pi \text{Sqrt}[(l^2 / 12 + b^2) / g b]$$

Symbol	Concept	Units	Fundamental Units	Vector Scalar	Equations
(a)	Acceleration	meters / sec ²	meters / sec ²	V	$a = \Delta v / \Delta t$
(θ)	Angular Displacement	radians or degrees		S	$\theta = S / r = \text{arc length} / \text{radius}$
(ω)	Angular Velocity	rad / sec		V	$\omega = v / r ; \omega = 2 \pi f ; \omega = 2 \pi / T$
(α)	Angular Acceleration	rad / sec ²		V	$\alpha = a / r ; \alpha = d\omega / dt$
(L)	Angular Momentum	kg - m ² / sec	kg - meter ² / sec	V	$L = I \omega ; L = r \times p = r \times m v = m (r \times v)$
(I)	Moment of Inertia	no units	kg - meter ²	S	Obtained from Tables of Objects $I = MR^2$
(τ)	Torque / Moment	N - m	kg meter ² / sec ²	V	$\tau = I \alpha ; \tau = r \times F = r \times m a = m (r \times a)$
(q)	Charge	Coulombs (C)		S	FUNDAMENTAL UNIT
(I)	Current	Ampere = Coulombs / sec		V	$I = q / t$
(s)	Displacement	meters (m)	meter	V	$s(t) = s_0 + v_0 t + \frac{1}{2} a t^2$
(E)	Energy	Joules = Newton - meter	kg meter ² / sec ²	S	$E = P t$
(KE)	Kinetic energy	Joules (J)	kg meter ² / sec ²	S	$KE = \frac{1}{2} m v^2$
(PE)	Potential Energy	Joules	kg meter ² / sec ²	S	$PE = m g h$
(W)	Work	Joules	kg meter ² / sec ²	S	$W = F d$
(Q)	Heat	Joules	kg meter ² / sec ²	S	$Q = m c \Delta T + m L$
(L)	Latent Heat	Joules / Kg	meter ² / sec ²	S	Vaporization or Fusion
(f)	Frequency	Cycles / sec	1 / sec	S	$f = 1 / T$
(F)	Force	Newton (N)	kg meters / sec ²	V	$F = m a$
(P)	Pressure	Pascal (Pa) = N / m ²	kg / meter sec ²	V	$P = F / A ; P = \rho g h$
(m)	Mass	Kilograms (kg)	kg	S	FUNDAMENTAL UNIT
(p)	Momentum	N - sec	kg meter / sec	V	$p = m v ; F = dp/dt ; F = m dv/dt + v dm/dt$
(I)	Impulse	N - sec	kg meter / sec	V	Change in Momentum $I = \Delta p = F \Delta t$
(s)	Position	Meters	meter	V	FUNDAMENTAL UNIT
(P)	Power	Watts = Joules / sec	kg meter ² / s ³	S	$P = E / t = W / t = F d / t = F v = \tau \omega$
(P)	Electrical Power	Watts = Joules / sec	kg meter ² / s ³	S	$P = I V ; P = I^2 R = V^2 / R$
(R)	Resistance	Ohms = Volts / Amperes		S	$R = \rho L / A$
(v)	Speed	Distance / time	meter / sec	S	$v = \Delta d / \Delta t$
(T)	Period	Seconds	sec	S	$T = 1 / f$
(t)	Time	Seconds (sec)	sec	S	FUNDAMENTAL UNIT
(T)	Temperature	degrees		S	degK = degC + 273
(v)	Velocity	Displacement / time	meter / sec	V	$v = \Delta s / \Delta t$
(V)	Voltage	Volts = Joules / Coulomb		S	$V = I R ; V = E / q$

Fundamental Unit Comparisons

$p = m v = F \Delta t$	$= \text{kg m / s}$	Momentum
$F = m a$	$= \text{kg m / s}^2$	Force (Newton)
$W = F d$	$= \text{kg m}^2 / \text{s}^2$	Work / Energy (Joule)
$P = F d / t$	$= \text{kg m}^2 / \text{s}^3$	Power (Watt = Joule/sec)
F / m	$= \text{m / s}^2$	Field Strength
	$= F / m = F / \text{kg} = \text{J / m kg} = \text{kg [m / s}^2\text{] m / kg m} \Rightarrow \text{m / s}^2$	
PE / m	$= \text{m}^2 / \text{s}^2$	Field Potential Energy = $F \times d$
	$= PE / m = \text{J / kg} = \text{kg [m / s}^2\text{] m / kg} = [\text{m / s}^2] \text{ m}$	

Thermal Physics

Pg 1

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Work W Energy transfer between physical macroscopic bodies

Heat Q Energy transfer given to a cold body from a hot body
by Conduction, Convection or Radiation

Temperature T a measure of the average KE of all the molecules

degKelvin = degCelsius + 273 ; degFahrenheit = 9/5 degC + 32

Internal Energy U the sum of the KE of all the molecules

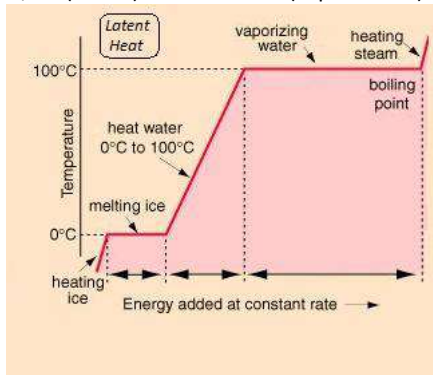
Joule's Law Energy $U = f(T)$ $U = 3/2 PV = 3/2 nRT$

Latent Heat L Amount of heat needed to change state without a ΔT

Latent Heat of Vaporization ; change liquid $\leftrightarrow$ gas

Latent Heat of Fusion ; change solid $\leftrightarrow$ liquid

For H₂O; L (fusion) = 3.35e5 L (vaporization) = 2.27e6 J / Kg



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N_A = Avogadro's # = # particles/mole = 6.022×10^{23} particles/mole

Mass of Hydrogen Atom = $1 / N_A$ in grams 1 gram H = 1 mole

Mass of Hydrogen Atom = 1.66×10^{-24} grams 12 gram C = 1 mole

1 mole = mass number of the atom in grams [A = Protons + Neutrons]

Mole has equal # molecules/particles N_A but not equal mass

1 mole @ RoomTemp & 1 atm ALWAYS has Volume = $RT/P = 24$ liters

Absolute Zero = -273 degC = 0 degK

$Q = m c \Delta T + m L$ $C = Q / \Delta T$

Q = Heat in Joules ΔT = change in temperature

m = mass, kg; c = Specific Heat Capacity; for H₂O = 4200 J / kg degC

C = Thermal Capacity J / degC L = Latent Heat J / kg

Avogadro's law 1/ "Equal volumes of all gases, at the same temperature and pressure, have the same number of molecules."

2/ For a given mass of ideal gas, the volume and amount (moles) of the gas are directly proportional if the temperature and pressure are constant.

Three States of Matter; Solid, Liquid, Gas [Plasma]

Liquid $\rightarrow$ Gas 1/ **Boiling**; all of the liquid is at same Temp

2/ **Evaporation**; only at the surface; can be at any Temp

Rate of evaporation proportional to

1/ surface area 2/ blowing the vapor cloud away

3/ increasing Temp [KE] **Evaporation is a cooling processes !**

Ideal Gas Identical, perfectly elastic, very small spheres

No forces between $\rightarrow$ constant velocity motion

Molecules hit the wall of container causing Pressure $P = F / A$

Triple point [all states in equilibrium] H₂O is 0.01 degC at P = 4.6 mm Hg

*****Linear relation Laws which led to the Ideal Gas Law*****

Charles' Law V vs T; Boyle's P vs V; Gay-Lussac's P vs T $\rightarrow$

Ideal Gas Law $PV = n R T$ **Molar View**

n = # moles R = Molar Gas Constant = $8.31 \text{ J mol}^{-1} \text{ degK}^{-1}$

Boltzman Gas Law $PV = N k T$ **Molecular View**

N = # molecules k = Boltzman Constant = $1.38 \times 10^{-23} \text{ J / deg K}$

$N = n N_A$ $N k = n R$ $k = R / N_A$ $R = k N_A$

Molecular view vs Molar view

Ideal Gas Law also called **Characteristic Gas Law**

0th Law of Thermodynamics All diathermal walls are equivalent

All heat is of the same nature All thermal equilibriums are equivalent

Allows Temperature to be a valid State variable and a measure of the Work value of Heat

1st Law of Thermodynamics **Conservation of Energy**

$$\Delta U = Q + W$$

ΔU = change in internal energy,

Q = Heat added to system W = work done on the system

$\rightarrow$ Heat and Work are forms of energy transfer $\leftarrow$

2nd Law of Thermodynamics Entropy $\Delta S > 0$ [Eq1]

All isolated systems spontaneously evolve towards thermodynamic equilibrium

Nature abhors differences in P, T, U Internal Energy

Natural processes have a preferred direction of progress

It is not possible to convert Heat completely to Work OR

Work completely to Energy. [W $\leftrightarrow$ ΔU with Q = 0 not possible]

Entropy Measure of Disorder $\Delta S = \Delta Q / T$ J / K [Eq2]

Entropy is the measure of Disorder of a system

Entropy is the measure of the # of States of the system $S = k \ln N$ [Eq3]

Total Entropy always increases [for an isolated or closed system]

Energy always spreads out. Systems tend toward equilibrium

Energy always flows naturally from hot to cold; Water flows downhill

Nature tends toward more disorder [chaos] & lower energy states

& takes path of least resistance / energy dissipation

Entropic processes are **NOT reversible**;

Disorder more Probable than Order Can not re-collect heat energy

Perpetual motion machines are "not possible"

Heat Energy is Transferred by

Conduction physical contact molecular agitation without material transfer

Convection intermediate material mass motion of a fluid like air or water leads to circulation currents due to expansion

Radiation emission of Infrared EM waves; no material involved

Heat Capacity, C, amount of energy/heat a substance can hold. It depends on the material AND its mass / quantity / size.

Specific Heat Capacity, c, is the heat required to raise Temperature of 1 gram of material by 1 deg. Depends on material only ! Water has much higher Specific Heat than other substances

Liquids have higher C than solids due to degrees of freedom of motion

Thermal Conductivity

What we perceive as hot or cold is NOT its temperature but its rate of energy transfer; Metals will feel hotter or colder than plastic even though both are at the same Temp.

High rate of energy transfer $\rightarrow$ Good conductor

Good Conductors; Metals

Poor Conductors; Most Liquids and Gases, Air, Vacuum, Wood, Cork, Glass, Most Ceramics, Plastics

Convection occurs in all Fluids [liquids & gases]

Winds near bodies of water result from convection currents due to

differences in Temp and density of air. These are called

Thermals and cause vertical motion of the air. The differences in

Heat Capacity between land and water cause horizontal motion;

morning, land to sea, evening, sea to land; California Santa Ana winds.

Thermal Expansion / Contraction

Matter expands when heated, contracts when cooled

Matter contains Internal Energy, not Heat

Liquids expand more than solids; Gases more than liquids

Water is very unusual; it contracts when heated from 0degC

to 4degC ; is less dense than its solid form, ice. It is the

ONLY substance to have its solid less dense than its liquid.

ONLY substance to have its solid float in its liquid.

Newton's Law of Cooling; Rate of cooling is proportional to

Temp difference of the object and its surroundings

This means it has an exponential decay

Good absorbers of radiant energy are also good emitters

Units Pressure 1 atm = 1.01 Bar = 14.7psi = 760 mm Hg = 10 m H₂O

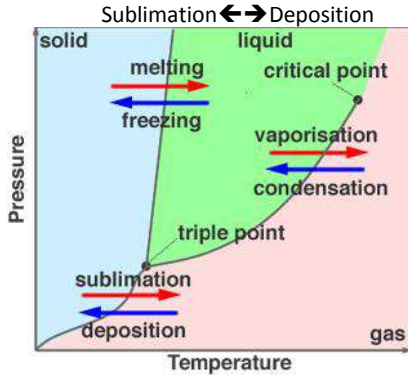
1 atm = 101325 Pa = 1.01×10^5 Pa = 1.03 kg / cm² = 10,356 kg / m²

1 Pascal = 1 N / m² ; 1 Torr = 1 mm Hg , 760 Torr = 760 mm Hg = 1 atm

Units Heat Joules OR Calories 4.2 Joules = 1 Calorie ; 4200 J = 1 KiloCalorie which what is found on food labels. 1 Calorie = Energy to raise 1 gm H₂O 1degC

Phase Diagrams P vs T

Shows the State/Phase [solid, liquid, gas] vs P T points
Boiling & Melting points change with Pressure & Temp
Triple Point is where all 3 States/Phases can co-exist
Melting $\leftrightarrow$ Freezing; Vaporization $\leftrightarrow$ Condensation



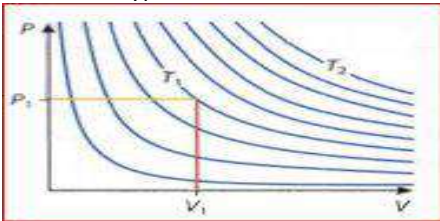
Change of Phase

Energy is absorbed; Solid $\rightarrow$ Liquid $\rightarrow$ Gas/Vapor
Energy is released; Gas/Vapor $\rightarrow$ Liquid $\rightarrow$ Solid
Heat of Vaporization H_2O $L(\text{vap}) = 540 \text{ Cal/gr} = 2.27 \text{ KJ/gr}$
Heat of Fusion H_2O $L(\text{fusion}) = 80 \text{ Cal/gr} = 335 \text{ KJ/gr}$

PV Diagrams of State P vs V with isotherms

Isotherms are lines of constant Temperature

Isotherms; the hyperbolic curves of constant $PV = nRT$



Thermodynamic State a point on the PV Diagram = **Energy**

Thermodynamic Process Transition from one **State** to another **State** on the PV Diagram

Thermodynamic Cycle 4 processes (or more) which end at the starting point and continually repeat

Joule's Law Internal Energy $U = f(T)$ $U = 3/2 PV = 3/2 n$
Point $V, P \rightarrow U \rightarrow T$; **Path** $pt - pt \rightarrow$ **Work done**
on or by the gas

Work = Force x Distance $\rightarrow$ **piston must move**

Decreasing volume $\rightarrow$ piston does work on the gas $W > 0$

Increasing volume $\rightarrow$ gas doing work on the piston $W < 0$

Work $W = -P \Delta V$ [$PV = (F/A)V = F s = \text{Work}$]

Gas can be heated or cooled [$Q > 0$ or $Q < 0$]

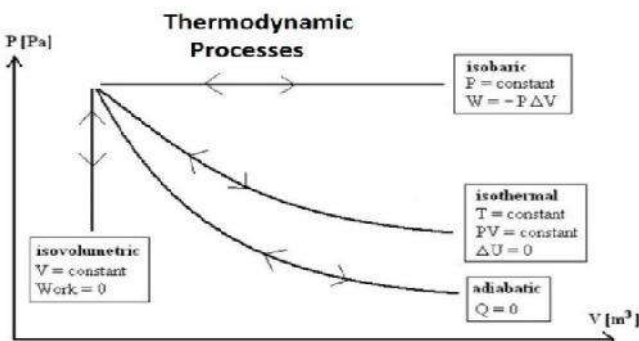
compressed or expanded [$\Delta V = V \uparrow$ or $V \downarrow$]

$\Delta V \rightarrow$ work done **by** or **on** the gas

**** If $\Delta V = 0 \rightarrow$ No work is being done $W = 0$ ****

$V \uparrow \rightarrow W < 0 \rightarrow$ **Work done by the gas**

$V \downarrow \rightarrow W > 0 \rightarrow$ **Work done on the gas**



Processes on PV Diagram $\Delta U = Q + W$ Equation

**** Joule's Law $U = f(T) = 3/2 nRT$ Energy is a function of T alone ****

$PV = nRT \rightarrow T$ **up or down** $\rightarrow U$ goes **up or down**

If $\Delta T = 0 \rightarrow \Delta U = 0$ No change in internal energy

$Q > 0 \rightarrow$ Heat is gained by gas; $Q < 0 \rightarrow$ Heat is lost from the gas

$W > 0 \rightarrow$ Work done on the gas; $W < 0 \rightarrow$ Work done by the gas

1/ Isobaric; $\Delta P = 0$ constant pressure P $P = k$

Expansion; $V \uparrow T \uparrow \Rightarrow \Delta U > 0, V \uparrow \Rightarrow W < 0, Q > 0$

Compression; $V \downarrow T \downarrow \Rightarrow \Delta U < 0, V \downarrow \Rightarrow W > 0, Q < 0$

2/ Isochoric; $\Delta V = 0$ constant volume V $V = k$

$\Delta V = 0 \Rightarrow W = 0, Q = \Delta U$ $P \uparrow T \uparrow \Rightarrow \Delta U > 0 \Rightarrow Q > 0$

$\Delta V = 0 \Rightarrow W = 0, Q = \Delta U$ $P \downarrow T \downarrow \Rightarrow \Delta U < 0 \Rightarrow Q < 0$

3/ Isothermal; constant temperature $\Delta T = 0 \Delta U = 0$ $PV = k$

Expansion; $\Delta T = 0 \Rightarrow \Delta U = 0, Q = -W, V \uparrow \Rightarrow W < 0, Q > 0$

Compression; $\Delta T = 0 \Rightarrow \Delta U = 0, Q = -W, V \downarrow \Rightarrow W > 0, Q < 0$

4/ Adiabatic; $Q = 0$ no heat is exchanged $\rightarrow \Delta U = W$ $PV^\gamma = k$

occurs so quickly $\rightarrow$ heat has no time to exchange

Expansion; $V \uparrow \Rightarrow W < 0, \Delta U = -W \Rightarrow \Delta U < 0, T \downarrow \Rightarrow$ int E $\downarrow$

Compression; $V \downarrow \Rightarrow W > 0, \Delta U = +W \Rightarrow \Delta U > 0, T \uparrow \Rightarrow$ int E $\uparrow$

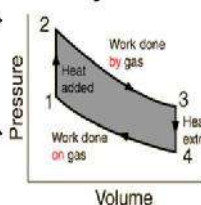
Thermodynamic Cycle

Thermodynamic Cycles

A cyclic thermodynamics process is a closed path on a PV diagram.

The most efficient thermodynamic cycle is called the Carnot cycle.

It consists of two adiabats and two isotherms.



$\rightarrow$ Examples of Thermodynamic Cycles 1/ The Internal Combustion Engine [your car], 2/ Refrigerator and 3/ Air Conditioner

For any Thermodynamic Process $\rightarrow$ **Work done = Area under**

For any Thermodynamic Cycle $\rightarrow$ **Net Work done = Area inside the curve of PV Diagram** [done on or by the gas]

*****For a Process; Work = the Area under the PV Curve *****

*****For a Cycle; Work = the Area inside the PV Loop *****

CW $\rightarrow$ net work done by the gas $W < 0 \rightarrow$ heat engine

CCW $\rightarrow$ net work done on the gas $W > 0 \rightarrow$ heat pump

Heat pump is a refrigerator OR air conditioner

Refrigerators and air conditioners; cool through refrigerant phase change

Expansion forced vaporization AND Compression forced condensation.

Heat taken from cold, given to something hot by the work added

Contrary to what Nature would do 2nd Law / Entropy

Isothermal expansions not possible [except at 0 deg K]

It implies heat can be transferred directly thru the gas

to the piston without a T increase. This violates the 2nd Law

which says it is not possible to convert heat directly in to work

Secret to all heat engines is that the gas is cooled before it is compressed

back to original volume. The gas must be made hotter than the environs

Carnot Cycle; Perfect Engine Theoretical Limit

Only use Isothermal, $\Delta T = 0$, or Adiabatic $\Delta Q = 0$ processes

Otto Cycle; Internal Combustion Engine

An Otto cycle consists of 4 processes: 2 isentropic (reversible adiabatic $\Delta Q = 0$)

2 isochoric (constant volume $\Delta V = 0$)

Diesel Cycle; The Diesel Cycle differs from Otto cycle by using an increased

compression of the fuel to ignite the fuel rather than using a timed spark

*****compression ignition" vs "spark ignition*****

Other Cycles; Stirling, Rankine, Ericsson

Waves & Simple Harmonic Motion Pg 1

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Oscillation a continuing repetitive motion ; periodic motion

Amplitude (x_m) meter SHM maximum displacement

Cycle one complete oscillation (revolution, 360 deg, 2π radians)

Period (T) sec/cycle ; = time to complete one cycle

$T = 2\pi \sqrt{X}^{1/2}$ Simple Pendulum $X = L/g$ Spring-Mass $X = m/k = d/g$

Physical Pendulums; $X = I / m g d$ Rod $X = 2L/3g$ Ring $X = 2L/g$

$d = \text{dist CM to Pivot}$ $I = \text{Moment of Inertia}$ Disc $X = 3R/2g$

Frequency (f) cycles per sec ; $1 \text{ cy/sec} = 1 \text{ Hertz (Hz)}$ $f = 1/T$

Wavelength (λ) meters / cycle $\lambda = v T \rightarrow v = f \lambda$ m/sec

Velocity (v) meters/sec is constant but different in every media

Angular frequency (ω) radians/sec $\omega = 2\pi f = 2\pi / T$

used for circular motion where 2π is one cycle

Equilibrium position the position where the system

is normally at rest; also the point of highest speed and KE

Forced oscillation: an oscillation caused by a repeating external force

[compared to a one time disturbance / impulse]

Natural frequency: the oscillation frequency of a disturbed system

determined by its natural physical characteristics

Resonance: increase in amplitude of a system because it is

being forced to oscillate near its natural frequency

Loud rattles heard while driving a car; $\Delta \text{ speed} \rightarrow \text{rattle stops}$

Damping: friction causing a loss of energy to a system

Wave : disturbance which travels from 1 location to another

A **transfer of energy, NOT mass**. Usually thru a medium like water or a gas like air. EM waves need no medium / no 'ether'

Simple Harmonic Motion [SHM]

Restoring force is proportional to the displacement from the equilibrium position

Differential Equation of Motion ... $dx^2/dt^2 + \omega_0^2 x = 0$

$\rightarrow$ SHM produces pure sinusoid solutions for position, velocity and acceleration $\leftarrow$

x_m = maximum displacement / amplitude ; $x' = dx/dt$

position $x(t) = x_m \cos \omega t$

velocity, $v(t)$ $x'(t) = -\omega x_m \sin \omega t$

acceleration, $a(t)$ $x''(t) = -\omega^2 x_m \cos \omega t$

velocity $v(x) = \frac{\omega (x_m^2 - x^2)^{1/2}}$

acceleration $a(x) = -\omega^2 x$

For circular motion $a(\text{rad}) = \frac{\omega^2 r = v^2 / r}{}$

KE = $1/2 m v^2 = 1/2 m \omega^2 (x_m^2 - x^2)$; Total energy = $1/2 m \omega^2 x_m^2$

Examples of SHM; Circular motion at constant speed,

Mass - Spring set , Pendulum ($\sin \theta = \theta$ for small θ) , Tuning fork

Object floating in a fluid, LC Resonant Oscillations

Waves propagation of a disturbance / energy

Sound, Water, Light, Electromagnetic Energy, Gravitational

1, 2, or 3 dimensional

No medium is needed for EM waves; Any solid, liquid or gas for sound waves

4 Properties of Waves

1/ **Reflection** reversal of direction when meeting a new medium

Law of Reflection ; **Angle in = Angle out**

Specular reflection from a smooth or wet surface

Diffuse reflection from a rough surface

2/ **Refraction** change of speed entering new medium.

A change of direction or bending also occurs; See Snell's Law

3/ **Interference** waves observe superposition

2 waves can occupy the same space and thus add/subtract amplitudes. Particles cannot do this !

Interference can be Constructive or Destructive

4/ **Diffraction** spreading out or bending of waves which

interact with objects; apertures, slits/pupils , corners, lenses

Huygen's Principle: Wave = Many synchronized point sources

Plane uniform wave = Many Huygens wavelet point sources

Fermat's Principle: Light follows the **path of least time**

Used to derive Law of Reflection & Snell's Law

Huygen's Principle: Wave = Many synchronized point sources

Reflection **Reversal of Direction when wave meets a new medium/material**

Law of Reflection ; **Angle in = Angle out**

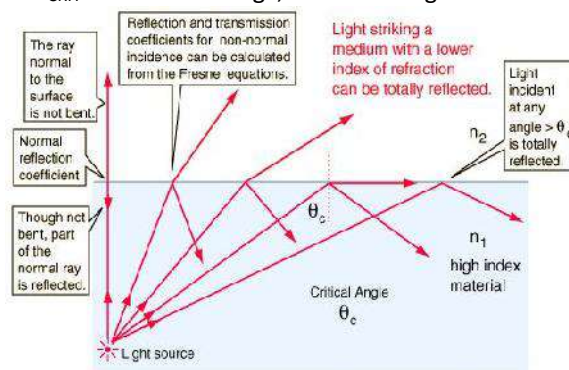
Total Internal Reflection [TIR] $\theta_{\text{CRIT}} = \text{Arcsin } n_2/n_1$ $n_1 > n_2$

Wave travelling from refractive index medium; high n_1 to low n_2

TIR Occurs when refraction angle $>$ or $= 90$ degrees

$\theta >$ or $= \theta_{\text{CRIT}} \rightarrow$ **TIR** ; Fiber optic cables, Diamonds

θ_{CRIT} = Water 49deg , Glass 42deg



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Note; Any energy going up at θ_{CRIT} or greater is totally reflected

Refraction 1 / Change of speed of a wave entering a new media.

2 / Change of direction [bending of the wave] due to Huygen's Principle

3 / Frequency does NOT change when wave enters a new medium

Refractive Index n is a measure of the change of speed.

The **Index of Refraction** is defined as the speed of the wave in vacuum divided by the speed of the wave in the medium

$n = c / v$ $n > 1$; $n_1 v_1 = n_2 v_2$ $v = f \lambda \rightarrow n_1 \lambda_1 = n_2 \lambda_2$

and from Snell's Law $n_1 \sin \theta_1 = n_2 \sin \theta_2$

$\rightarrow$ **Bending of the wave does not occur if incidence angle is 0 degrees, but the change of speed of the wave does occur.** $\leftarrow$

Sound waves travel faster in physically dense media like water

EM waves travel slower in optically dense media

Optical Density is NOT same as Physical Density

Refracted wave bends **toward / away** from the **Normal**

FST = Fast to Slow $\rightarrow$ **Toward** ; **SFA** = Slow to Fast $\rightarrow$ **Away**

Fast Media = Low index refraction = Optically low density

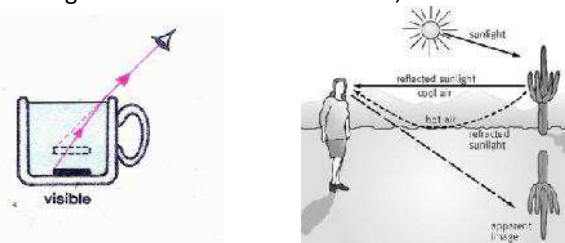
Slow media = High index refraction = Optically high density

Consequences of Refraction

Objects appear closer or farther , distorted or bent , larger or smaller when looking thru a translucent media.

Refractive Index $n = \text{Real Depth} / \text{Apparent Depth}$

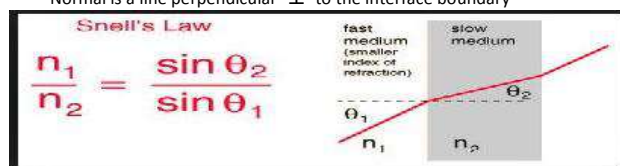
Apparent magnification is $\times 1.33$ for Water, $\times 1.5$ for Glass



Snell's Law quantifies relation of bending angle to refractive index due to the change in velocity

$\rightarrow$ Angles measured with respect to the **Normal** $\leftarrow$

Normal is a line perpendicular $\perp$ to the interface boundary



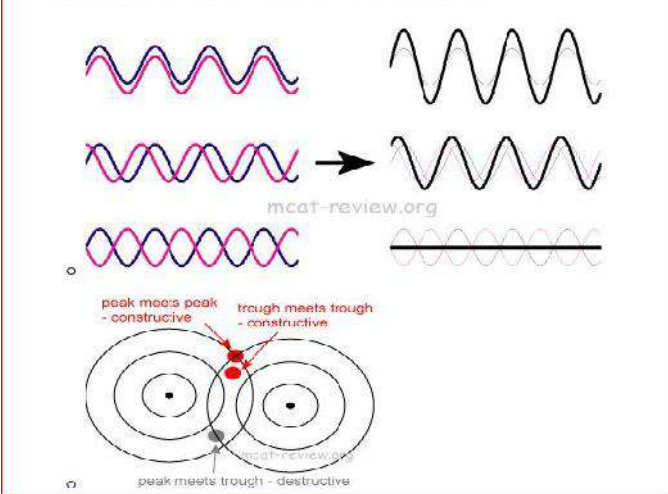
Interference

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Superposition of Waves

Constructive (addition) & Destructive (subtraction) Interference

- Supposition of waves, interference, addition



2 Circular Wave Sources; No obstacle ; Interference calculations

Maxima $d = 1x \ 2x \ 3x \ 4x \ \lambda$ Minima $d = 0.5x \ 1.5x \ 2.5x \ \lambda$

d = path length difference Phase angle $\phi = 2\pi d / \lambda$

If d = whole # wavelengths $m \lambda \rightarrow$ in phase $\rightarrow$ **Constructive** / add

If d = odd # $\frac{1}{2}$ wavelengths $(m + \frac{1}{2}) \lambda \rightarrow$ out of phase $\rightarrow$ **Destructive** / cancel

Diffraction Waves spread out as they propagate

Many Huygens wavelets + Interference $\rightarrow$ Max & Minima

Single Slit Diffraction; **Narrow slits diffract more than wider slits** because larger angle is needed to obtain the same path length difference

Longer wavelength red light diffracts more than shorter blue

Narrow slit acts like a point source $\rightarrow$ circular wave fronts

Slit diffraction..... (at) small angles $\rightarrow \tan \theta = \sin \theta = \theta$

Calculating half width 'y' of a maximum on a screen or retina or film

From screen geometry $\tan \theta = \theta = y/D$ From slit $\Delta L = a \sin \theta = m \lambda$

$\sin \theta = m \lambda / a$ and $\theta = y/D \rightarrow y = m \lambda D / a$

y = half width of maximum D = screen distance

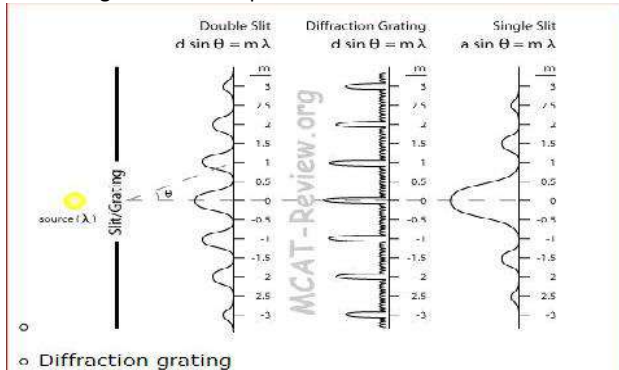
λ = wavelength a = slit width m = mode #

Central Maximum Width $2y$; For **Circular Aperture** the central maximum width is 1.22 times wider than for a slit.

Slit Interference Max / Min Plane wave incident

	Maximums (λ)	Minimums (λ)	Equation for a Maximum
1 slit	$0 \pm 1.5 \ 2.5$	$\pm 1 \ 2 \ 3 \ 4$	$m \lambda = a \sin \theta$
2 slits	$\pm 0 \ 1 \ 2 \ 3$	$\pm .5 \ 1.5 \ 2.5$	$m \lambda = d \sin \theta$
Diff Grating	$\pm 0 \ 1 \ 2 \ 3$	$\pm .5 \ 1.5 \ 2.5$	$m \lambda = d \sin \theta$

λ = wavelength d = slit separation a = slit width m = mode order



Rayleigh Criterion & Diffraction Limited Eye Resolution

Non Resolvability of 2 sources due to the 2 maxima converging $\theta_R \geq 1.22 \lambda / a$

Two point sources are resolved when the principal diffraction maximum of one image coincides with the first minimum of the other. 2 Objects seen thru a lens, mirror, pupil or aperture making an **angle** smaller than the angular resolution θ_R cannot be resolved. As distance object to slit/pupil $\uparrow$ so $\theta \downarrow$ converging the 2 maxima This equation also gives the angular spreading of a source of light having a beam diameter 'a'. Resolving Power = $1 / \text{Resolution}$

Diffraction Limit of resolution of any imaging system

is about equal to the imaging wavelength

Optical microscope resolution limit 200 nm

Electron microscope limit 0.1 nm (size of an atom)

Effective $\lambda = 0.02 \text{ nm}$

Polarization Brewster's Angle Ω ;

If angle between reflected and refracted rays is 90 deg,

polarization of reflected wave is 100%

Ω = the angle of incidence necessary for this condition

$$\Omega = \text{Arctan } n_2 / n_1 \quad n_2 > n_1$$

wave travels from n_1 medium $\Rightarrow$ n_2 medium

$\Omega = 53 \text{ deg air : water ; } \Omega = 56 \text{ deg air : glass}$

Note; These angles will vary with wavelength λ of light

Malus' Law $\rightarrow I = I_0 \cos^2 \theta$

I = Intensity of light passing thru 2 polarizers

θ = angle between the polarizers

Polarizers reduce intensity by 50% at $\theta = 45 \text{ deg}$

For $\theta = 90, 60, 45, 30, 0 \text{ deg ; } I_0 \cos^2 \theta = 0, 0.25, 0.5, 0.75, 1.0$

Types of Waves

Progressive / Travelling Waves ; plane or circular wave fronts

Transverse Wave; disturbance is perpendicular to direction of energy flow; Ex; Water, EM Radiation

Longitudinal Wave; disturbance is in direction of energy flow. Ex; Sound. Can not be polarized

Standing Waves / Stationary Waves SW

Incident and Reflected waves Interfere (add/subtract) to form a SW

Reflection + Interference at the right frequency $\rightarrow$ **Resonance**

Many "harmonic / right" frequencies $\rightarrow$ **Resonant Modes**

Can exist in Solids, Liquids or Gases

Node (Not Moving); place on SW where amplitude stays at zero

Antinode; place amplitude changes from + max to - max

Speed of wave on a string $v_{\text{string}} = [T / u]^{1/2}$

T = tension, u = mass per unit length = grams / meter

$f = v / \lambda$ v = velocity λ = wavelength

Wavelength (λ) for Acoustic Resonance vs (L) Physical Length

Harmonic	1 st	2 nd	3 rd	4 th	5 th
String	$2L$	L	$2/3 L$	$1/2 L$	$2/5 L$
Closed Pipe	$4L$	-	$4/3 L$	-	$4/5 L$
Open Pipe	$2L$	L	$2/3 L$	$1/2 L$	$2/5 L$

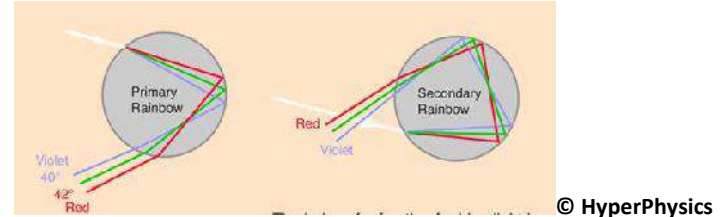
String $\rightarrow$ Node both sides ; Open $\rightarrow$ antinodes both sides ; Closed $\rightarrow$ node at 1 end

Dispersion; $v = f(\lambda)$ & $n = f(\lambda)$

Change in wave speed & refractive index n with frequency.

Dispersive devices; Water, Prism, Diffraction Grating

$\rightarrow$ A Rainbow is an example of Refraction + Dispersion + Reflection



Doppler Effect change in apparent freq due to motion

1/ Sound

$$c = 330 \text{ m/s}$$

f_{ob} = observed freq

f_s = source frequency

c = speed of sound/light

v = velocity of the wave

Source [transmitter]

Observer [receiver]

$$f_{ob} = f_s \frac{c \pm v[o]}{c \pm v[s]}$$

Moving Source $\rightarrow v[ob] = 0$; Moving Observer $\rightarrow v[s] = 0$

2/ Light & EM Radiation

$$c = 3 \times 10^8 \text{ m/s}$$

$$f_{ob} = f_s + \Delta f$$

$$\Delta f = [v/c] f_s$$

$$f_{ob} = f_s \pm [v/c] f_s$$

$$v \ll c$$

Electric Fields & Forces

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Electric Field & Electric Force [+ & - charge]

Field A place where you find something [potato, soccer, rice, football]

Electric Force Field is a region where a + charged object experiences a

Force and moves in the direction of the Force Field.

Charge +q creates radial Electric Force Field that accelerates a + charge away

Like charges repel Unlike charges attract

Static Charge → Electric Field

Moving charge / current → Magnetic Field

Accelerating Charge → EM Radiation

Maxwell's Equations, displacement current dE/dt **predicted radio waves**

and showed that light was an electromagnetic wave

Hertz confirmed it 8 yrs later—Tesla wireless xmission—Marconi TransAtlantic

Gauss → there are no magnetic monopoles

Ampere's Law → $B = \mu_0 I$ (current), [missing Displacement Current]

Biot-Savart Law → relates magnetic field **B** at position **r**

generated by a *steady* current **I**

Kirchoff's Laws are a special case of Faraday's Law omitting the magnetic

induction of electric field

Our economy runs on these principles & Faraday's Law to make Electricity.

Need Energy to move the magnet or the coil

Coal, Gas → Steam, Water/Hydro, Wind, Nuclear

Electrostatics → Force F_E

Triboelectric Effect is charge transfer due to contact/friction

+ Glass rod ** - Silk cloth + CatFur ** - Rubber rod

+ Hair ** - Comb + Wool ** - Plastic Balloon

Attraction by **Induction**; charged comb with paper bits

Note some bits fall back down after hitting the comb and

some stick to it. **Gravitational Force vs Static charge Force**

Electrical force $\gg$ Gravitational force; Ratio of F_E to $F_G = 10^{36}$

Conservation of Charge *charge is neither created nor destroyed*

Fields and Forces add as vectors PE and Voltage add as scalars

For point sources, the following Equations apply

F_E Electric Force Coulomb's Law

$F_E = k Q_1 Q_2 / r^2 \rightarrow$ Force units Newtons **N**

$k = 1/4 \pi \epsilon_0 = 9 \times 10^9 \text{ N m}^2 / \text{C}^2$ **Force = Gradient of PE**

$\epsilon_0 = 8.85 \times 10^{-12} \text{ C}^2 / \text{N m}^2 =$ vacuum permittivity

E Electric Field Strength → Force per unit charge

$E = k Q_1 / r^2 = F_E / q$ units **N / C = V / m**

$E = dV_E / dh$ **Field Strength = Gradient of Potential**

→ Field lines show direction a + charge will move

Gravitational Analog = $g = GM/r^2 = F / m = \text{N / Kg}$

PE Potential Energy → Force x Distance units **Joules J**

$PE = k Q_1 Q_2 / r$ Work done $W = k Q_1 q / r$

Work done taking a point charge +q in from infinity [zero potential]

$PE = \int F dr$ PE = work done against the Field PE = Integral of Force

V_E Electric Potential → Work done per unit charge

$V_E = k Q_1 / r = PE / q = E r$ units **Volts = J / C**

$q = 1.6 \times 10^{-19} \text{ J / ev [CV/e-]} \quad [\text{kg m/s}^2 \times \text{m / kg}]$

$V_E = \int E dr = \Delta EPot$ Potential = Integral Field Strength

V = Work done *by* charge moving thru E Field across Equipotential lines

$V = \text{Work}/q = F d/q = E d \quad W = q \Delta EP = q E \Delta h \text{ Joules}$

In a uniform field between 2 plates, the Force & Field are the same

everywhere for a given charge q [Joule = $6.24 \times 10^{18} \text{ ev}$]

Equations Summary

$F = qE = dPE/dr \quad PE = F \times r = qV \quad \Delta V = W/q = (F/q)d = Ed$

$E = F/q = dV/dr \quad V = E \times r = PE/q$ and the definitions for point q ...

Electric Dipole $E(\text{on axis}) = 2kp/r^3 \quad E(\perp \text{axis}) = kp/r^3 \quad p = q d$

Gauss Law Electrostatics $E = \text{Flux Density} = \phi/A$

$\phi = \int E \cdot dA = Q / \epsilon_0 \rightarrow \Sigma \text{Flux Out} = \text{Charge enclosed in the volume}$

$\phi = E A$ [if $E \parallel$ to A 's Normal vector] Flux $\phi = E A \cos \theta = Q / \epsilon_0$

ϕ Electric Flux = N-m²/Coulomb = Volt-meter [Magnetostatics $\int B \cdot dA = 0$]

E Flux lines begin on a + Charge/North Pole and end on a - Charge/South Pole

Electric Circuits

Kirchoff's Laws

KVL; The sum of the voltages around a loop = 0

KCL; The sum of the currents in to any node = 0

Parallel vs Series Circuit

A parallel circuit; the voltage across the components is the same

Note; parallel does not mean to be geometrically parallel

A series circuit; the current thru the components is the same

Ohm's Law $V = I R$ Voltage = Current x Resistance

Power Laws $P = I V = I^2 R = V^2 / R$ Watts = Joules / sec

Resistor Combinations

Series; $RT = \text{Total } R = R_1 + R_2 + R_3 \dots + R_n$

Total Series Resistance is the sum of the resistors

Parallel; a/ **Reciprocal Rule**

$1 / RT = 1/R_1 + 1/R_2 + 1/R_3 \dots + 1/R$

This works for any number of *R*'s but is cumbersome Math

The following 2 Rules work **ONLY** for 2 Resistors

But Any combination of 'n' resistors can be calculated

With multiple steps, 2 *R*'s at a time.

b/ Product Over Sum Rule $RT = [R_1 R_2] / [R_1 + R_2]$

c/ N + 1 Rule $N = R(\text{large}) / R(\text{small}) \quad RT = R(\text{large}) / N + 1$

This works even if 'N' is a fraction, not an integer

Capacitor Combinations are opposite of Resistor Combination rules

R Series laws = *C* Parallel Laws AND *R* Parallel Laws = *C* Series Laws

Series; Charge same, Voltage adds --- Parallel; Charge adds, Voltage same

Capacitor Equations $q = C V \quad C = \epsilon_0 A / d$

$E = q / \epsilon_0 A \quad V = q / (\epsilon_0 A / d) \quad V = E d$

Conservation of Energy

Capacitor $E = \frac{1}{2} C V^2 = \frac{1}{2} Q^2 / C = \frac{1}{2} Q V; \quad I = C dV/dt$

Inductor $E = \frac{1}{2} L I^2 \quad \phi = L I \quad (q = CV) \quad V = L di/dt$

Since energy can not be created or destroyed

Energy can not move/change instantaneously, thus

The voltage across a capacitor can not change instantaneously

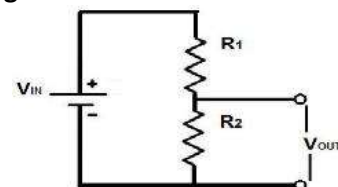
The current in an inductor can not change instantaneously

DC current seeks path of least Resistance; AC current least Inductance

Dielectric → $C \uparrow \quad E_{\text{field}} \downarrow$ Energy density $n_E = \frac{1}{2} k \epsilon_0 E^2 \downarrow$

Polarization of the dielectric molecules reduces the total E Field

Voltage Divider Rule



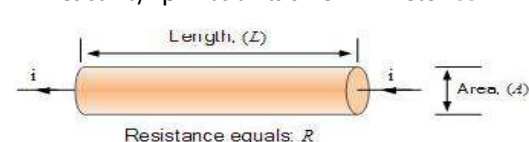
For Resistors; $V_{out} = V_{in} [R_2 / R_1 + R_2]$

For Capacitors; $V_{out} = V_{in} [C_1 / C_1 + C_2]$

Replacing the R_1, R_2 with C_1, C_2

Resistivity ρ Resistance $R = \rho L / A$

Resistivity ρ has units of Ohm - meter CC



1/ Lines of Electric Flux begin at + charge and terminate on - charge.

2/ The direction of the electric field and electric force is tangent to the field/flux line.

3/ Electric Field Vector (lines of force) always normal to the surface of charged body.

4/ Two electric lines of force cannot intersect each other.

5/ Electric lines of force in the same/opposite direction repel/attract each other

6/ Electric Potential exists due to charge. Voltage = EP difference between 2 pts

7/ Electric Field inside a conductor = 0 (if $I = 0$). E Potential = constant - $dV/dr = E$

8/ Free charges on a conductor reside only on the surface due to mutual repulsion

9/ Inside a surface charged hollow spherical volume, E Field = 0 due to symmetry

Magnetic Fields & Forces

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Magnetostatics

F_M Magnetic Force & B Magnetic Field

Units for B Tesla = Newton - sec / Coulomb - meter

Magnets exist as dipoles with a North and South pole

Like poles repel and unlike poles attract $\rightarrow$ Force F_M

Opposing fields from magnets or currents $\rightarrow$ Force F_M

Static charge $\rightarrow$ Electric field

Moving charge [current] $\rightarrow$ Magnetic field

Accelerating Charge $\rightarrow$ Radiation

Earth's geographic North Pole is a Magnetic South Pole ☺

Pauli Exclusion Principle explains magnetism with idea of e- spin

Ferromagnetism; permanent magnets,, Iron, Nickel, Cobalt

Paramagnetism; materials attracted to a magnetic field; Fridge door

Diamagnetism; materials repulsed by magnetic field; **most materials**

Antiferromagnetism; no net magnetic field, low temperatures

Substances that are *non-magnetic* include copper, aluminum, gases, and plastic. A material may exhibit more than one form of magnetism depending on its temperature.

Similarities of Magnetic Field B with Electric Field E ;

Superposition, attraction & repulsion, penetrates matter

Differences of Magnetic Field B w Electric Field E ;

Interacts with only certain materials, circular, closed field lines

B Magnetic Field $\rightarrow$ Force per unit current units Tesla

Magnetic Field Strength B = Flux density = ϕ / A

$\rightarrow$ Flux lines are the Field / Force vector $\leftarrow$

Lorentz Force Law Interaction of 2 Magnetic Fields $\rightarrow$ **Force**

F_M (due to current) = $I L \times B$ L = length vector, I = current

F_M (due to motion) = $q v \times B$ v = charge velocity vector

$\rightarrow$ To go from $q v B$ to $I L B$; $q v = q (L / t) = (q / t) L = I L$

Cross Product Direction Rules

Positive charge $\Rightarrow$ Right Hand Rule; Negative charge $\Rightarrow$ Left Hand Rule

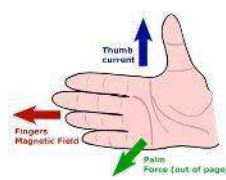
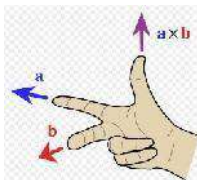
Right Hand Grip/Fist Rule $\rightarrow$ Current (thumb) vs Magnetic Field (fingers)

Solenoid Rule $\rightarrow$ Current (fingers) vs Magnetic Field (thumb)

RH Rule; v (index finger **a**) $\times$ B (middle finger **b**) $\Rightarrow F$ is (thumb **a $\times$ b**)

RH Slap Rule v (thumb) $\times$ B (index finger) $\Rightarrow F$ is (middle finger)

All 3 vectors F, v, B OR F, I or L, B are Perpendicular



Fist / R H Grip Rule

R H Rule

R H Slap/Palm Rule

- 1/ Magnetic forces are perpendicular to both the velocity of charges or current, and to the magnetic field. Since magnetic forces are perpendicular to the velocity, they do no work !!! ($W = F \cdot r$)
- 2 / Speed of particles moving in a magnetic field remains constant in magnitude, the direction changes. Kinetic energy is constant! (no work).
- 3 / $FB = q v \times B$ $r = mv / qB$ for circular / helical motion of q in uniform magnetic field.
- 4 / Magnetic dipoles align with the magnetic field same as electric dipoles

2 Parallel Wires; current in **same** direction **attract** each other

in **opposite** directions, **repel** each other. Why? Current in wire 1 creates B field that wire 2 sees. The Lorentz Force on wire 2 points to wire 1 ; $F_M = q v \times B$. Similarly, Force on wire 1 points to wire 2

The Force decreases with distance d using Biot-Savart Law

$B_1 = \mu_0 I_1 / (2\pi d)$ & $F = I_2 L B_1 \rightarrow F = \mu_0 I_1 I_2 L / (2\pi d)$

Note how this Force is different from Gravitational or Electric Forces between a Field and a Mass / Charge. Since there is no Magnetic monopole, the Force arises from the interaction of the two Dipoles created by the B Fields.

Electric Dipole Torque = $p \times E$ $p = q d$ d = dipole separation p direction + charge

Magnetic Dipole Torque = $N I A \times B$ on wire loop of Electric Motor

Gauss Law Magnetostatics $\int B \cdot dA = 0$ Σ Flux Out = 0

There are *no Magnetic Monopoles* like there are Electric Charges

Electromagnetic Induction $dB/dt \rightarrow$ emf & current is induced

Magnetic Flux $\phi = \oint B \cdot dA$ = Lines of B Field Webers [Tesla-m²]

An idea for understanding Mag or Elec field. Flux lines never cross

each other. Make closed loops thru the magnetic material

Flux lines proportional to Charge [Electric] OR Current [Magnetic]

E Flux lines begin on a + Charge/North Pole and end on a - Charge/South Pole

B = Flux density = $\phi / A \rightarrow \phi = B \cdot A = B A \cos \theta$

For 'N' turns of coil $N \phi = N A B \cos \theta = N A B \cos \omega t$

Faraday's Law / Maxwell's 3rd $V_{EMF} = \oint E \cdot dl = - d\phi/dt$

$V_{EMF} = - d\phi / dt = - d/dt \int B \cdot dA = - d(A B \cos \omega t) / dt = A B \omega \sin \omega t$

Rate of change of Flux $\rightarrow$ Induces E Field $\rightarrow$ Voltage/Emf $\rightarrow$ Current

.... If a Conductor is moving in a Magnetic Field OR

a Magnetic field is moving/changing near a conductor

No matter how the change is produced, voltage/EField will be generated.

The Flux change could be produced by

1 / changing the magnetic field strength or frequency

2/ moving a magnet toward or away from the coil,

3/ moving the coil into or out of the magnetic field,

4/ rotating coil or magnet relative to each other

Lenz's Law The direction of the induced current creates

a field that opposes the field that produced it

$\rightarrow$ It is the minus sign in Faraday's Law $\leftarrow$

Eddy Currents and Magnetic Braking

Units Tesla = Newton-sec/Coulomb-meter $T = N \cdot s / C \cdot m$

Units Flux ϕ = Webers 1 Tesla = 1 Weber / m²

Ampere's Law / Maxwell's 4th

$\oint B \cdot dl = \mu_0 I$ $\mu_0 = 4\pi \times 10^{-7} T \cdot m / A = N / A^2$ π

Biot-Savart Law; Relates B Field (ϕ/A) to the

magnitude, direction, length and proximity (r) of **current** producing it.

It is the Magnetostatic equivalent to Coulomb's Law for Electrostatics

For a constant current I $B = \mu_0 q I$ Integral $r \times dl / (4 \pi r^2)$

Magnetic Field Equations for Specific Geometries

B (near a wire) = $\mu_0 I / 2\pi r$ B (center of 1 loop) = $\mu_0 I / 2 r$

B (center solenoid) = $\mu_0 N I / L$ [$r \ll L$] B (toroid) = $\mu_0 N I / 2 \pi r$

B (dipole on axis) = $\mu_0 I / 2 \pi z^2$ L (solenoid) = $\mu_0 N^2 A / \text{length}$

B (Ring / Current loop on z axis) = $\mu_0 I r (2 \pi r) / 4 \pi (z^2 + R^2)^{3/2}$

Coulomb's Law Relates E Field = F/Q to the **charge** producing it

Gauss Law Magnetism; Flux out = Flux in, No Mag monopoles

Fleming Rules; Motor-Generators 3 Fingers at right angles

Fingers; Index - Middle - Thumb

Left Hand Rule; Motors; Field Current Force

Right Hand Rule; Generators; Field Current Motion

Electric / Magnetic Force Field Similarities

- 1) Both Force Fields are entities of Electromagnetism $\rightarrow$ electricity and magnetism are not separate subjects but are intimately related.
- 2) A time varying electric field gives rise to a magnetic field and a time varying magnetic field generates an electric voltage / current in a conductor. If one of them varies with time or space, the other one is induced.
- 3) Both are vector fields and exert forces on electric charges

Major Differences - Miscellaneous Points

- 1) Electric field has sources and sinks (+ and - charge). Begin on + and terminate on -. But, Magnetic fields have no sources or sinks (are solenoidal) **No Magnetic monopole exists.** North & South Poles come in pairs called a Dipole. The Field begins on N Pole and ends on S Pole by convention.
- 2) For static charges and steady currents, Electric fields have Divergence and zero Curl; Magnetic fields have zero Divergence and non zero Curl.
- 3) Electric field force on charges are independent of velocity while magnetic field forces vary with charge velocity; $F_M = q (v \times B)$
- 4) A static charge will move in an Electric Field BUT not move in a Magnetic Field
- 5) Charge motion is parallel to E field lines but moves perpendicular to B field lines
- 6) Electric fields exert force parallel to charge motion and thus perform work on them while Magnetic fields exert forces perpendicular to motion so they **do no work** on them. $W = F d \cos \theta = 0$ ($\cos 90 \text{ deg} = 0$)
- 7) No work $\rightarrow$ No change in KE or speed. BUT, direction & velocity both change.
- 8) A flowing electric current gives rise to a Magnetic Field that circles the current
- 9) A time-changing Electric Field gives rise to a Magnetic Field that circles the E field

Pascal's Law (Blaise Pascal) P [pressure]

$$P = \text{Force} / \text{Area} = F / A = \text{Energy/Volume} \rightarrow E = PV$$

$$P_1 - P_2 = \rho g [h_2 - h_1] \quad P = \rho g \Delta h \rightarrow P = \rho g h$$

$$\rho = \text{density fluid} \quad g = 9.8 \text{ m/s}^2 \quad h = \text{height in fluid} = \text{depth}$$

→ **Pressure increases with depth** ←

→ **Pressure at any depth is the same** ←

Absolute pressure = Atmospheric pressure + Gauge pressure

$$P = p_0 + \rho g h \quad p_0 = P (\text{depth} = 0)$$

P is independent of Direction, Volume, Weight or Shape of the vessel

Change in pressure between two elevations is due to

the weight of the fluid between the elevations

Atmospheric pressure is due to gravity. We live under the weight of the fluid that is the atmosphere.

Fluids are Liquids, Gases, Plasmas [Shape of their container → Volume]

Air pressure at sea level is $\sim 1 \text{ kg} / \text{cm}^2 = 10,000 \text{ kg/m}^2$

100 kg pressure on your hand both top and bottom

Mercury barometer is 0.76 m high = 760 mm high

Water barometer is 10.33 m high ; 13.6x higher than Hg

10 meters of water produce an overpressure of 1 atm

Units Standard Atmospheric Pressure (1 atm = 6 different units)

$$1 \text{ atm} = 14.696 \text{ psi} = 10.33 \text{ m H}_2\text{O} = 760 \text{ mm Hg} = 1.01325 \text{ Bar}$$

$$1 \text{ atm} = 101325 \text{ Pa} = 1.01 \times 10^5 \text{ Pa} = 101.325 \text{ kPa} = 1.01325 \text{ Bar}$$

$$1 \text{ atm} = 760 \text{ mm Hg} = 10,356 \text{ kg} / \text{m}^2 = 1.01325 \text{ Bar}$$

$$1 \text{ Torr} = 1 \text{ mm Hg} = 133.3 \text{ Pa}, 760 \text{ Torr} = 760 \text{ mm Hg} = 10.3 \text{ m H}_2\text{O}$$

$$1 \text{ Pa} = 1 \text{ Pascal} = 1 \text{ N} / \text{m}^2 \quad \rho [\text{mercury}] = 13.6 \times 10^3 \text{ kg} / \text{m}^3$$

$$1 \text{ Bar} = 10^5 \text{ Pa} ; \quad \rho [\text{water}] = 1000 \text{ kg} / \text{m}^3 = 1.0 \text{ gram} / \text{cm}^3$$

Pressure vs Height in the Atmosphere

$$P(h) = P_0 e^{-(h / H_0)} \quad H_0 = kT/mg = 8000 \text{ m}$$

$$h = 2400 \text{ m} \rightarrow P = 0.75 \text{ atm} \quad \text{H}_2\text{O boils at } 92^\circ\text{C}$$

$$h = 8.9 \text{ km} \rightarrow P = 0.33 \text{ atm} \quad \text{H}_2\text{O boils } 70^\circ\text{C} \quad [\text{Mt Everest}]$$

$$h = 30 \text{ km} \rightarrow P = 1/45 \text{ atm} = 17 \text{ mm Hg} \quad \text{H}_2\text{O boils } 20^\circ\text{C}$$

$$e^{-(2400/8000)} = 1.349 \rightarrow 1/1.349 = 0.74 \dots \text{ checks ok}$$

Snorkel at depth of 1 m is impossible given hydrostatic pressure

To inhale, you must expand your chest & overcome H_2O and

Atmospheric Pressure. You can not inhale (suck in/expand your chest)

even at a depth of only 1 meter → need for pressurized air tank

Letting air out underwater is easy. H_2O pressure helps.

Siphon; Atmospheric pressure down on the liquid in upper tank

pushes fluid up into the top of the siphon [as it does in a barometer]

where the pressure is lower. So it flows up and over the top.

The "Chain-pulling + tensile strength" explanation is incorrect.

By equating the PE at the top tank to the KE at the bottom tank

yields outflow velocity to be $v = [2gh]^{(1/2)}$ [Note no Volume or Mass]

This is the same equation for fluid leaving the bottom of any tank.

Straw sucking height limit calculation

Sucking up thru a straw from a height ...how high can one go ?

The **work done** to raise the column to h is $W = F \cdot s = P A h/2$

The **potential energy** PE of the column = $m g h/2 = (\rho A h) g (h/2)$

PE of the column = The work done moving the fluid →

$$W = P A (h/2) = PE = \rho g h^2 A/2 \quad [h/2 = \text{avg distance}] \rightarrow h = P_{1\text{atm}} / \rho g$$

$$h = 1.01 \text{ e5} [\text{N/m}^2] / [1000 \text{ kg/m}^3 \cdot 10 \text{ m/s}^2] = 10.1 \text{ m} \text{ at sea level}$$

This is the height of a water H_2O barometer at sea level

Pascal's Barrel Experiment; He attached a narrow vertical tube

10 m long to a barrel of H_2O , then filled it with water. It caused the barrel to

explode. Why ? Internal P now 2 atm >> External P of 1 atm ... **see Pascal's Vases**

Torricelli's Law solves Bernoulli Equation for the speed of water

leaving a tank → speed of emptying liquid = $\text{Sqrt} [2 g h]$ h = height of tank

Laplace's Law; Wall Tension $T = PR$ Vessel radius ↑ → Surface area A ↑.

If $P = F / A$ is constant & A ↑ → F , wall tension force ↑ (must go up)

Poiseuille's Law; Laminar Flow rate in cylindrical pipe **drops inversely with**

length and viscosity, and with **linearly with 4th power of the radius and Pressure drop**.

For blood flow, a 19% decrease in radius due to cholesterol **will cut flow rate in half !**

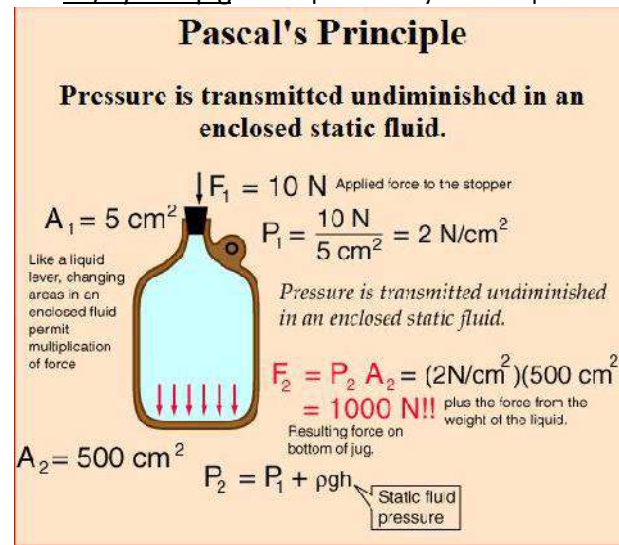
Pressure in the pipe drops linearly with length just as voltage drops along a wire.

Pascal's Principle; **Pressure change in an 'enclosed' incompressible fluid at rest is transmitted to all points in the fluid & acts in all directions. [perpendicular to the walls]**

$$P (\text{scalar}) = \text{Force} / \text{Area} = F / A \rightarrow F d / A d = \text{Work} / V$$

$$P = \text{Work} / \text{Volume} = \text{Energy Density}$$

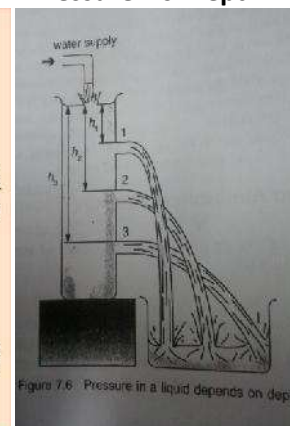
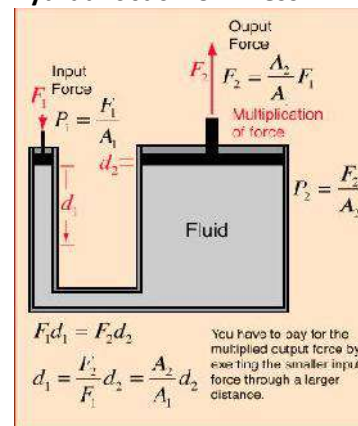
$$dP/dy = -\rho g \quad \rho = \text{density of the liquid}$$



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Hydraulic Jack or Press

Pressure vs Depth

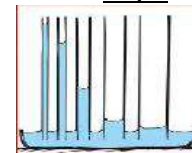
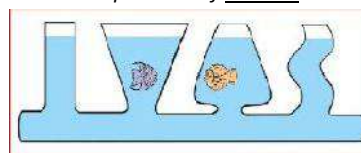


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Note that if the vessel at right is dropped and thus is in free fall, the Equivalence Principle adds an upward pseudo gravity equal to the downward gravity, so there is no longer a differential pressure on H_2O . Thus water will fall as one solid body. **No net Gravity → No Pressure differential with depth** → Not only is there no difference in stream outflow velocity, BUT **All 3 streams will stop flowing out. ! !**

Applications; Auto braking system, Siphon, Hydraulic Jack/Press

Pascal's Vases demonstrate fluid pressure depends only on **height**; and is **independent of volume** of fluid above it or **shape** of the vessel.



Capillary Action / Capillarity the rise of a liquid in a fine tube or narrow spaces. Caused by cohesive/adhesive forces. Height determined by Gravity.

Examples 1/ Towel or hair dipped in water 2/ The wick of an oil lamp 3/ Paint brush 4/ Transport of water from a plant root to its leaves

Vacuum levels vs Method Mechanical Pump 1 Pa;

Vapor Diffusion / Jet 10^{-8} Pa Sublimation Pumps 10^{-12} Pa

U Tube with 2 fluids. The difference in heights → relative densities

Fluid Statics Pg 2

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Archimedes Principle & Buoyant Force 3rd century BC

→ **Buoyant Force = weight of displaced fluid** ←

$$Wt [\text{displaced fluid}] = mg = [\rho (\text{fluid}) V (\text{displaced fluid})] g$$

ρ & g are constants → **Displaced Fluid Volume** ↔ **Buoyant Force**

WHY ??? Buoyancy arises **due to Gravity** causing fluid pressure

to increase with depth (Pascal's Law) [liquid or gas] thus

The bottom of a submerged object has more pressure than the top.

So there is a **Net Upward Force** on any submerged object

called the Buoyant Force. **Downward Force** = Wt of object

If the object floats → Object Wt down < Buoyant Force Up

**** Floating is an Equilibrium ; $F[\text{buoyant}] = F[\text{gravity}]$ ****

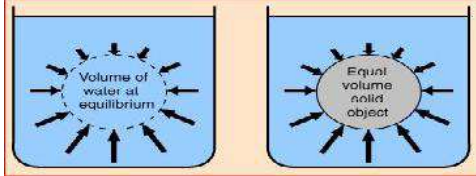
**** Equal submerged volumes feel the same Buoyant Force ****

**** The Buoyant Force is the same at all depths for given Volume ****

**** Submerged objects displace their volume BUT**

Floating objects displace their weight **

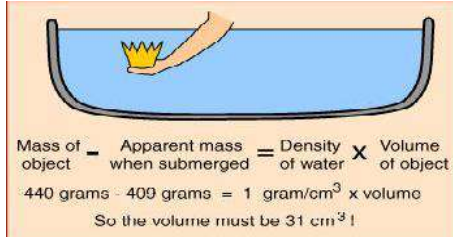
**** Fraction of object submerged = density ratio of object / fluid ****



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Archimedes Solution of King's Crown Density Problem

The King's crown is suspected of not being made of Pure Gold



Mass of object - Apparent mass when submerged = Density of water x Volume of object

$$440 \text{ grams} - 409 \text{ grams} = 1 \text{ gram/cm}^3 \times \text{volume}$$

So the volume must be 31 cm³!

$$\frac{440 \text{ grams}}{31 \text{ cm}^3} = 14.2 \text{ grams/cm}^3$$

Wait a minute! The density of solid gold is 19.3 gm/cm³!!

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Problem: to find the density of the material in the King's crown

Weight crown in water = Wt of crown - Buoyant Force Upward

Buoyant Force Upward = Wt of crown - Weight crown in water

Buoyant Force = Weight of displaced fluid

Wt of displaced fluid = Wt of crown - Weight crown in water

$$\rho_w V_{cr} g = \rho_{cr} V_{cr} g - [\text{known by measurement}]$$

$$V_{cr} = \text{volume of displaced water [measured]} \rightarrow \rho_{cr}$$

Iceberg in water; $\rho[\text{ice}] = 0.92 \text{ g/cm}^3$ $\rho[w] = 1 \text{ g/cm}^3$

Floating → No motion → Wt of object = Wt of displaced water, F_b

$$M_o g = M_{uw} g \rightarrow \rho_o V_o g = \rho_w V_{uw} g$$

$$V_{uw} = V [\text{ice under water}] \quad \rho_o V_o = \rho_w V_{uw}$$

$$V_{uw} / V_o = \rho_o / \rho_w = 0.92 / 1 = 0.92$$

so ... 92% of an iceberg's volume is underwater

Vertical Cylinder $A \times L$ in water; h = length under water

$$F_b = \text{Buoyant Force} = \text{Weight of displaced fluid} = \rho_w A h g$$

If floating / stable → $F_b = Mg + \text{Air Pressure}$

$$\rho[\text{fluid}] A h g = \rho[\text{object}] A L g + \text{Air Pressure}$$

independent of the shape or volume !!

To float → $\rho[\text{object}] < \rho[\text{fluid} = H_2O]$ & $h < L$

Classic Problem; Boat in a pool. A Rock in a is thrown overboard

Does the water line... of the pool ...go up or down ? Of the boat ? m = mass of rock

In boat rock displaces its Weight In the pool rock displaces its Volume

$$\text{Buoyant Force} = F_b = \text{Wt of displaced } H_2O = \rho[H_2O] V[\text{displaced}] g$$

Rock In water; Volume $[H_2O \text{ displaced}] = V \text{ of rock}$

Rock In boat; Floating → $F_b = F_G \Rightarrow \rho[H_2O] V[H_2O \text{ displaced}] g = \frac{mg}{\rho[H_2O] V[H_2O \text{ displaced}]} = \text{Wt rock}$

$$\text{Volume } [H_2O \text{ displaced}] = \frac{(\rho[\text{rock}] / \rho[H_2O]) V[\text{rock}]}{\rho[H_2O]} \text{ Comparing Volumes ;}$$

If $\rho[\text{rock}] \gg \rho[H_2O] \rightarrow$ more H_2O displaced with rock in the boat so BOTH pool

& boat water lines goes **down**

Fluid Dynamics

Bernoulli's Principle An increase in the speed of a fluid occurs simultaneously with a decrease in pressure

OR a decrease in the fluid's potential energy.

Venturi Effect is the reduction in fluid pressure that results when a fluid flow speeds up through a constricted section of pipe

HIGH pressure area is where speed is LOW, not high

LOW pressure is where speed is HIGH * highly counter intuitive !! *

→ From Bernoulli Eq at same height h , If $v \uparrow$ then $P \downarrow$ and visa versa

Law of Mass Conservation / Continuity → $A_1 v_1 = A_2 v_2$

If **Area $\downarrow$ then fluid velocity must increase** [Finger on a garden hose]

And to satisfy the **Law of Conservation of Energy**, since

Energy = Work = $P V$ must not change, → if $V \uparrow$, then P must $\downarrow$

$$P_1 - P_2 = \frac{1}{2} \rho (v_2^2 - v_1^2) \quad \text{from Bernoulli Equation, } h=0$$

Bernoulli's Equation Conservation of Energy`

$$\frac{1}{2} m v^2 + m g h + P V = \text{Constant} \quad V = \text{Volume}$$

$$\text{OR } \frac{1}{2} \rho v^2 + \rho g h + P = \text{Constant} \quad v = \text{velocity}$$

Trade speed for height, h , or pressure, P , in a tube of fluid

$$P = \frac{1}{2} \rho v^2 = \text{KE} / V \quad P = \rho g h = \text{PE} / V$$

Examples; 1/ Airplane wing / air foil; air on top travels farther and thus faster due to mass continuity creating lower pressure on top. 2/ NASCAR spoiler stabilizer bar pushes the car down with inverted air foil. 3/ Blow up on a funnel with a ping pong ball inside and you can NOT get it up or out due to the speed causing a low pressure in the narrow region between wall and ball. Turn it upside down and you can keep it in the funnel against gravity; hard to believe until seen 4/ Vacuum hose blow ping pong ball vertically will be very **stable horizontally** due to the Bernoulli Effect as long as LP region is below C of Mass. Even at angle of 30 degrees 5/ Spray can **Atomizer**; air flow over a tube going down to the perfume creates LP and the perfume rises and flows out with the air flow.

Balloons Air is a fluid → Buoyancy forces, as with water

Think of a water tank holding an ice cube & rock halfway down and let go. The ice rises and the rock sinks.

Same with He balloon & apple in air. One rises one falls

Specific Gravity = Ratio of densities of the object to the fluid

**** If density of the object < density of the fluid → object rises**

Hot Air Balloon **and visa versa, sinks ****

$$M[\text{total}] g = \text{Mass } M \text{ of gas} + M[\text{rest of the materials}]$$

$$F_b = \text{weight of displaced air} = V \rho[\text{air}] g$$

To rise ; $F_b > Mg$ ρ is density

$$V \rho[\text{air}] g > V \rho[\text{gas}] g + M[\text{rest}]$$

density of air > density of gas

necessary but not a sufficient condition

This is why hot air balloons are so large; Need big V

Since air ρ decreases with height, a He balloon rises only to the

height where $\rho[\text{air}] = \rho[\text{He}]$ = approx. 20 miles. But a hot air balloon

can go higher by further reducing balloon air density with more heat.

Acceleration Buoyancy

****Acceleration of box in horizontal [or any] direction simulates gravity****

Einstein's Equivalence Principle, Gravity - Acceleration

Acceleration creates a pseudo force due to inertia

→ **Imagine a hanging Apple & He balloon on strings in a box in outer space**

1/ Accelerating the box in outer space → no gravity or air. The apple & balloon **both stand still due to inertia**. But in the box they both seem to move in a direction, opposite the box acceleration vector.

2/ Same accelerated box, but with Air added inside

Air inertia creates higher density in back of Apple/ He balloon & thus

a differential air[fluid] pressure → buoyancy

Apple moves opposite acceleration vector [sinks in the fluid/ gravity]

He balloon moves with acceleration vector [rises] due to buoyancy

made possible by inertia & acceleration

Box + Air + acceleration → differential pressure → buoyancy

No air [fluid] → no buoyancy ; No accel → no buoyancy

No box → no buoyancy ; No gravity → no buoyancy

3/ Back on Earth, in a car with gravity or acceleration & air

Hanging apple & He balloon will go in opposite directions

if you hit the gas OR the brakes → $\pm$ Acceleration

4/ Driving around a bend, you & apple seem to move to the outside

but He balloon moves to the inside due to Centripetal Acceleration and the inertia of the air

Atomic & Nuclear Physics

Pg1

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Plum pudding Model of the atom JJ Thompson 1897

Rutherford Model experiment firing alpha particles at a thin Au sheet;
most went right thru, but some reflected straight back. → small positive nucleus.

Bohr Model of a mini Solar system flawed; can not explain why electrons do not fall in to the nucleus OR nuclear protons do not fly out of the nucleus

Three Theories of Light / Electromagnetic Energy λ = wavelength

1/ $\lambda \ll$ object size → Ray Optics ; Geometric Optics

1/ λ approx. = object size → Wave Theory Optics

1/ $\lambda \gg$ object size (atomic dimensions) → Quantum Mechanics

Light $\lambda = 5e-7$ m Atom $d = 1e-10$ m Nucleus $d = 1e-15$ m

Photoelectric Effect / Experiment Heinrich Hertz 1887

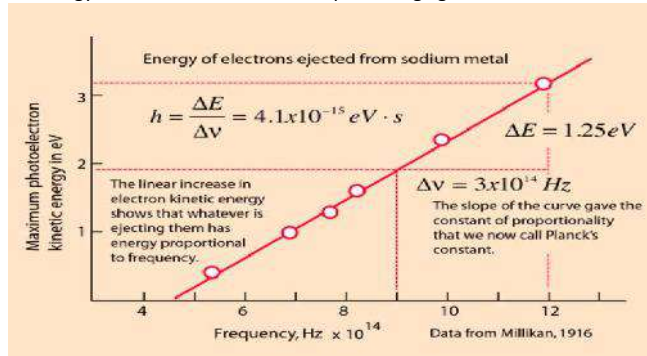
Light shining on metal => electrons are more easily with UV light

Predictions; 1/ change in the intensity of light would induce changes in the kinetic energy & velocity of the electrons emitted 2/ there is a delay in electron emission at low light intensities/energies

Results; 1/ electron energy depended on light frequency, not intensity
2/ below a threshold frequency, no electrons are emitted regardless of light intensity
3/ weak violet light creates higher energy electrons than intense red or yellow light
4/ There was no delay in electron emission at low light intensities/energies → Light Waves have particle like properties thus a new model is needed. The Quantum Model

KE (electron) = $h f - W$ W = Work function = Y intercept

KE = Energy of emitted electron f = freq incoming light, h = Planck's constant



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Max Planck's Quantum Model → **$E = hf$, $c = f \lambda$ → $E = hc / \lambda$**

h = Planck's Constant = $6.63 e-34$ J-sec = $4.1 e-15$ eV-sec

Photon is a packet of energy; a particle as well as a Wave

Louis DeBroglie 1924 Electron/particle is like a wave

Related electron momentum p to wavelength λ_e

Wavelength $\lambda_e = h / p$ $p = h / \lambda_e = hf / c = h v / c$ [$f = v$]

Electrons/Particles have a wavelike nature; superposition & interference

→ Wave - Particle Duality - Einstein believed light is a particle (photon) and the flow of photons is a wave much like a water wave is a flow of atoms

Erwin Schrodinger Wave Equation ; Quantum Wave Function

$$h^2/2m \partial^2 \Psi / \partial x^2 + V \Psi = i \hbar \partial \Psi / \partial t$$

Max Born Psi Ψ^2 = Probability of finding the electron at point x in space

Electron in Potential Well KE = $\frac{1}{2} m [h / m \lambda]^2$

Standing Wave metaphor for the electron trapped in a box model is too simple

Edwin Schrodinger replaced Standing Wave with Probability Wave Function Ψ^2

Electrons/ Protons as Mass & Charge Probability waves

Werner Heisenberg Uncertainty Principle Limit to precision

Pairs of physical properties of a particle such as position x and momentum p , can NOT be known simultaneously.

$$\Delta p \Delta x = \Delta E \Delta t > h / 4 \pi \quad \hbar = h / 2 \pi \quad [h \text{ bar}]$$

Albert Einstein $E = m c^2$ Energy and Mass are equivalent

1 kg = $9e12$ Joules 1 u = 935.1 MeV u = 1/12 M [C^{12}]

$$E = m^2 c^4 + p^2 c^2 \quad p = mv \text{ not valid near } c$$

Wave Properties Reflection, Refraction, Interference/Superposition, Diffraction, Polarization. **Particle Properties** can not explain the last three

Compton Effect demonstrated that inelastic scattering of light by collision with a charged particle, produces light of longer wavelength [less energy] than that of the incident radiation. ... The effect is significant because it showed that light cannot be explained purely as a wave phenomenon

Other Important Early Quantum Theory Experiments;

1/ Davisson - Germer 2/ Franck - Hertz 3/ Stern - Gerlach

Atomic Emission & Absorption Spectra →

Electron Orbit Radii → Quantized Electron Energy Levels

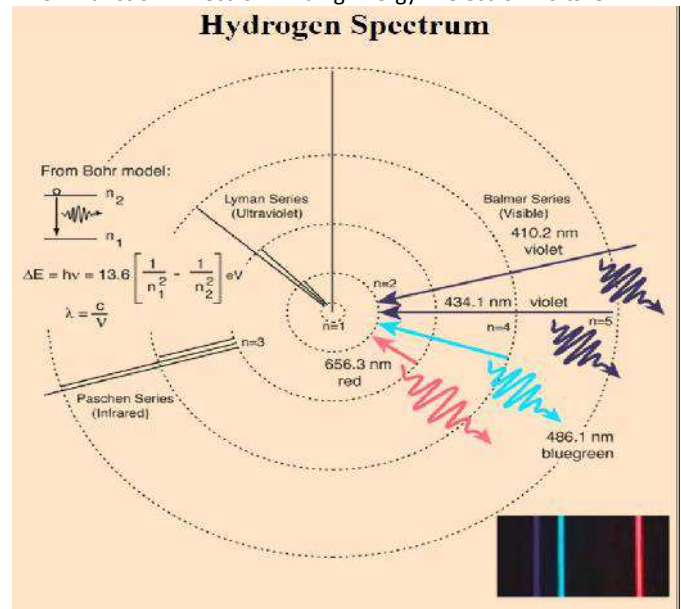
Atoms have discrete electron energy levels / states

Spectral Absorption & Emission lines → Quantized electron energy levels

Size of the change in electron energy levels in the atom →

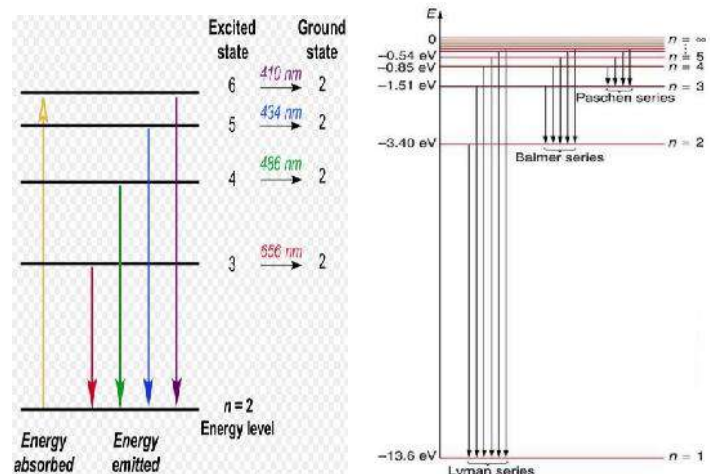
→ frequency of light/ electromagnetic energy emitted or absorbed

Work Function = Electron Binding Energy in electron Volts eV



Balmer Series

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Negative energy levels in electron levels => it is bound to the atom

Work is done => PE, but done by the charge, not on/to the charge

Analogous to satellite negative Potential Energy PE

Energy level comparisons ...

Per Atom	Energy	Per Atom	Energy
KE gas @RT	0.02 eV	Chemical	50 eV
Red Light	1.75 eV	Nuclear	200 MeV
Blue Light	3.1 eV		

Ex; Δ of 6 eV → $f = E / h = 6 (1.6e-19 \text{ J/eV}) / 6.63e-34 = 1.45 e15 \text{ Hz} \Rightarrow$ UV 207 nm

Visible Light Photon 1.5 - 3.5 eV, Medical Xrays 200KeV, Gamma/Beta 0-3MeV

Alphas 2-10MeV, Cosmic Ray 10MeV - 1kTeV, Kinetic Energy mosquito = 1 TeV

Energy electron **1eV = 1.6e-19 Joule**; # electrons / Coul = **6.24 e18 q / C**

Unified Atomic Mass Unit 1 u = 931.5 MeV/c^2 = 1.66054e-27 kg

Mass Electron $9.1 e-31 \text{ kg} = 0.00054858 \text{ u} = 0.511 \text{ MeV/c}^2$

Mass Proton $1.672e-27 \text{ kg} = 1.00727647 \text{ u} = 938.28 \text{ MeV/c}^2$

Mass Neutron $1.675 e-27 \text{ kg} = 1.00866490 \text{ u} = 939.57 \text{ MeV/c}^2$

Photons interact with the atom, alphas bounce off

Ionization; electron(s) removed leaving a positive + charged ion

Absorption / Emission Spectra => additional evidence of quantum energy levels

Phosphorescence Electron gun and Zinc Sulfide detector screen emits light if electrons excite the ZnS electrons

Electrons thru a thin graphite film give a pattern that looks like a

wave diffraction pattern. Particles appear to behave as a diffracting wave

Atomic Symbols and Terms

A = Z + N **A** = Mass Number = # Protons **Z** + # Neutrons **N**
Z = Atomic Number = # Protons = # Electrons
Atomic Mass/Weight = the average mass [grams] of all of the naturally-occurring isotopes of an element; e.g Carbon C 12.01 Lithium Li 6.941 Magnesium Mg 24.31

Experiments show all atoms have mass and charge that were multiples of the same number → nucleus was made of unit masses and unit charges.

Nucleon Neutron or Proton **Nuclide** Unique Nucleus species
Isotope Nuclides with same Z, but different N & A
Symbol X^A_Z or ${}_AZX$ Ex Carbon14 C^{14}_6 or ${}^{14}_6C$
Z is often omitted; element => Z, e.g. U_{235} $U \Rightarrow Z=92 A=235$
Neutrino Properties 1/ Zero electrical charge
2/ Mass much smaller than the electron, recent experiments indicate definitely some mass 3/ Spin of 1/2 4/ Very weak interaction with matter

Radioactive Decay

Emission of α β or γ Rays (Alpha, Beta or Gamma rays) from a nuclide that decreases Mass & increases Binding Energy per nucleon => making it more stable
Ex; Alpha Decay $\Delta m = 0.0304 u$ $BE = 28.3 Mev$
 $M[\alpha] = 4.00153 u$, $M[2 \text{ protons} + 2 \text{ neutrons}] = 4.03188$
Fe56 (Iron) has 3rd highest BE/A → so stable → abundant
**** Generally, stable nuclei have more neutrons than protons in order to compensate for the repulsion of protons ****
No nuclei are stable for A > 83 [Lead is 82, Bi is 83]
Light nuclei most stable if N = Z ; Heavy nuclei if N > Z

Note; Nuclear BE >> Electron BE by factor of 10⁶
Conservation of Energy & Momentum → the decay particle will get the highest speed [KE] in the decay. The sum of both the Mass Numbers and Atomic Numbers do not change

1/ Alpha Decay α 2 protons + 2 neutrons 4He2 [Helium nucleus]

$226Ra_{88} \Rightarrow 222Rn_{86} + 4He_2$ [Transmutation]
 $238U_{92} \Rightarrow 234Th_{90} + 4He_2$
Typical energy of 5 Mev Highly ionizing
Range of 2-3 cm of air Can not penetrate paper

2/ Beta Minus Decay $\beta(-)$ electron [Transmutation]

Neutron → Proton + electron $\beta(-)$ + Antineutrino
 $14C_6 \Rightarrow 14N_7 + {}^0_{-1}e + \bar{\nu}_e$
 $209Pb_{82} \Rightarrow 209Bi_{83} + {}^0_{-1}e + \bar{\nu}_e$
Range of 30cm air, 1 mm in Al Not highly ionizing

3/ Beta Plus Decay $\beta(+)$ positron [Transmutation]

Proton → Neutron + positron $\beta(+)$ + neutrino
 $22Na_{11} \Rightarrow 22Ne_{10} + {}^0_1e + \nu_e$
 $19Ne_{10} \Rightarrow 19F_9 + {}^0_1e + \nu_e$
Range of 30cm air, 1 mm in Al Not highly ionizing

4/ Gamma Ray Decay = No Change in Element

Gamma Rays are photons of very high Energy
Range centimeters of Pb (Lead) Not ionizing

5/ Neutrino No charge, little mass, unreactive,
Range 1000's km of Pb (Lead) !! Not ionizing

Antimatter = positive electrons + negative positrons
Changes in Z, N & A numbers with Decay Radiation Type

#	Z	N	A
Alpha α	-2	-2	-4
Beta $\beta -$	+1	-1	=
Beta $\beta +$	-1	+1	=
Gamma γ	0	0	=

Radiation protection; Distance, Lead or Concrete Shielding
Radiation The unit of activity, R, is the Curie, Ci
 $1Ci = 3.7 \times 10^{10}$ decays/second The SI unit of activity is the Becquerel, Bq
 $1Bq = 1$ decay / second → $1 Ci = 3.7 \times 10^{10} Bq$
The most commonly used units of activity are the mCi and the μCi
Average Nucleus radius is $r = r_0 A^{(1/3)}$ $r_0 = 1.2 \times 10^{-15} m$
Average Atomic radius = $1.0 \times 10^{-12} m$

Half Life The time it takes for the mass to halve by Decay

Rate of decay $\propto$ Mass M [# of Nuclei] → $dM/dt \propto \lambda M$
Exponential Growth or Decay → Rate of change is proportional to size
Decay follows an Exponential decay curve
 λ = Decay constant = Rate of decay in percent per unit time [years]
 $dM/dt = -\lambda M$ Solution is $M[\text{final}] = M[\text{initial}] e^{-\lambda t}$
Exponential Decay Equation $M(t) = M_0 e^{-\lambda t}$
Decay Constant λs^{-1}
The Log of both sides yields $\ln [M] = \ln [M_0] - \lambda t$
 $\ln [M/M_0] = -\lambda t$
 $\ln [1/2] = -\lambda t$

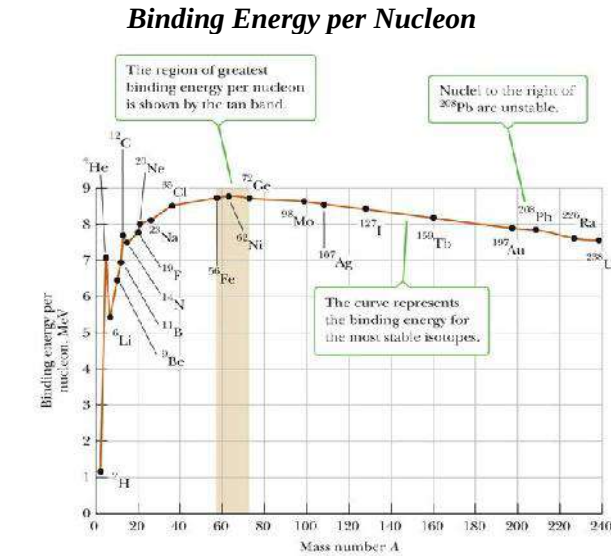
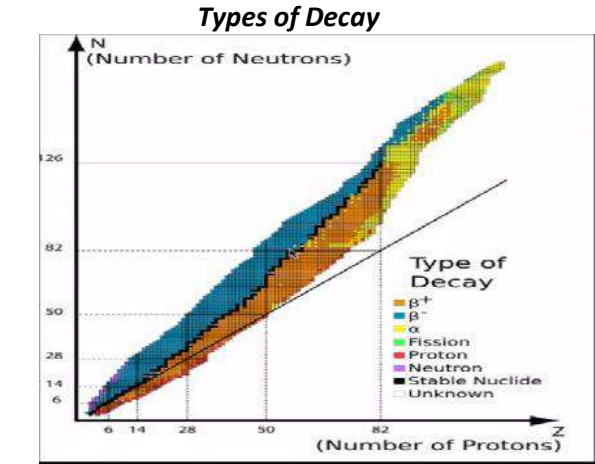
$t [\text{half life}] = \ln (1/2) / -\lambda = 0.693 / \lambda$
Plot of Log of M vs time is linear with slope = $-\lambda$
Carbon Dating measures ratio of radioactive C14 to C12 ratio.
C14 Half Life is 5730 years; Organic materials only
Potassium-40 → Argon Dating $t [\text{half}] = 1.25e9 \text{ yrs}$

Nuclear Binding Energy = $\Delta m c^2$ = Energy to split the nucleus
Binding Energy BE = Mass Defect → $M[\text{separate}] = M[\text{bound}] + BE$

1/ Energy not present in the separate masses is what holds the nuclei together. 2/ Energy given to KE of decay products 3/ Nuclear binding energy is used to determine whether fission or fusion will be a favorable process
**** In Decay, Fission or Fusion processes, BE/nucleon ↑ goes up ****

Nuclear Reactions Fission & Fusion

$14N_7 + 1n_0 \Rightarrow 14C_6 + 1p_1$ Nitrogen to Carbon
To find Energy released, subtract masses A before and after the reaction in units of "u" and x by 931.5 MeV/u
Energy came from Binding Energy of nucleus, NOT from Mass
Fusion and Fission increase Binding Energy per Nucleon
Binding Energy Nucleus = $10^6 \times$ Binding Energy of an Atom
Nuclear Strong Force holds nuclei together only over short range
Neutrons are stable in nucleus but unstable outside; half life = 15 min



Electron Configurations Shells, Subshells & Orbitals

- Shell

1st number of a subshell like 2 in 2p⁴

The Energy Level with a known Work function,

Principle quantum number 'N'

Mean radial distance of the electron from the nucleus
- Subshell

The letter after the Shell # p in 2p⁴

There are 4 subshells at present s , p , d , f

Angular quantum number 'L' the shape of the orbit or standing wave order

Some Physicists Don't Find Giraffes Hiding In Kitchens [G,H,I,K not needed yet.]
- Orbital

Divisions of the Subshell each of which can contain 2 electrons of opposite spin

There is 1 s orbital, 3 p orbitals, 5 d orbitals, and 7 f orbitals

Regions within an atom that the electron will most likely occupy.

Each orbital can hold two electrons. One spin-up and one spin-down
- Electron Configuration

A single string of orbital names and superscripts 1s² 2s² 2p⁶

The superscript = # electrons in that subshell // the sum of exponents = Atomic Number

An electron configuration for an atom with every orbital completely filled would be written: 1s² 2s² 2p⁶ 3s² 3p⁶ 4s² 3d¹⁰ 4p⁶ 5s² 4d¹⁰ 5p⁶ 6s² 4f¹⁴ 5d¹⁰ 6p⁶ 7s² 5f¹⁴ 6d¹⁰ 7p⁶ sum of exponents = 118 No 8s yet

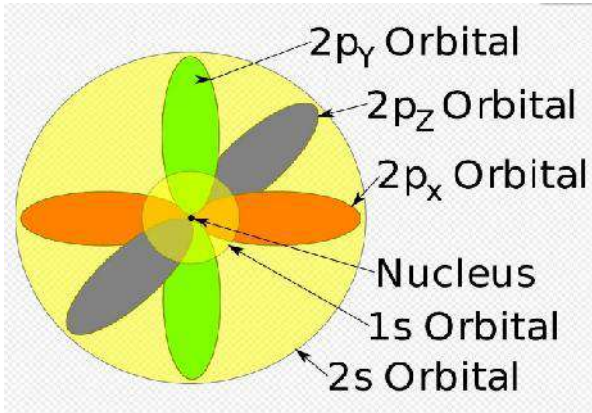
Note that the above list would be the electron configuration for (Oganesson), 118, Og,

the highest-numbered atom on the periodic table –

So this electron configuration contains every currently known electron shell for a neutrally charged atom

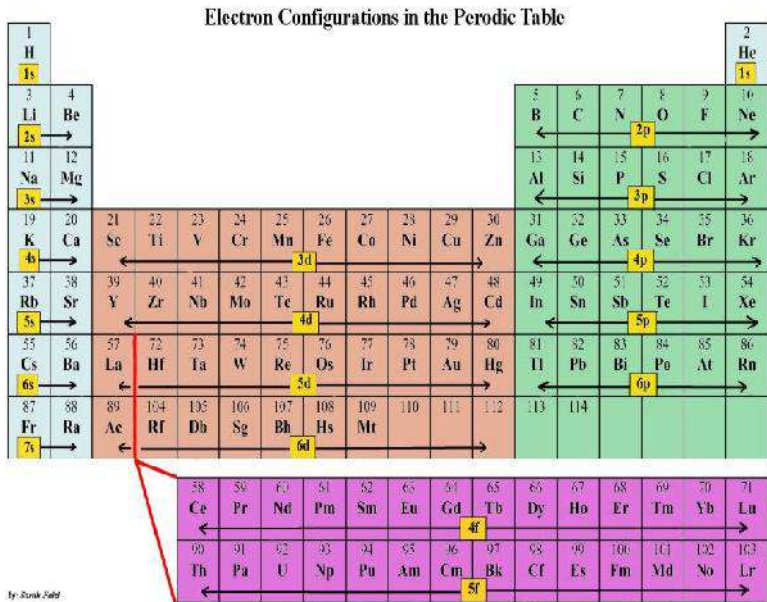
- Each shell can contain only a fixed number of electrons
- The 1st shell can hold up to 2 electrons,
- The 2nd shell can hold up to 8 (2 + 6) electrons,
- The 3rd shell can hold up to 18 (2 + 6 + 10) and so on.
- The nth shell can in principle hold up to 2(n²) electrons

- Each subshell has 2 electrons in each orbital.
- s subshell has 1 orbital that can hold up to 2 electrons,
- p subshell has 3 orbitals that can hold up to 6 electrons,
- d subshell has 5 orbitals that hold up to 10 electrons,
- f subshell has 7 orbitals that can hold 14 electrons.



- Four quantum numbers can describe an electron in an atom completely.
- Principal quantum number (n) Shell
- Azimuthal quantum number (l) Subshell 0,1,2,3 for s p d f
- Magnetic quantum number (m) Energy shift (orientation of the subshell's shape)
- Spin quantum number (s) Spin of the electron +1/2 or -1/2

- The electrons do not orbit the nucleus in the manner of a planet orbiting the sun, but instead exist as standing waves. Thus the lowest possible energy an electron can take is similar to the fundamental frequency of a wave on a string
- The 4th shell of any atom can hold a maximum of 32 electrons = 2n² n=4
- There are four subshells --- 4s, 4p, 4d, and 4f - that can hold a maximum of 2 times 1,3,5,7 = 2, 6, 10, and 14 electrons, respectively, for a total of 32
- 1st = max of 2 2nd = max of 8 3rd = max of 18 4th = max of 32 5th = max of 50
- The subshells s , p , d , f sharp, principal, diffuse and fundamental, respectively.
- The letters and words refer to the visual impression left by the fine structure of the spectral lines
- Quantum numbers L 0, 1, 2, 3 correspond to s , p , d , f
- Octet Rule: Atoms tend to gain or lose electrons to achieve an outer shell of 8 electrons [s² p⁶ orbitals] which is highly stable.
- Atoms in Group 1 & 2 tend to mate with atoms in Groups 16 – 17.
- Group 18 are the Noble Gases which are also highly stable

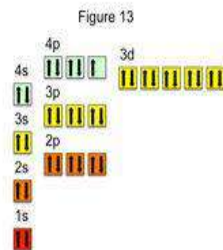


- The shape of the periodic table corresponds to the order of orbital sets in electron configurations
- When writing an electron configuration for Chlorine,.... think: "This atom is in third row (or "period") of the periodic table. It's also in the fifth column of the periodic table's p orbital block. Thus, its electron configuration will end with ...3p⁵"
- Examples
- Na11 or Al13⁺ 1s² 2s² 2p⁶ 3s¹
- O8 or N7⁻ or F9⁺ 1s² 2s² 2p⁴
- Sc21 or Ti22⁺ or Ca20⁻ 1s² 2s² 2p⁶ 3s² 3p⁶ 4s² 3d¹
- Cd48 or Sn50²⁺ 1s² 2s² 2p⁶ 3s² 3p⁶ 4s² 3d¹⁰ 4p⁶ 5s² 4d¹⁰
- Cs55 1s² 2s² 2p⁶ 3s² 3p⁶ 4s² 3d¹⁰ 4p⁶ 5s² 4d¹⁰ 5p⁶ 6s¹
- Lr103 or No102⁻1s² 2s² 2p⁶ 3s² 3p⁶ 4s² 3d¹⁰ 4p⁶ 5s² 4d¹⁰ 5p⁶ 6s² 4f¹⁴ 5d¹⁰ 6p⁶ 7s² 5f¹⁴

- Shorthand Notation Ca20 = [Ar] 4s² O8 = [He] 2s² 2p⁴
- Fe26 = [Ar] 4s² 3d⁶ Cd48 = [Kr] 5s² 4d¹⁰ Nd60 = [Xe] 6s² 4f⁴
- Pb82 = [Xe] 6s² 4f¹⁴ 5d¹⁰ 6p² Bk97 = [Rn] 7s² 5f⁸

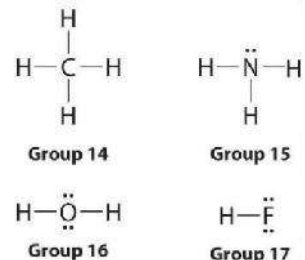
- Electron / Gilbert Lewis Dot Diagrams
- a shorthand ways to represent how atoms form covalent or ionic bonds to form molecules. Lewis dot diagrams use dots arranged around the atomic symbol to represent the electrons in the outermost energy level of an atom.
- Single bonds are represented by a pair of dots or one line between atoms.
- Double bonds are represented by a pair of lines between atoms
- Electron Filling Diagrams representation of the Electronic Configuration using boxes with 2 electrons of opposite sign per box.
- One box = one of the subshell orbitals

Bromine Electron Filling Diagram



bromine

Electron Dot Diagrams



Optics History

Before 1700 $\lambda \ll f$ Geometric / Ray Optics focal length f
Around 1800 $\lambda \sim f$ Wave Theory
Around 1900's $\lambda \gg f$ Quantum Theory

Law of Reflection ; Angle In = Angle Out

Law of Refraction ; $n_1 \sin \theta_1 = n_2 \sin \theta_2$

θ = angle theta = angle from the Normal to the surface

Both Laws can be derived from Fermat's Principle of Least Time

Frequency f does not change in a new medium, but λ does

Index of Refraction $n = c/v \Rightarrow n v = \text{constant} = \text{speed of light } 3e8 \text{ m/s}$

Since $v = f \lambda$ and $n = c/v = c/f \lambda \Rightarrow f = c/n \lambda \Rightarrow n \lambda = \text{constant}$

$n_1 v_1 = n_2 v_2$ & $n_1 \lambda_1 = n_2 \lambda_2$ & $n_1 \sin \theta_1 = n_2 \sin \theta_2$

Geometrical optics does not account for optical effects such as [diffraction](#) and [interference](#)

Definitions

R = Real V = Virtual Real & Inverted OR Virtual & Upright

I = Inverted U = Upright f = focal length

L = Larger S = Smaller do = object distance from center

If $do > f \Rightarrow di$ Real Inverted Image, Opposite side

If $do < f \Rightarrow di$ Virtual Upright Image, Same side

If $do = f \Rightarrow$ No image / Image is at Infinity

Object anything being viewed by an optical system or device

Image the likeness of an object from using an optical device where light rays cross or focus [mirror or lens]

Real Object rays physically emanate from the Object

Virtual Object rays appear to physically emanate from the Object but do NOT

Real Image where the light is, in front of mirror, behind a lens, can be seen on a screen / retina

Virtual Image where the object "seems to be" coming from

1/ diverging mirror 2/ object inside focal length of converging lens

A real image occurs where rays converge, whereas a

virtual image occurs where rays only appear to converge

Virtual images are formed by diverging lenses

or by placing an object inside the focal length of a converging lens

Plane/Flat mirrors, convex mirrors, and diverging lenses can

never produce a Real Image

Convex lens is **thicker in the center** than the ends

Concave lens is **thicker at the ends** than in the center

A **concave** mirror converges light to a focal point on the **same side** as the object.

A **convex** lens converges light to the focal point on the **other side** of the object

For a Thin Lens, the power is approx. the sum of the surface powers

Surface Power = $(n_2 - n_1) / R$

Lenses have two focal points, one on either side of the lens.

Mirrors have one focal point;

A concave mirror OR convex lens $\Rightarrow$ converges light to a focal pt $\Rightarrow$ Real Image.

A convex mirror OR concave lens $\Rightarrow$ diverges light from a focal pt $\Rightarrow$ Virtual Image

Concave or Flat Mirror $f > 0$ positive Convex Mirror $f < 0$ negative

Spherical lens or mirror is an approximation to a Parabolic

Geometry for small angles. The Focal Point is one half the Radius of the Spherical Curvature; $f = R/2$

Thin Lens $\Rightarrow$ refracts but NO dispersion/chromatic aberration OR spherical aberration

Differences between lenses and mirrors

1/ Light reflects from a mirror

2/ Light goes through, and is refracted by, a lens.

3/ Lenses have two focal points, one on either side of the lens.

Similarities between lenses and mirrors

1/ The equations we used for mirrors all work for lenses.

2/ A convex lens acts a lot like a concave mirror.

Both converge parallel rays to a focal point, have positive

focal lengths, and form images with similar characteristics.

Geometric Analysis

Convex Lens Ray Tracing Rules

The PARALLEL RAY goes through the lens and passes through the focal point on the far side

The CHIEF RAY passes through the center of the lens [for thin lens]

The FOCAL RAY goes through the focal point on the object side and emerges from the lens parallel to the principal axis.

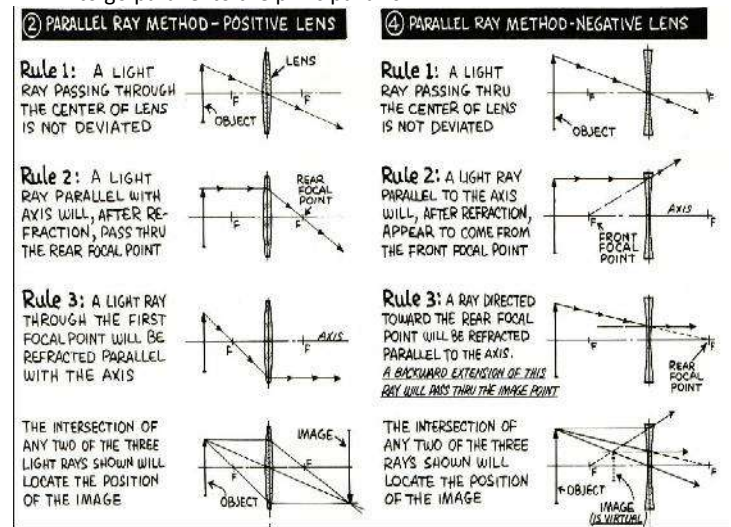
It is a mirror image of the parallel ray.

Concave Lens Ray Tracing Rules

The PARALLEL RAY goes through the lens and diverges away from the principal axis going directly away from the focal point on the object side of the lens.

The CHIEF RAY passes through the center of the lens [for thin lens] SAME AS FOR CONVEX LENS

The FOCAL RAY goes through the lens heading toward the focal point on the far side of the lens. It is re-directed by the lens to go parallel to the principal axis



Algebraic Analysis

LENSMAKER'S FORMULA

Spherical surface

$$1/f = (n - 1) (1/R_1 + 1/R_2)$$

R radius of curvature f Focal Length do object distance di image distance
 R_1 = front surface $R_1 > 0$ R_2 = back surface $R_2 < 0$ $n = n_2/n_1$ $n_1 = 1$ for air
 m = Magnification P = Lens Power in diopters h = height

$$f = do di / do + di \quad di = do f / (do - f) \quad f = R/2 \text{ for Spherical surface}$$

$$m = -di / do = hi / ho$$

$$P = 1/f \text{ in Diopters}$$

MEANING OF NEGATIVE QUANTITIES

do distance, di distance, f focal length, m magnification

Negative image distance $di \Rightarrow$ virtual image

Negative Image distance $di \Rightarrow$ object and image are on same side of lens

Negative focal length or Power $\Rightarrow$ diverging lens OR mirror

Negative magnification $m \Rightarrow$ image is inverted compared to the object.

A positive lens focal point is on the other side of the lens from where the object is placed.

Negative lens focal point is on the same side of the lens from where the object is placed.

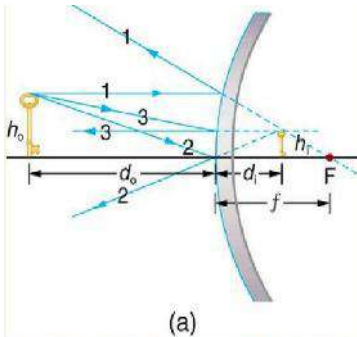
Converging lens or mirror $f > 0$ always

Diverging lens/mirror $f < 0$ always

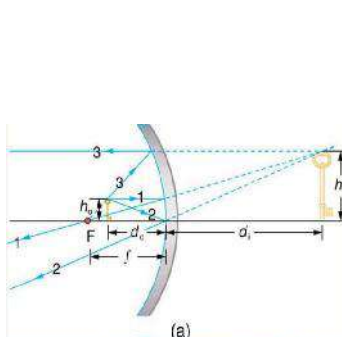
USES FOR LENSES AND MIRRORS

Concave Lens corrects for Nearsightedness Myopia
 Convex Lens corrects for Farsightedness Hyperopia
 Concave Lens $d_o > f$ glasses/contacts/ nearsighted, myopia
 Concave Lens $d_o < f$
 Convex Lens $d_o > f$ glasses/contact/ farsighted, hyperopia
 Convex Lens $d_o < f$ Magnifying glass
 Correction w converging lens; flashlights, binoculars, telescopes, photography
 Correction w diverging lens; cameras, microscopes
 Concave Mirror; Focusing Shaving mirrors, Head mirrors, Ophthalmoscope, Satellite dish, Astronomical telescopes, Headlights, Solar furnaces & collectors
 Convex Mirror; Fish eye/Diverging Mirror; Inside buildings, Sunglasses, Vehicle mirrors, Magnifying glass, Security
 Flat Mirror; Personal hygiene
 Reflecting telescopes make use of a concave mirror, a plane mirror, and convex lens
 Refracting telescopes use two convex lenses.
 Microscopes make use of a concave mirror, a plane mirror, and a convex lens
<https://byjus.com/physics/uses-of-convex-mirror/>

Convex Lens



Concave Lens



Cartesian Sign Convention

All figures are drawn with light traveling from left to right.
 All distances are measured from a reference surface, such as a wavefront or a refracting surface.
 Distances to the left of the surface are negative
 Angles measured clockwise from the optic axis are negative.
 The refractive power of a surface that makes light rays more convergent is positive.
 The focal length of such a surface is positive.
 The distance of a real object is negative.
 The distance of a real image is positive.
 Heights above the optic axis are positive.

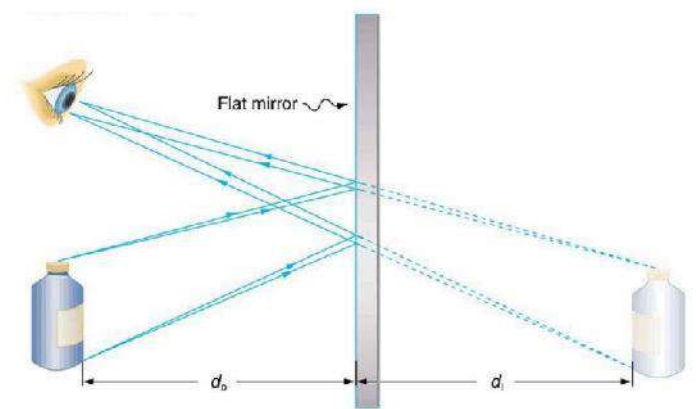
Hyperphysics Lenses

<http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/raydiag.html>

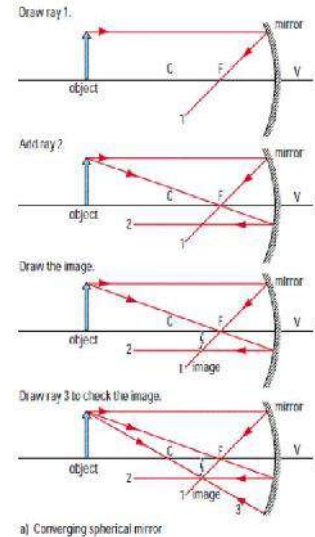
Hyperphysics Mirrors

<http://hyperphysics.phy-astr.gsu.edu/hbase/geoopt/mirray.html#c2>

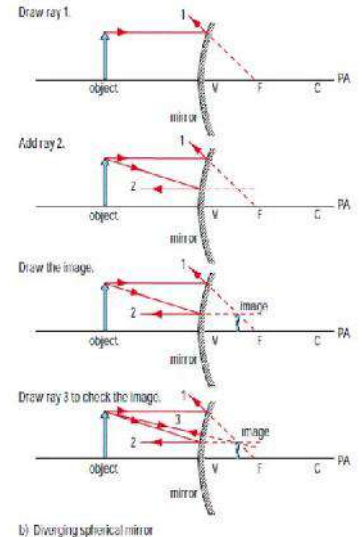
Flat Mirror showing Virtual / Apparent Image



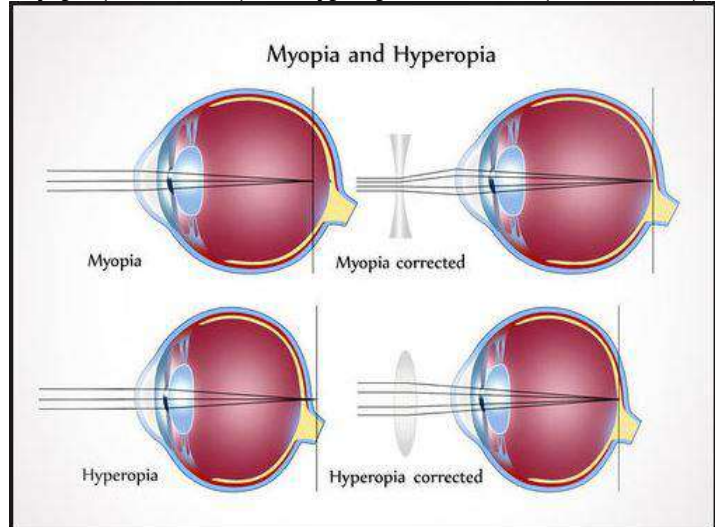
Converging Spherical Mirror



Diverging Spherical Mirror



Myopia (Convex Lens) and Hyperopia Corrections (Concave Lens)



Astronomy Pg 1

@pfreda@gmail.com 1/14/2021

Astronomical Unit AU is Distance $\text{Earth-Sun} = 1.5e11 \text{ m}$

Light Year 1 yr is Distance λ travels in 1 yr $1 \text{ yr} = 9.46e15 \text{ m}$

Parsec "parallax = 1 arc – second" $1 \text{ pc} = 3.26 \text{ yr} = 3.1e16 \text{ m}$

1 arcsec distant object in triangle with Sun & Earth

1 arcsec = $1/3600 \text{ deg}$!! (See "Stellar Distances" below)

Time for light to travel to Earth from

Moon 1 sec, Sun 8 min, Nearest star 5 yrs,

Nearest Galaxy $2e6 \text{ yrs}$

Light travels $10e16 \text{ meters/yr}$, $3 \times 10^8 \text{ m/s}$

Looking back in time

Stars are observed in constant relative position, but

Planets move about ; Planet in Greek = wanderer

Planet orbit is an ellipse with Sun at one focus

40 dwarf Planets one of which is Pluto

Energy Conservation $\rightarrow \text{PE} + \text{KE} = \text{constant}$

plus ellipse $\rightarrow$ radius and thus PE changes

so as $R \downarrow$, $\text{PE} \downarrow \rightarrow \text{KE} \uparrow$, $v \uparrow$ and visa versa

Earth period 23 hr 56 minutes; Moon period = 27.3 days

Star is a massive plasma

Asteroid $R < 100 \text{ km}$

Comet; loose particles of ice and rock with a tail

Distances; Star-star 1 yr; Diameter of galaxy 10^5 yr ;

galaxy to galaxy 10^6 yr

Fusion inside the sun balances the gravitation forces

Proton proton chain fusion of H to He

Apparent Brightness (b) $\text{Energy} / \text{Area}$ $b = L / 4 \pi d^2$

Luminosity (L) total **Power** in Watts

[energy radiated per sec] $L[\text{Sun}] 3.839 \times 10^{26} \text{ W}$

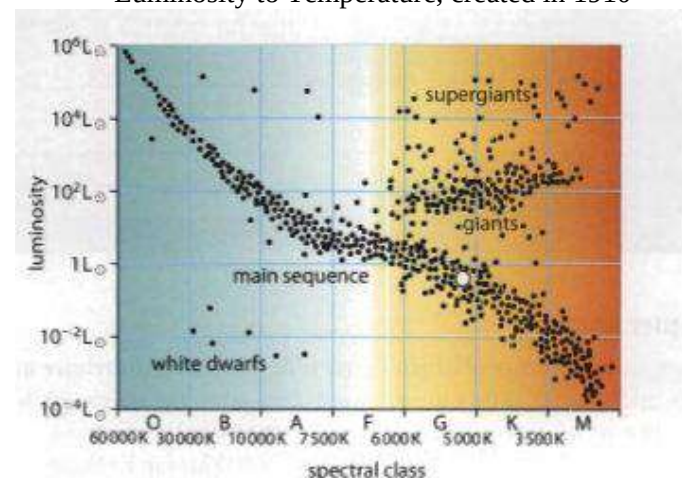
Stephan-Boltzman Law $P / m^2 = \sigma T^4$

$\sigma = 5.6e-8 \text{ W} / \text{m}^2 \text{ K}^{-4}$ Power emitted = $L = \sigma A T^4$

Wein Displacement Law $\lambda [\text{max}] = 2.9 \times 10^{-3} \text{ km} / T$

Hertzsprung-Russell Diagramrelates a star's

Luminosity to Temperature, created in 1910



Main Sequence HR Diag approx linear L vs T

Stars 90% of Stars seen on this line

Stars off the Main Sequence

Red Giant ; cool, large, $L = 100x L[\text{Sun}]$

$R = 10x R[\text{Sun}]$

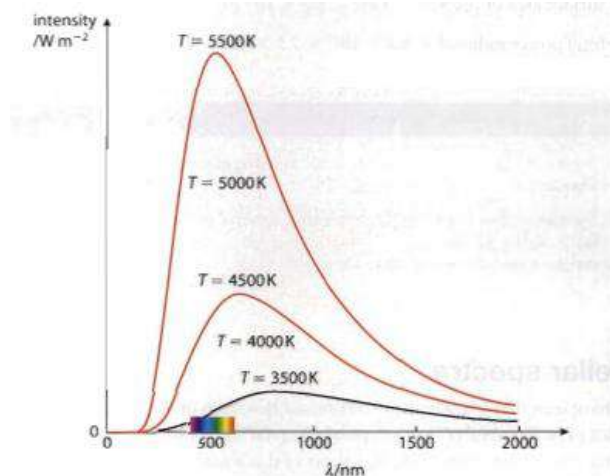
SuperGiants ; cool, very large, $L = 10^6 x L[\text{Sun}]$

$R = 1000x R[\text{Sun}]$, very rare, Betelgeuse

White Dwarf ; hot, small, $R \cong R[\text{Earth}]$, Low L

Neutron Star; Result of SG collapse, hot $R \cong 12 \text{ km}$

Black Body Radiation Intensity Distribution



Area under Intensity-Wavelength curve = Power

Black Body Radiation absorbs all, reflects none

Binary Stars

Accretor – Donor pair

For an orbiting body about a mass M

Gravitational Force = Centripetal Force

$$G M m / r^2 = m \omega^2 r$$

so if we measure ω and $r \rightarrow M$

Binary Stars almost half the stars out there

$$T^2 = 4 \pi^2 d^3 / G (M_1 + M_2) \quad d = \text{separation}$$

T & $d \rightarrow$ Mass of the 2 stars

Types of Binaries; Visual, Eclipsing, Spectroscopic

Donor gives up mass to the Accretor

Magnitude : Brightness Classifications

Harvard Classification; OBAFGKM

2000 degK to 60000degK

$$(2)^5 = 32 \quad (2.512)^5 = 100$$

Apparent Magnitude m [viewed from Earth]

Greek scale was x2 each step for a range of x32

1 (brightest) to 6 (dimpest) $(2.512)^5 = 100$

Modern scale uses x2.512 each step, range = x100

Dimmest Star seen with; eye $m=6$, binoculars 10,

large telescope 20, photographic telescope 25

$$\text{Brightness ratio } b_1 / b_2 = 2^{(m_2 - m_1)}$$

Absolute Magnitude M [viewed from dist 10 Pc]

$$\text{Distance from Earth } d = 10 \times 10^{(m - M)/5} \text{ pc}$$

$$\text{If } p = \text{parallax in arcsec, } M = m + 5 (1 + \text{Log } p)$$

$$\text{Distance modulus } u = m - M \quad d = 10^{(1 + u/5)}$$

Apparent (m) & Absolute (M) Magnitudes

	m	M
Sun	-26.8	4.8
Full Moon	-15.6	
Venus	-4.4	
Sirius	-1.47	1.4
Vega	0.04	0.5
Betelgeuse	0.41	-5.14
Polaris	1.99	-3.6
Pluto	15.1	

Stellar Distance Measurements

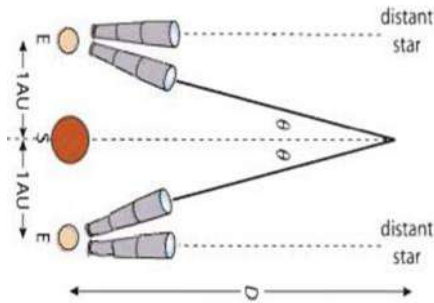
1/ Stellar Parallax Method

For small angles $\tan \theta = \sin \theta = \theta$ in radians

$$\tan \theta = \theta = \text{AU} / D \rightarrow \underline{D = \text{AU} / \theta}$$

AU is distance Sun-Earth for measurements

6 months apart to create the Parallax



1 Parsec = Distance if $\theta = 1$ arcsecond

so then $\text{Distance (pc)} = 1 / \text{Angle in arcseconds}$

smallest angle measurable from Earth = 0.01 arcsec

due to atmosphere distortion. Satellites 0.001 arcsec

360 arcsec per arc deg; 1 arcdegree = $2\pi / 360$ rad

2/ HR Diagram; Given b & λ [max] $\rightarrow$ Find L and d

measure λ [max] $\rightarrow$ T from Wein's Law $\rightarrow$

L from HR; with $L \rightarrow d$ from Apparent brightness

$$b = L / 4\pi d^2 \rightarrow d = \text{Sqrt} [L / 4\pi b]$$

3/ Cepheid Variables; **Important Standard Candles**

expand $\Rightarrow$ bright [fast] & contract $\Rightarrow$ dim [slowly]

L vs T is linear on a log-log plot

$T \rightarrow L$, $+ b \rightarrow d$

Stellar Evolution & Processes

Birth dust particle gravitation not enough; a force like a Supernova needed for compression. 10^5 yr process

1/ Protostar; contracts, $T \uparrow$, λ emitted but not visible due to dust

2/ Pre main sequence; More contraction, $T \uparrow$, fusion

3/ Main sequence; contraction stops, fusion forces now balance gravitational forces

Star life process moves along & across HR Diagram

Small Star Heating/Fusion stops when Hydrogen runs out He then fuses to Carbon, until Fe iron is left in the core and fusion stops; Fe has max Binding Energy per nucleon, so it can not provide more energy for reaction. Smaller stars never make it to Fe and stop at Carbon

Chandrasekhar Limit; White Dwarfs can form only from stars w Mass $< 4 M_{\text{Sun}}$

Big Star Layers of fusing elements; H, He, C, Ne, O, Si, Mg, Fe SuperNova to Neutron Star will be stable if Mass $< 3 M_{\text{Sun}}$. the **Oppenheimer-Volkoff Limit**

Black Holes ; If the Neutron star has $M > 3 M_{\text{Sun}}$ it will continue to collapse [increasing density] until not even light can escape and becomes a Black Hole.

Pulsars; Stars rotate (1 cycle/month)

As they collapse they gain speed due to Conservation of Angular Momentum. Since the magnetic field also then intensifies, exciting EM radiation near the poles that give a flashing of light with period 0.3 to 1.5 sec.

Discovered in 1967 by Jocelyn Bell

Cosmology

Models of The Universe

Newton; Infinte in size & age. Static. Uniform

Olber's paradox; ∞ # stars $\Rightarrow$ why is sky black ?

Galaxies found in clusters, not randomly

Red Shift killed Newton's Static Universe

Big Bang explosion model

universe is expanding vs stars moving thru the universe; Balloon surface analogy for separation

Red shift $= \Delta \lambda / \lambda = v / c$ v = relative velocity

Hubble's Law Recession Speed vs Distance

He measured Recession speed & Distance of many

Galaxies and found **they are linearly related.**

Hubble's Constant H_0 is the slope of this line

$H_0 = 72 \text{ km/sec} / \text{Mpc}$ Divide by $3.09 \times 10^{19} \text{ km/Mpc}$

yields $H_0 = 2.33 \times 10^{-18} \text{ sec}^{-1}$

Age of Universe $= 1 / H_0 = \frac{s}{r} = \frac{d}{v}$

$1 / H_0 = 4.29 \times 10^{17} \text{ sec} = \underline{1.36 \times 10^{10} \text{ yrs}}$

Age of Universe is ≈ 13.6 Billion Years

Calculation assumes velocity is constant

Atoms did not form until 10^9 yrs when $T = 4000\text{K}$

which is $\approx 0.4 \text{ eV} <$ ionization energy of Hydrogen.

Cosmic Microwave Background Radiation CMB

Radiation from the Big Bang

Discovered/Confirmed in 1960's Penzias & Wilson

COBE satellite showed CMB not uniform $\rightarrow$

galaxies can form

Future Possibilities of the Universe's Expansion

Open; keeps expanding

Flat; rate of expansion tends toward zero

Closed; expansion stops, contraction begins

Critical density is that which will cause Closed Universe

$$\rho [\text{critical}] = 3 H_0^2 / 8 \pi G = \underline{10^{-26} \text{ kg/m}^3}$$

this is just 6 H atoms per cubic meter ! !

Hubble's Law $\Rightarrow v = H_0 r$

Dark Matter does not emit or interact with light

MACHO Massive Astro Compact Halo Objects

WIMPS Weakly Interacting Massive Particles

Dark Energy is what can explain the accelerating recession of the galaxies.

	R km	M M_{Sun}	ρ gm/m^3
White Dwarf;	$e4$	0.5	$e6$
Neutron Star	$e1$	1.5	$e19$
Black Hole	0	3.0	∞

Speed of Light is an Absolute Maximum

A constant of Nature

STR = Special Theory of Relativity

➔ **Position, velocity, energy, momentum all relative**

..... Galileo's Principle of Relativity

All Inertial motion [non accelerated, uniform] is relative

No absolute and well-defined state of rest

No privileged or absolute reference frames

Einstein extended this principle so that it included

the constant speed of light [in inertial frames]

STR is defined in the absence of Gravity or acceleration

STR is based on two postulates:

1/ Relativity Principle: The laws of nature are the same in all non accelerated (inertial) or non gravitational reference frames

2/ **The speed of light in a vacuum is the same in all inertial frames. It is an absolute of Nature and NOT relative.**

STR is about both relative and high velocities

Mass–Energy equivalence $E = m c^2$

Mass–Energy Equivalence => Light bending

Not about Gravity or accelerated frames of reference

At speeds near speed of light 'c' other frames see

Time Dilation [TD] Length Contraction [LC]

Non Simultaneity [NS]

Gamma γ is the factor $[1 / \sqrt{1 - [v/c]^2}]$

v	0	0.5 c	0.8 c	0.9 c	0.99c	1.0 c
γ	1	1.15	1.66	2.3	7	∞

Lorentz Transformation gives the parametric value in

Special Relativity. One multiplies or divides by Gamma

STR Consequences

Mass–energy equivalence $E = mc^2$ => Light bending

Time dilation [in the moving frame seen by rest frame]

Length contraction [in moving frame seen by rest frame]

Relativistic mass increasing with speed

Different reference frames disagree about Simultaneity

Relativity of simultaneity disappears near c

Relativistic Doppler effect Thomas precession

Twin Paradox - travelling twin ages more slowly; which one travelled ? Ans; Neglect accel and decell [thus not inertial frames] is what causes the paradox

More STR Consequences and Paradoxes

Bell's spaceship paradox - 2 rockets w string between them going in same direction snapping due to LC

Ehrenfest paradox - Rotating disc; radius 'r' does not contract being perpendicular to motion, but circumference = $2 \pi r$ does

Ladder - Garage Paradox - which contracts ? Incorrectly assumes absolute simultaneity

Mass–energy equivalence is a consequence of special relativity's speed of light limitation on mass
=> Equivalence of mass and energy,

$E = mc^2$ => photon energy behaves as mass; can bend in a G field.

Galileo Galilei had already postulated that all uniform motion is relative AND already showed gravitational motion is independent of mass ! !

Speed of Light 'c' is not just the velocity of a certain phenomenon—namely the propagation of electromagnetic radiation (light) — but rather a fundamental feature of the way space and time are unified as Spacetime.

Newton's other great achievement, the Universal Law of Gravitation, is not compatible with Special Relativity

Curvature of Spacetime is the Absolute of Nature

GTR = General Theory of Relativity

➔ **Acceleration & Force are relative concepts too**

Matter tells spacetime how to curve ... and ...

Spacetime geometry tells matter how to move.

Einsteinian Equivalence Principles

1/ Gravitational Field on Earth = Accelerated Frame in Space

You feel a Force in either situation = [Normal force]

It is a pseudo Force in that it is a result of Inertia

2/ Free Fall in GField = Inertial Frame in Space [$\Delta v = 0$]

You feel NO Force ... in either situation

General relativity is a geometric theory of gravitation

STR = Relativity for velocity [inertial frames]

GTR = Relativity for acceleration [non-inertial frames]

GTR removes an asymmetry from Newtonian Gravity, namely the distinction between inertial and non inertial [accelerated] frames of reference.

General relativity is the generalization of special relativity to include gravitation and/or acceleration (non inertial reference frames)

Special relativity is restricted to flat spacetime and constant velocity inertial frames.

Curvature itself => change in velocity direction

=> acceleration => Force = m a

GTR Predictions/Implications/Consequences

differ significantly from those of classical physics

geometry of space is curved; Riemannian

gravitational time dilation, - , processes close to a massive body run more slowly

gravitational time delay light signals take longer to move through a gravitational field

gravitational lensing ; light bends thru a G Field

gravitational redshift of light

gravitational waves analogous to EM Waves

orbital decay caused by emission of gravitational waves

predicts existence of black holes

Increases in Velocity or Gravity slow down time

Moving clocks run slower than clocks at rest STR

Clocks on Earth run slower than clocks in space GTR

Light slows down in gravitational fields GTR

$$E^2 = (mc^2)^2 + (pc)^2$$

E^2 = rest energy squared + momentum squared

$$v E / c = pc$$

Einstein deduced that free-fall is actually inertial motion

Free fall hides G force; You do not feel any Force

Acceleration creates an opposite pseudo Force

Gravity is a geometric property. Curvature is what creates the force called gravity. If you're driving along the highway and you hit a steep hill, or a curve in the road, that geometric curvature is going to change your velocity ➔ acceleration ➔ Force

The curvature of spacetime is directly related to the energy and momentum of whatever matter and radiation are present; spacetime is a relativistic manifestation of the existence of mass

Evolution of Ideas in Physics History

Earth flat => Earth curved;

Space Euclidean flat => SpaceTime Riemannian curved