A Planned Course of Study

for

AP Calculus-BC

ASHS Course # 0360

Abington School District

Abington, Pennsylvania

September, 2016

AP CALCULUS-BC PLANNED COURSE OF STUDY

I. Objectives

Students will demonstrate the appropriate level of proficiency in each of the following areas:

A. Limits
- a. The Limit of a Function
- b. Calculating Limits Using the Limit Laws
- c. Continuity

B. The Derivative
- a. Definition of the Derivative
- b. The Derivative as a Function
- c. Derivative Rules
- d. Derivatives of Trigonometric Functions
- e. The Chain Rule
- f. Implicit Differentiation
- g. Position, Velocity, Acceleration
- h. Related Rates

C. Applications of the Derivative
- a. The Extreme Value Theorem, Finding Extrema on a Closed Interval
- b. The Mean Value Theorem
- c. The First Derivative Test, Concavity and The Second Derivative Test
- d. Graphing Polynomial, Rational and Radical Functions
- e. Optimization
- f. Anti-Derivatives, Particle Moving on a Line

D. Integration
- a. Areas and Distances
- b. The Definite Integral
- c. The First and Second Fundamental Theorems of Calculus
- d. Indefinite Integrals and the Net Change Theorem
- e. Integration by Substitution (*u*-substitution)
- f. Numeric Integration, Trapezoidal Rule

E. Applications of Integration
- a. Area Between Curves

b. Volume by Discs and Washers, Volume by Known Cross Section
c. Average Value of a Function and the Mean Value Theorem for Integrals
d. Arclength

F. Inverse Functions and Calculus

a. Differentiation of Exponential and Logarithmic Functions with Applications
b. Integration of Exponential and Logarithmic Functions with Applications
c. Models of Exponential Growth and Decay
d. Inverse Trigonometric Functions
e. Indeterminate Forms and L'Hopital's Rule

G. Techniques of Integration

a. Integration by Parts
b. Integrating Powers of Sine and Cosine, Secant and Tangent
c. Partial Fraction Decomposition
d. Improper Integrals

H. Series

a. Sequences, Limits of Sequences
b. Series, Types of Series
c. Tests for Convergence and Divergence
d. Power Series
e. Taylor and MacLaurin Series

I. Differential Equations

a. Modeling With Differential Equations
b. Direction Fields
c. Euler's Method
d. Logistic Differential Equations

J. Curves in Parametric, Vector and Polar Form

a. Plane Curves and Parametric Equations
b. Vector Valued Functions
c. Polar Coordinates and Polar Graphs

II. Major Concepts

Students will demonstrate the appropriate level of proficiency in each of the following areas:

A. Limits

a. The Limit of a Function

i. Understand the limit definition and symbolic representation of a limit.
ii. From a given graph, evaluate a limit at a point. Include one-sided limits.
iii. Use numeric and graphic information with and without the graphing calculator to estimate a limit. Include one-sided limits, limits at +/− infinity, and functions that are unbounded, oscillating, and contain two different one-sided limits.
iv. Sketch a function given a set of limit conditions.

b. Calculating Limits Using the Limit Laws

i. Use limit theorems to evaluate limits analytically, including limits of piecewise functions.
ii. Evaluate limits analytically. Include finding limits of sums, differences, products, quotients, and composite functions that use limit theorems and algebraic rules.
iii. Evaluate the limit of a function involving algebraic manipulation and special trigonometric functions.
iv. Use one-sided limits to recognize and find vertical asymptotes.
v. Use limits as x tends toward +/− infinity to recognize and find horizontal asymptotes.
vi. Use limits to describe asymptotic and unbounded behavior.

c. Continuity

i. Explore continuity visually, and understand and apply the definition of continuity.
ii. Explore different types of discontinuity (i.e., removable, jump, infinite) using the graphing calculator.
iii. Determine types of discontinuity analytically. Include polynomial, rational, power, exponential, logarithmic, and trigonometric functions.
iv. State and explore the meaning of the Intermediate Value Theorem analytically and graphically.

B. The Derivative

a. Definition of the Derivative

i. Visually understand the difference between average rate of change and instantaneous rate of change.
ii. Make connection between the limit of the difference quotient and rate of change at a point.

iii. Recognize the use of the limit of difference quotients. Use the limit of difference quotients when evaluating rate of change, slope of tangent line, velocity, acceleration, or derivative value at a point.
iv. Define a derivative using limits. Understand the multiple notations for representing derivatives.

b. The Derivative as a Function
i. Find equations of tangent and normal lines.
ii. Use linear approximations to approximate a function's value at a point.
iii. Use the graphing calculator to find the derivative of a function at a given point.

c. Derivative Rules
i. Find derivatives by the power, product, and quotient rules.

d. Derivatives of Trigonometric Functions
i. Determine derivatives of trigonometric functions.

e. The Chain Rule
i. Find derivatives of composite functions by applying the Chain rule.

f. Implicit Differentiation
i. Define the differences between explicit and implicit functions.
ii. Find derivatives of functions implicitly.

g. Position, Velocity, Acceleration
i. Recognize and use difference quotients when evaluating average rate of change, average velocity, average acceleration, and approximation of slope or derivative.
ii. Solve rectilinear motion problems involving position, speed, velocity, and acceleration.

h. Related Rates
i. Apply the concepts of implicit differentiation and the chain rule to solve related rates problems.

C. Applications of the Derivative

a. The Extreme Value Theorem, Finding Extrema on a Closed Interval
i. Define and apply the Extreme Value Theorem
ii. Find absolute extrema of a function on a closed interval.

b. The Mean Value Theorem
i. Explore the connection between continuity and differentiability.
ii. Determine differentiability of a variety of functions using limits.
iii. Define and apply the Mean Value Theorem.

c. The First Derivative Test, Concavity and The Second Derivative Test

i. Use the first derivative to determine critical points, horizontal and vertical tangent lines, intervals of increase or decrease, and relative extrema.
ii. Use the second derivative to determine intervals of upward/downward concavity and points of inflection.

d. Graphing Polynomial, Rational and Radical Functions

i. Given various representations of f'or f'' (e.g., by graph, table, and formula), state the relationship these functions have to the function f and sketch a possible graph of f.
ii. Use the graphing calculator to graph f or f' to find critical points; horizontal and vertical tangent lines; intervals of increase or decrease; and relative extrema, concavity, and points of inflection.
iii. Use the graphing calculator to confirm characteristics (e.g., concavity) of graphs of functions.
iv. Explore and apply the second derivative test for relative extrema.

e. Optimization

i. Apply the rules of differentiation and the concepts of the First Derivative Test to solve problems involving the optimization of a variable quantity.

f. Anti-Derivatives, Particle Moving on a Line

i. Define the concept of anti-differentiation.
ii. Determine the formula for a function given one or more of its derivatives.
iii. Define the concepts and conventions of a particle moving on a line.
iv. Determine the position, velocity and acceleration of a particle as well as total distance traveled along a line.

D. Integration

a. Areas and Distances

i. Translate the symbolic representation of the definite integral to a graphical representation.
ii. Make a connection between the definite integral and area. Calculate the definite integrals of linear functions over a given interval using geometry.

b. The Definite Integral

i. Formulate and use properties of definite integrals. Include the integral of a constant times a function, the integral of the sum of two functions, reversal of limits of integration, and the integral of a function over adjacent intervals.
ii. Use technology to explore a Riemann sum with different partition sizes on non-linear functions.
iii. Define the definite integral as a limit of a Riemann sum.

iv. From functions represented graphically, numerically, algebraically, and verbally, use a left Riemann sum, a right Riemann sum, and a midpoint Riemann sum with different partition sizes to find approximations for definite integrals.
v. Recognize a limit of a Riemann sum as the definite integral and translate to its symbolic form.

c. The First and Second Fundamental Theorems of Calculus
i. Understand and use the Fundamental Theorem of Calculus [If f is continuous on [a,], then $\int (x)$x $=F(b)-F(a)$] to evaluate definite integrals.

d. Indefinite Integrals and the Net Change Theorem
i. Define and use the process of anti-differentiation to find the expression of an indefinite integral.

e. Integration by Substitution (*u*-substitution)
i. Develop techniques for finding antiderivatives by algebraic manipulation and substitution of variables.

f. Numeric Integration, Trapezoidal Rule
i. From functions represented graphically, numerically, algebraically, and verbally, use a trapezoidal sum with different partition sizes to find approximations for definite integrals.
ii. Use the graphing calculator to perform numerical integration.

E. Applications of Integration

a. Area Between Curves
i. Define the concept of the area of the region between two functions.
ii. Define and evaluate a definite integral that represents the area of the region between two functions.
iii. Use the graphing calculator to determine the area between two functions.

b. Volume by Discs and Washers, Volumes by Known Cross Section
i. Define the concept of the volume of a solid of revolution (both disc method and washer method).
ii. Define and evaluate a definite integral that represents the volume of a solid of revolution.
iii. Define the concept of a volume of known cross-section.
iv. Define and evaluate a definite integral that represents the volume of a solid of known cross-section.
v. Use the graphing calculator to determine the volumes of solid shapes (disc, washer and known cross-section).

c. Average Value of a Function and the Mean Value Theorem for Integrals

i. Apply definite integrals to problems involving average value of a function on an interval.

d. Arclength

i. Define and evaluate a definite integral that represents the length of a finite arc.
ii. Use the graphing calculator to evaluate a definite integral that represents the length of a finite arc.

F. Inverse Functions and Calculus

a. Differentiation of Exponential and Logarithmic Functions with Applications

i. Define the concept of inverse functions.
ii. Derive the rules for differentiation of exponential and logarithmic functions of any base.
iii. Apply the concepts of differentiation to sketch exponential and logarithmic functions.

b. Integration of Exponential and Logarithmic Functions with Applications

i. Determine the antiderivatives of functions containing exponential or logarithmic expressions.
ii. Use the rules of anti-differentiation to evaluate definite integrals representing the volume of a solid of revolution defined by an exponential or logarithmic function.

c. Models of Exponential Growth and Decay

i. Use separation of variables to find general solutions of differential equations to model the exponential growth or decay of specific variable quantities.
ii. Use exponential models to evaluate population growth.

d. Inverse Trigonometric Functions

i. Define the inverse trigonometric functions.
ii. Derive the rules for differentiation of inverse trigonometric functions.
iii. Apply the rules of inverse trigonometric derivatives to sketch inverse trigonometric functions.

e. Indeterminate Forms and L'Hopital's Rule

i. Evaluate limits of indeterminate form 0⁄0 and ∞⁄∞ using L'Hospital's Rule.
ii. Define other indeterminate forms and use algebraic manipulations to rewrite expressions into a form to which L'Hospital's Rule may be applied.
iii. Determine whether one function grows faster than another using limits.

G. Techniques of Integration

a. Integration by Parts

i. Derive the formula for integration by parts.
ii. Apply the techniques of integration by parts to anti-differentiate indefinite integrals and to evaluate definite integrals.

b. Integrating Powers of Sine and Cosine, Secant and Tangent

i. Define the concepts associated with integrating powers of trigonometric functions.
ii. Apply the concepts of trigonometric integration to anti-differentiate indefinite integrals and to evaluate definite integrals.

c. Partial Fraction Decomposition

i. Define the concepts of decomposing rational functions into partial fractions.
ii. Apply the concepts of partial fraction decomposition to anti-differentiate indefinite integrals and to evaluate definite integrals.

d. Improper Integrals

i. Define an improper integral and determine its value, if it exists, using limits of definite integrals.

H. Series

a. Sequences, Limits of Sequences

i. Define a sequence using proper symbolic representation
ii. Explore a variety of types of sequences. Include alternating, geometric, and arithmetic sequences.
iii. Use limit properties when determining convergence or divergence of a sequence.
iv. Explore partial sums using a graphing calculator.

b. Series, Types of Series

i. Define infinite series. State that an infinite series of numbers converges to a number if and only if the limit of its sequence of its partial sums exists and equals that number.
ii. Determine the value of a telescoping series.
iii. Review geometric series and determine whether they converge or diverge.
iv. Find the sum of a convergent geometric series.
v. Recognize the harmonic series and the alternating harmonic series.

c. Tests for Convergence and Divergence

i. Use the nth-term test for divergence.
ii. Identify when a series is a p-series and use its rules to determine convergence or divergence.
iii. Use the integral test to determine convergence or divergence.
iv. Use the direct comparison test and limit comparison test to determine convergence or divergence.
v. Use the alternating series test to determine convergence or divergence.
vi. Use the ratio test to determine convergence or divergence.

d. Power Series

i. Define a power series and create power series for $\sin x$, $\cos x$, and e^x.
ii. Determine the interval of convergence of a power series.

e. Taylor and MacLaurin Series

i. Construct Taylor polynomials for a variety of functions centered at any point $x=a$.
ii. Define a Maclaurin polynomial and use it to construct a Taylor polynomial centered at $x=0$.
iii. Use the Taylor polynomial centered at $x=a$ to approximate a function's value near $x=a$.
iv. Use the Lagrange error bound to bound the error of a Taylor polynomial approximation to a function on a given interval.

I. Differential Equations

a. Modeling With Differential Equations

i. Define differential equations.
ii. Verify solutions to non-separable differential equations.

b. Direction Fields

i. Define slope fields.
ii. Create slope fields and sketch particular solutions through slope fields to approximate function values using given differential equations and initial conditions.

c. Euler's Method

i. Define Euler's Method.
ii. Apply the concepts of Euler's Method to approximate function values using given differential equations and initial conditions.

d. Logistic Differential Equations

i. Define Logistic Differential Equations.
ii. Determine the carrying capacity of a variable quantity by applying concepts of logistic differential equations.

J. Curves in Parametric, Vector and Polar Form

a. Plane Curves and Parametric Equations

i. Define plane curves and parametric equations.
ii. Sketch plane curves by eliminating the parameter.

b. Vector Valued Functions

i. Review vectors and vector operations.
ii. Define vector-valued functions.
iii. Define the rules for differentiating and anti-differentiating vector-valued functions.

iv. Apply the concepts of vector-valued functions to solve problems involving position, velocity and acceleration.
v. Determine the total distance traveled by an object whose motion is defined by a vector-valued function.

c. Polar Coordinates and Polar Graphs

i. Review polar coordinates and polar function graphs.
ii. Determine the area of a region defined by one or more polar functions.
iii. Determine the length of a finite arc defined by a polar function on a closed interval.

III. Instruction

A. Course Schedule

a. 5 days a week
b. 45 minute classes

B. Pacing

a. Marking Period 1
   i. Limits and Derivatives Assessment
   ii. Applications of Differentiation Assessment
   iii. Indefinite Integrals Assessment
   iv. Riemann Sums/ Integration Assessment
b. Marking Period 2
   i. Applications of Integration Assessment
   ii. Inverse Functions Assessment
   iii. Techniques of Integration Assessment
c. Marking Period 3
   i. Sequences and Series Assessment
   ii. Taylor Series Assessment
   iii. Parametric, Vector and Polar Function Assessment
d. Marking Period 4
   i. Differential Equations AP Quiz
   ii. Area and Volume AP Quiz

iii. Accumulated Change AP Quiz
iv. Riemann Sums AP Quiz
v. Vector-Valued Functions AP Quiz
vi. Taylor Series AP Quiz

C. Methods

a. Lecture
b. Cooperative learning
c. Mathematics software and internet resources such as applets and math websites will be incorporated into the course using computers and the Activeboard.
d. Exploration and discovery lessons with and without technology
e. Homework
f. Pre-class assignments
g. Graphing calculator activities
h. Formative assessments and differentiation
i. Summative assessments
j. Data analysis of student results

D. Resources

a. Stewart, J. *Single Variable Calculus With Vector Functions AP Edition*. Brooks/Cole, Cengage Learning: Belmont, California, 2012.
b. Released AP Free Response Questions and Multiple Choice Exams
c. TI-89 Titanium Graphing Calculator provided to each student
d. Ancillary materials from the text
e. Teacher made presentations, handouts, activities, practice, quizzes
f. Departmental chapter tests, midterm and final exam
g. Reference materials available in the mathematics office and the school library
h. Websites such as AP Central, Wolfram Alpha, Desmos, etc.
i. Google Classroom and Skyward
j. Apperson scan sheets and software for test analysis

IV. Assessment

A. Procedures for Evaluation

a. Summative assessments
    i. A departmental common assessment will be administered at the end of each unit.
    ii. A departmental common assessment will be administered at the end of the course.
b. Formative assessments will be administered in a variety of formats.
c. Accommodations aligned with those permitted for the PSSA/Keystone Exams and included in IEP's will be provided for Special Education students who are enrolled in this course.

B. Expected Levels of Achievement

Students are expected to achieve at least a minimum level of proficiency. Proficiency and related grades are defined as follows:

A……..90 – 100%
B……..80 - 89%
C……..70 - 79%
D……..60 - 69%