Line Segments, Distance and Midpoint

Partitioning a Segment

CC Standard

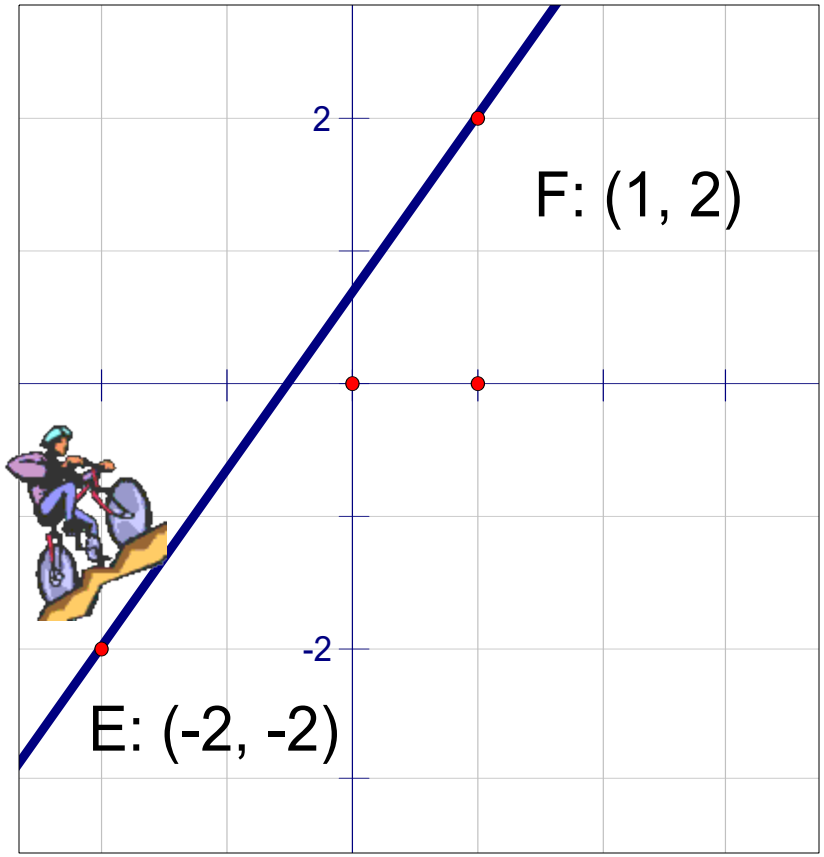
G-GPE.6 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

At the end of this lesson, you should be able to answer the following question:

How do you find the point on a directed line segment that partitions the segment in a given ratio?

Understanding Slope

- If a line rises as you move from left to right, then the slope is positive.

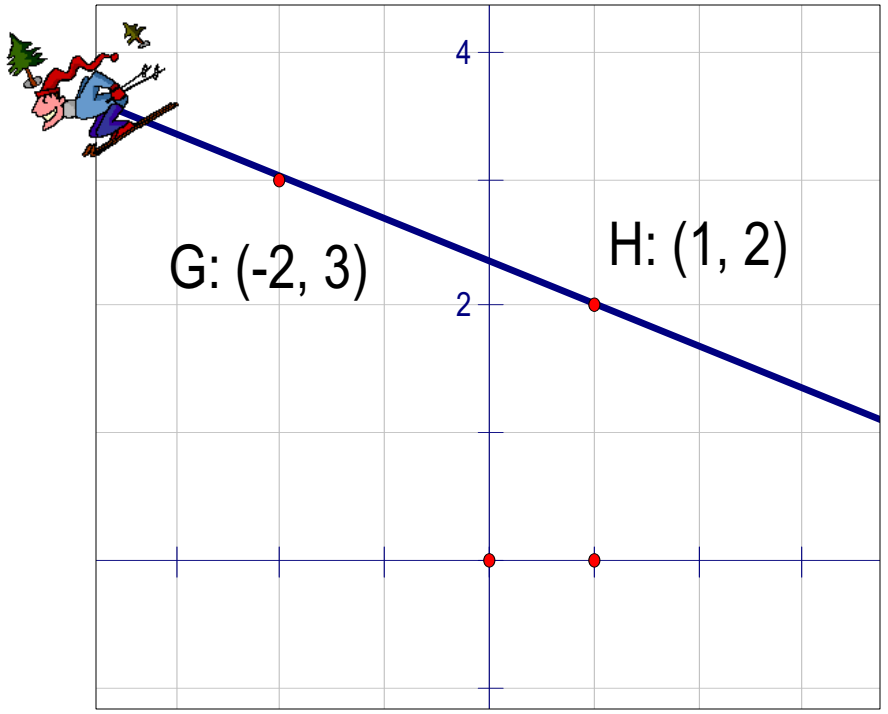


Riding a bike uphill

Understanding Slope

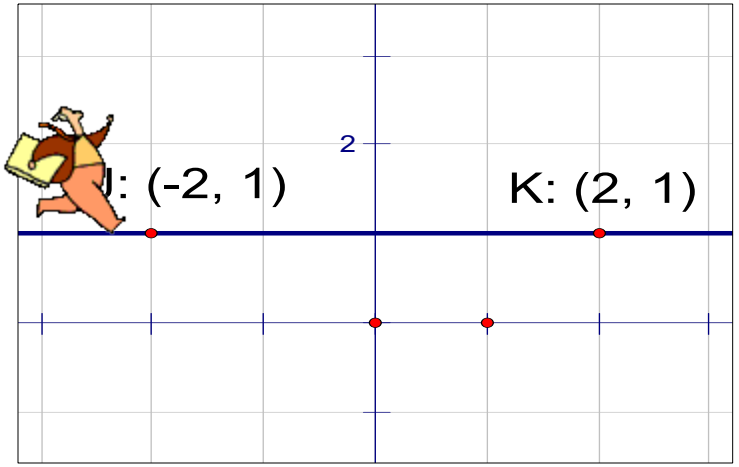
- If a line drops as you move from left to right, then the slope is <u>negative</u>.

Skiing Downhill



Understanding Slope

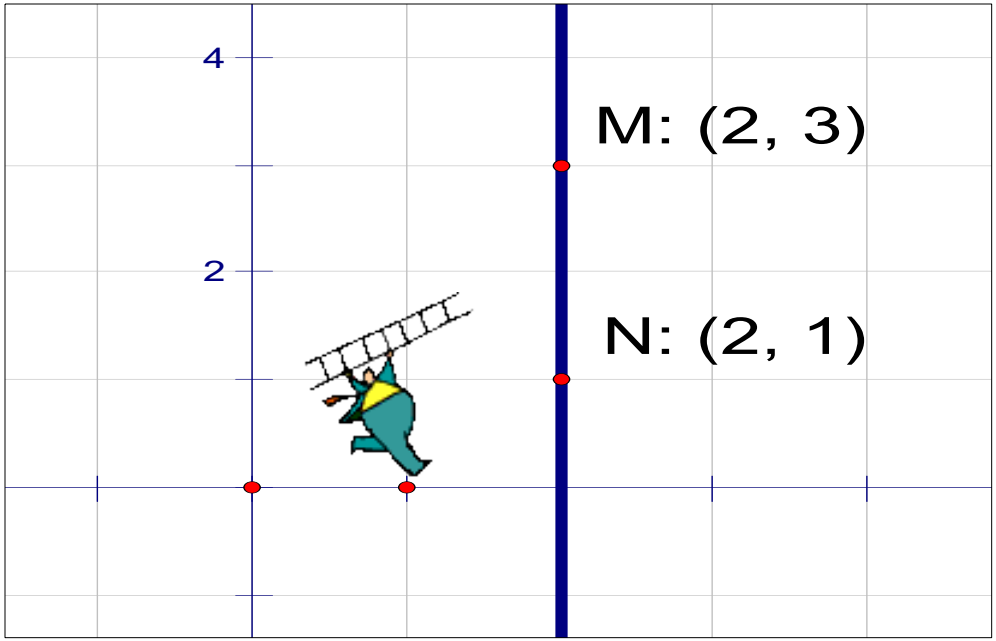
- A horizontal line has <u>zero slope</u>: $m = 0$



Running on a flat surface like a track
Or any athletic field

Understanding Slope

- A vertical line has <u>no slope</u>:
  m is undefined.

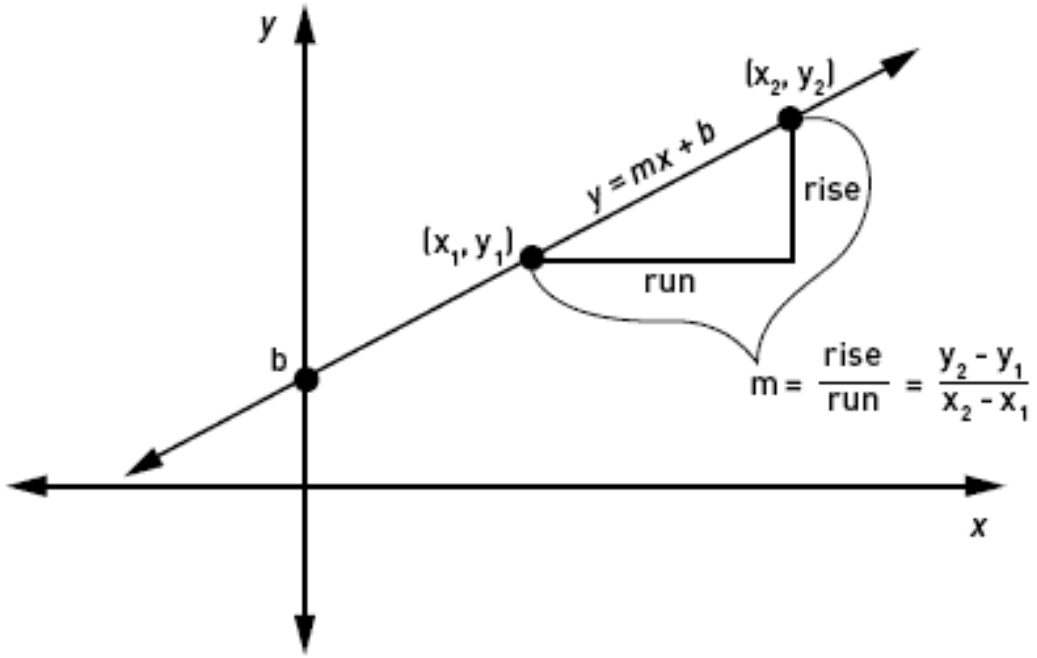


Running into a wall, you cant get past it

Slope Formula

The slope of a line through the points (x_1, y_1) and (x_2, y_2) is as follows:

$$m = \frac{y_2 - y}{x_2 - x}$$



Ex.

Find the slope of the line that passes through (–2, –3) and (4, 6).

Let (x_1, y_1) be $(-2, -3)$ and (x_2, y_2) be $(4, 6)$.

$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - (-3)}{4 - (-2)}$$

Substitute 6 for y_2, –3 for y_1, 4 for x_2, and –2 for x_1.

$$= \frac{9}{6} = \frac{3}{2}$$

The slope of the line that passes through (–2, –3) and (4, 6) is $\frac{3}{2}$.

*** Always reduce your fractions****

What is a ratio?

- A ***ratio*** is a comparison of two quantities
- The *ratio of a to b* can be expressed as:

a : b

or

$^{a}/_{b}$ or $\frac{a}{b}$

Connor has a wallet with:

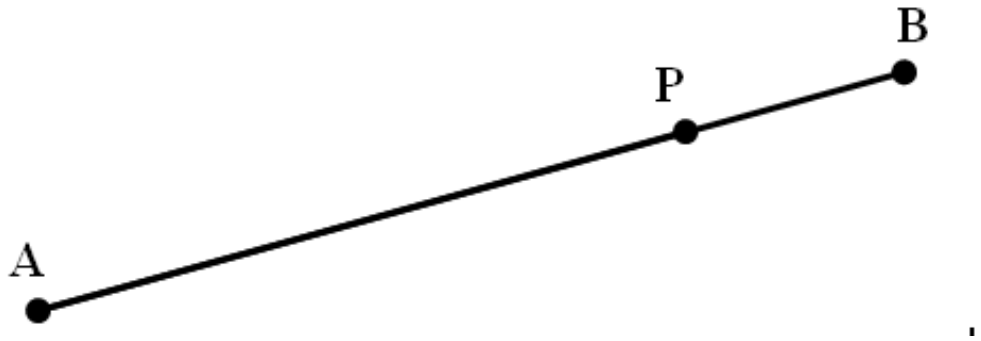
1-$20 bill
2- $10 bills
1- $5 bill
8-$1 bills

1) What is the ratio of $1 bills to $10 bills?

2) What is the ratio of $10 bills to the total number of bills in the wallet?

Point P divides $\overline{AB}$ in the ratio 3 to 1.

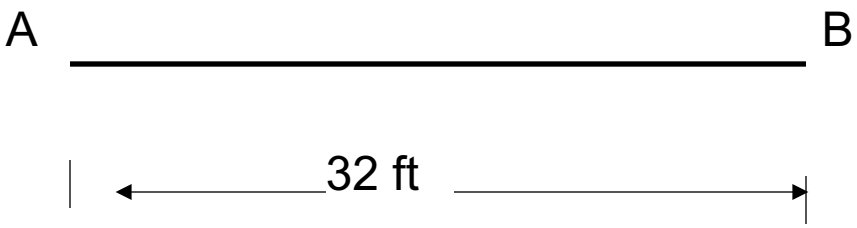
1. What does this mean?



2. Do you expect point P to be closer to A or closer to B? Why?

3. How does the slope of $\overline{AP}$ compare with the slope of $\overline{AB}$? Why?

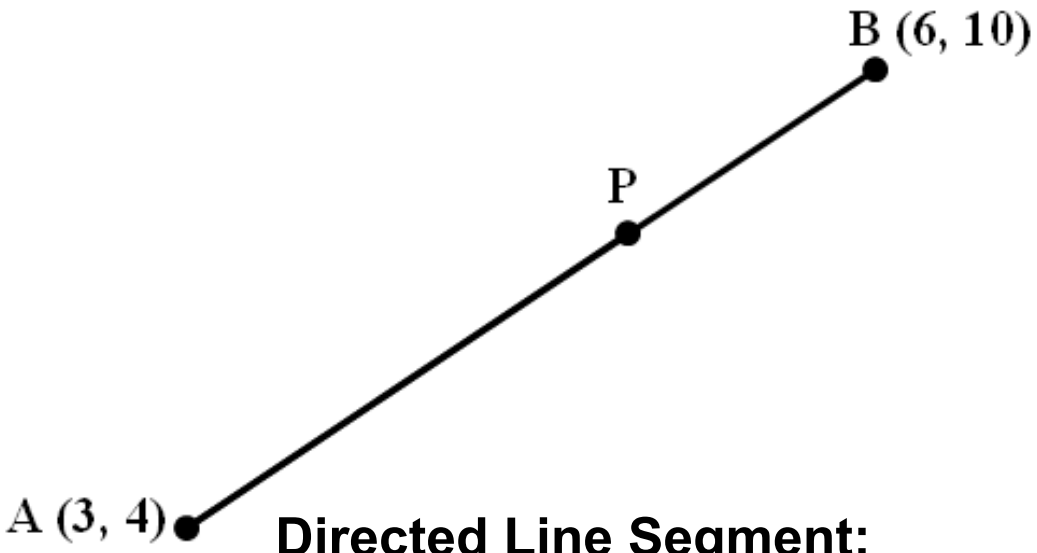
A 32 foot long piece of rope has a knot tied to divide the rope into a ratio of 3:5.



Where should the knot be tied?

Example 1:

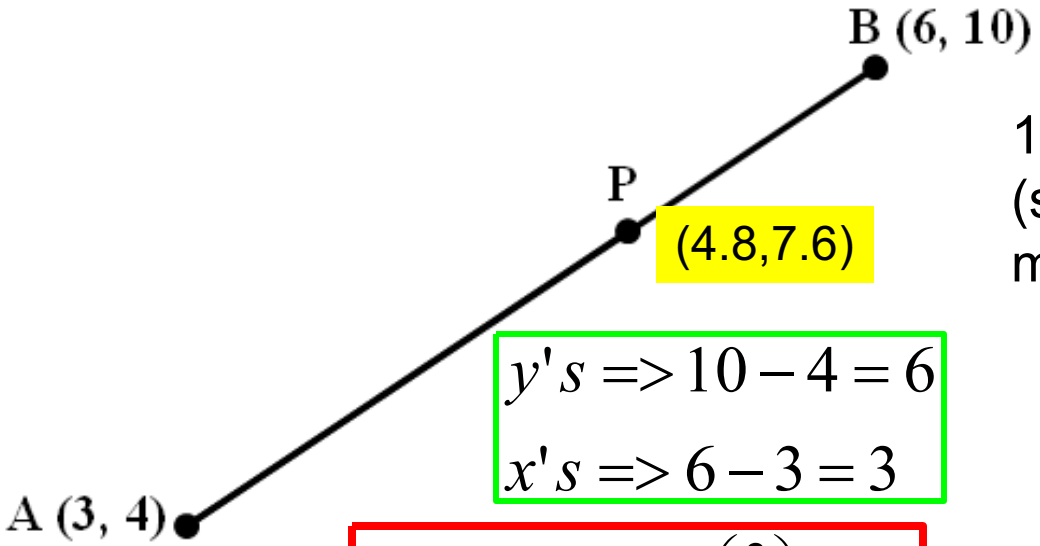
Find the coordinate of point *P* that lies along the directed line segment from *A*(3, 4) to *B*(6, 10) and partitions the segment in the ratio of 3 to 2.



Directed Line Segment:
Tells the direction in which from which point to start and end.
In this case, from Point A to Point B

What does that tell you about the distance AP and PB in relation to AB?

Find the coordinate of point *P* that lies along the directed line segment from *A*(3, 4) to *B*(6, 10) and partitions the segment in the ratio of 3 to 2.



$$y's => 10-4=6$$

$$x's => 6-3=3$$

$$y's => 10-4=6\left(\frac{3}{5}\right)=3.6$$

$$x's => 6-3=3\left(\frac{3}{5}\right)=1.8$$

1. Find the rise and run for AB (since you start at A and end at B, make Ax_1 and Bx_2)

2. Multiply the rise by the ratio from A to P, and the run by the ratio of A to P

3. Add/subtract these values to your starting point A

$$\frac{A\underset{+1.8}{(3},\ \underset{+3.6}{4)}}{P\,(4.8,\ 7.6)}$$

4. How can you use the distance formula to check that P partitions $\overline{AB}$ in the ratio of 3 to 2?

example

Find the point Q along the directed line segment from point *R*(–3, 3) to point *S*(6, –3) that divides the segment into the ratio 2 to 1

$$x's => 6 - {}^{-}3 = 9$$
$$y's => {}^{-}3 - 3 = {}^{-}6$$

$$x's => 6 - {}^{-}3 = 9\left(\frac{2}{3}\right) = 6$$
$$y's => {}^{-}3 - 3 = {}^{-}6\left(\frac{2}{3}\right) = -4$$

1. Find the rise and run for RS (since you start at R and end at S, make Rx_1 and Sx_2)

2. Multiply the rise by the ratio from R to Q, and the run by the ratio of R to Q

3. Add/subtract these values to your starting point R

$$\begin{array}{l} R\ (-3, 3) \\ \hline Q\ (3, -1) \end{array} \quad \begin{array}{l} {\scriptstyle +6 \quad -4} \end{array}$$

example

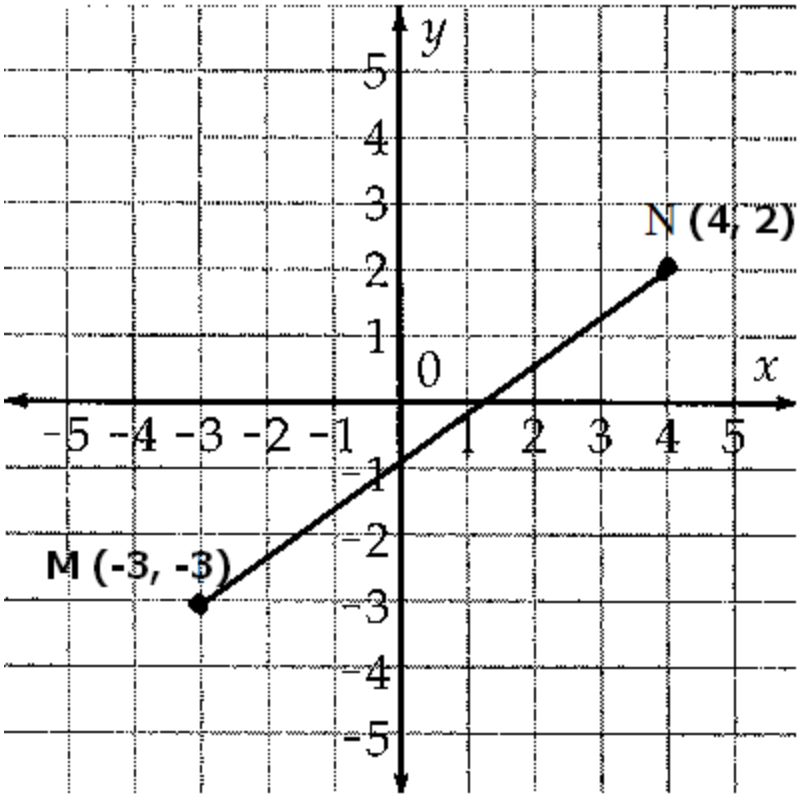
Find the point Q along the directed line segment from point $R(-2, 4)$ to point $S(18, -6)$ that divides the segment in the ratio 3 to 7.

example

Find the coordinates of the point P that lies along the directed segment from A(1, 1) to B(7, 3) and partitions the segment in the ratio of 1 to 4

Example

Find the coordinates of point P that lies along the directed line segment from M to N and partitions the segment in the ratio of 3 to 2



Essential question:

- How do you find the point on a directed line segment that partitions the segment in a given ratio?