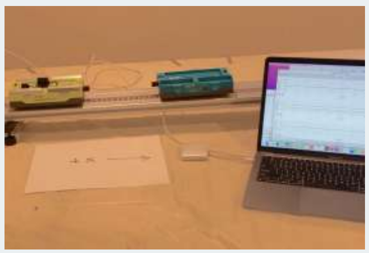
Lesson 6: Do our motion relationships help predict any of the interactions or outcomes in a collision?

Previous Lesson *We looked back at our speed versus time graphs and used the slope to name acceleration. We rearranged our equations to show that unbalanced forces cause change in motion. We analyzed graphs for vehicles stopping in rainy conditions and added possible design solutions to help drivers react sooner to our Engineering Progress Tracker. We completed an Electronic Exit Ticket.*

This Lesson

Investigation

3 days



We analyze sensor data from a collision of a cart with a barrier and another between two carts. We analyze fatality data from collisions between different-mass vehicles. We develop an equation for the outcomes of two-vehicle collisions and test it with data from a simulation. We develop and use alternate algebraic models to solve for the mass or velocity of an object before or after a collision.

Next Lesson *We will apply our ideas about momentum to an assessment about vehicles colliding with a stopped bus. We will look at new data to explore possible correlations with the trends we identified in Lesson 1. We will explore a simulation of a vehicle collision and add new questions about safety features to the Driving Question Board.*

BUILDING TOWARD NGSS

HS-ETS1-3, HS-PS2-2, HS-PS2-3, HS-PS2-1

What students will do

6.A Analyze data collected from speed and force sensors and use multiple mathematical representations to describe and make claims about the patterns that show the relationship between different variables in a collision (force applied to a vehicle, mass, and change in velocity of a vehicle). (SEP: 5.2; CCC: 1.4; DCI: PS2.A.1)

6.B Apply techniques of algebra to solve for an unknown initial condition or outcome of a collision in a two-object system using a version of Newton's second law, arranged to describe conservation of momentum. (SEP: 5.3; CCC: 4.2; DCI: PS2.A.2)

What students will figure out

- When two objects collide, the contact forces on each object are equal in magnitude and opposite in direction, acting over the same time period.
- In a collision between different-mass vehicles, the occupants of the larger-mass vehicle are at less risk of death and/or injury.
- The momentum of an object is the product of its mass and its velocity.
- During a collision, the total momentum of the colliding objects is conserved when there is no unbalanced external net force on the system.
- Momentum conservation provides a close approximation of collision outcomes when the magnitude of the external net force on the system (e.g., friction) is small compared to the average collision force.

Lesson 6 • Learning Plan Snapshot

Part	Duration		Summary	Slide	Materials
1	4 min		**NAVIGATE** Consider whether the relationships between the variables in our braking equations apply to a vehicle and a fixed barrier in a collision.	A-C	*Student Mindfulness Resource* from Lesson 1, Initial Consensus Model poster from Lesson 1, Force and Motion Relationships poster
2	13 min	☑	**ANALYZE AND INTERPRET DATA** Orient to the data collection system for the cart, track, and barrier. Interpret a velocity graph for the cart. Make a prediction for a different collision and analyze the related velocity data.	D-H	*Collision A/B Predictions*, *Motion: Collision B*, https://youtu.be/wmlCn22dTwk, dry erase marker (green), dry erase marker (red), dry erase marker (black)
3	5 min		**USE AN EQUATION MODEL TO MAKE PREDICTIONS** Use the motion relationship equations from Lesson 5 to make predictions for the magnitude of the force on the cart during the period of contact with the barrier.	I-K	calculator, Force and Motion Relationships poster, chart paper markers, 3 sticky notes or index cards, tape
4	8 min		**ANALYZE AND INTERPRET DATA** Compare the actual contact forces recorded in each collision using tables and graphs.	L-N	Force and Motion Relationships poster
5	6 min		**DESIGNATE VARIABLES** Brainstorm how to extend our model to apply to a two-vehicle collision. Identify the need for two equations and establish subscripts as a way to indicate which vehicle we are referring to.	O	Force and Motion Relationships poster
6	9 min	☑	**MAKE PREDICTIONS AND ANALYZE DATA** Watch a video of three collisions (D-F) between two different carts. Make predictions for how the magnitude of the forces on each cart would compare. Analyze the graphs of this data.	P-R	*Collision A/B Predictions*, *Motion: Collision B*, *Collisions D-F Forces*, Force and Motion Relationships poster, https://youtu.be/DXFVadiFb_c?si=1v9mpU4L_gRx8CbT
					End of day 1
7	3 min		**NAVIGATE** Question how what we figured out last time applies to vehicle safety outcomes.	S	

8	9 min	**MAKE EVIDENCE-BASED ARGUMENTS CONNECTED TO VEHICLE SAFETY** Argue for what is shown by fatality data from collisions with trucks.	T-V	
9	7 min	**ANALYZE AND INTERPRET VELOCITY GRAPHS** Analyze and interpret velocity graphs for collisions (D-F).	W-X	*Collisions D-F Velocities*
10	5 min	**USE MATHEMATICAL THINKING** Calculate and compare the exact values for Δv for each cart in each collision (D-F).	Y-Z	calculator, *Collisions D-F Velocities*
11	6 min	**DEVELOP AND USE A MATHEMATICAL MODEL** Review force symmetry and combine a system of two equations. Consider whether the new equation predicts the outcomes in other conditions.	AA-CC	dry erase marker (blue), dry erase marker (green)
12	15 min	**TEST IDEAS WITH A SIMULATION AND CHECK THE RESULTS** Use a simulation to collect data on an inelastic and an elastic collision and test the new equation model using the simulation results. Compare findings across groups.	DD-GG	calculator, whiteboard, dry erase markers, computer with access to https://openscied-static.s3.amazonaws.com/HTML+Files/Inelastic+and+Elastic+Collisions.html
				End of day 2
13	13 min	**CONSIDER CONSERVED QUANTITIES AND USE GEOMETRIC REPRESENTATIONS** Recall other examples of conserved quantities. Develop a geometric model using rectangles to represent the conserved quantity in different collision scenarios. Evaluate the affordances and limitations of the geometric model.	HH-KK	dry erase marker (black), dry erase marker (red), Force and Motion Relationships poster
14	11 min	**INTRODUCE MOMENTUM AND DEVELOP AN ALTERNATE FORM OF OUR EQUATION** Annotate the equation as showing a relationship for the change in momentum. Problematize what momentum is. Rewrite the equation to identify momentum for times before and after the collision.	LL	calculator, whiteboard, dry erase markers, paper towel or whiteboard eraser, Momentum Relationships poster on whiteboard or chart paper, M-E-F triangle poster, dry erase markers or chart paper markers
15	4 min	**ADD TERMS TO PERSONAL GLOSSARIES**	MM	

		Add *velocity*, *elastic collision*, *inelastic collision*, and *momentum* to students' Personal Glossaries.		
16	9 min	**PREDICTING OUTCOMES ACROSS A BROADER SET OF COLLISION PHENOMENA** Independently apply and practice the use of the momentum equation(s) to solve for an unknown condition or outcome in one or more related phenomena.	NN	*Different Momentum Cases*, calculator, Momentum Relationships poster, chart paper markers
17	5 min	**ASSIGN THE HOME LEARNING ON SELF-ASSESSMENT AND REFLECTION** Orient to a key to use for self-assessing understanding and reflecting on confidence in different approaches to using conservation of momentum to solve for an unknown condition or outcome.	OO-PP	*Different Momentum Cases*, *Momentum Self-Assessment Key*, calculator
18	3 min	**NAVIGATE** Identify that differences in velocity change for different-mass vehicles in a collision are correlated to differences in passenger safety. Ask questions related to how and why this affects passengers inside the vehicle.	QQ	

End of day 3

Lesson 6 • Materials List

	per student	per group	per class
Lesson materials	- science notebook *Student Mindfulness Resource* from Lesson 1 *Collision A/B Predictions* - calculator *Collisions D-F Velocities* *Different Momentum Cases* *Momentum Self-Assessment Key*	*Motion: Collision B* *Collisions D-F Forces* - whiteboard - dry erase markers - computer with access to https://openscied-static.s3.amazonaws.com/HTML+Files/Inelastic+and+Elastic+Collisions.html - paper towel or whiteboard eraser	- Initial Consensus Model poster from Lesson 1 - Force and Motion Relationships poster - https://youtu.be/wmlCn22dTwk - dry erase marker (green) - dry erase marker (red) - dry erase marker (black) - chart paper markers 3 sticky notes or index cards - tape - https://youtu.be/DXFVadiFb_c?si=1v9mpU4L_gRx8CbT - dry erase marker (blue) - Momentum Relationships poster on whiteboard or chart paper - M-E-F triangle poster - dry erase markers or chart paper markers - Momentum Relationships poster

Materials preparation (15 minutes)

Review teacher guide, slides, and teacher references or keys (if applicable).

Make copies of handouts and ensure sufficient copies of student references, readings, and procedures are available.

Make copies of the handouts for this lesson:

- *Collision A/B Predictions* (1 per student)
- *Different Momentum Cases* (1 per student)
- *Momentum Self-Assessment Key* (1 per student)
- *Collisions A/B Forces* (1 per student)
- *Collisions D-F Velocities* (1 per student)
- *Motion: Collision B* (1 per pair of students; these are to be collected for reuse)
- *Collisions D-F Forces* (1 per pair of students; these are to be collected for reuse)

Three-hole-punch all handouts so they can be added to students' science notebooks.

Prepare chart paper for the posters in this lesson, described below:

- Force and Motion Relationships
- Momentum Relationships

The Force and Motion Relationships poster summarizes the equations developed in Lesson 5. You will add to this poster in Lesson 9, so leave the bottom of the poster empty. You will cover up the Δspeed part of each of the equations with a sticky note or an index card that has "Δvelcoity" written on it in the middle of this lesson when you revise it as a class, and you will then remove these additions and reset the poster for the next class. Display it in the classroom to the left of the M-E-F triangle poster. Leave this poster up for the rest of the unit.

Force and Motion Relationships

$$\Delta t = \frac{m * \Delta \text{ speed}}{F}$$

$$F = \frac{m * \Delta \text{ speed}}{\Delta t} = m * \left(\frac{\Delta \text{ speed}}{\Delta t}\right) = m * \text{acceleration}$$

$$\text{acceleration} = a = \frac{F}{m}$$

To save paper, the Momentum Relationships poster is only to be prepared for your last-period class. This will serve as a permanent artifact for all of your classes to refer to in future lessons. For prior classes, use a whiteboard to record something similar.

Have the Initial Consensus Model poster from Lesson 1 displayed in the classroom.

Test the videos viewed in this lesson:

- https://youtu.be/wmlCn22dTwk
- https://youtu.be/DXFVadiFb_c?si=1v9mpU4L_gRx8CbT

Test the simulation students use in this lesson:

- https://openscied-static.s3.amazonaws.com/HTML+Files/Inelastic+and+Elastic+Collisions.html

Optional Collision Introduction describes a more-scaffolded sequence that can be used to introduce the sensor cart. If you choose to use this sequence, paste slides (C1-C4) between slide C and slide D above. Be sure to review all safety protocols on *Optional Collision Introduction*. This sequence will add 20-25 minutes to the lesson.

Materials needed for the more-scaffolded sequence are:

- sanitized safety glasses with side shields
- smart cart
- smart cart track
- wall or large brick
- sticky tack
- computer with data collection software (see https://youtu.be/1LLZEYo1Yzk or https://youtu.be/iJB1o6WOxqY

Lesson 6 • Where We Are Going and NOT Going

Where We Are Going

This lesson is designed to coherently build ideas related to the following disciplinary core idea (DCI):

- **PS2.A.1: Forces and Motion.** Newton's second law accurately predicts changes in the motion of macroscopic objects.
- **PS2.A.2: Forces and Motion.** Momentum is defined for a particular frame of reference; it is the mass times the velocity of the object.

Students interpret the structure of an equation, identify ways to rewrite it, and rearrange it to highlight a quantity of interest. Talk with your students' math teachers to identify students' prior experiences with these related ideas from the Common Core State Standards (CCSS) for mathematics, further detailed at the end of this *Teacher Guide*.

Students analyze bivariate data in multiple graphs to describe collision patterns over time. They have previously worked with graphs to identify trends and quantities over time in *OpenSciEd Unit P.1: How can we design more reliable systems to meet our communities' energy needs? (Electricity Unit)* and in nonlinear exponential models derived from data in *OpenSciEd Unit P.2: How forces in Earth's interior determine what will happen to its surface? (Earth's Interior Unit)*. The highly scaffolded analysis of velocity versus time graphs and force versus time graphs built into day 1 of this lesson lays the groundwork for needing less scaffolding to interpret these types of graphs in future lessons (e.g., Lessons 9 and 10).

If students have taken OpenSciEd Chemistry, this lesson provides an opportunity to make connections across courses by discussing how the incremental derivation of a suspected algebraic relationship is reminiscent of what they did in *OpenSciEd Unit C.1: How can we slow the flow of energy on Earth to protect vulnerable coastal communities? (Polar Ice Unit)* to derive thermal energy transfer and conservation of energy relationships. Students also used a similar approach for determining the efficiency of energy transfer systems in the *Electricity Unit* and relative amounts of matter due to radioactive decay in the *Earth's Interior Unit*.

The two core CCSS-related ideas targeted in this lesson are CCSS.MATH.CONTENT.HS.N-VM.1 and CCSS.MATH.CONTENT.HS.A-SSE.1b, described in further detail at the end of this *Teacher Guide*. Students worked with vectors in the prior unit. Some students may also draw on their prior work in high school CCSS with ideas about symmetry, scaling, and shifting of functions (building new functions from existing functions), when interpreting the graphs in *Collision A/B Predictions* and *Motion: Collision B*.

The idea that models have limitations but can still provide useful approximations is a fundamental part of the high school-level science and engineering practice (SEP) elements 2.2 and 2.4. In this lesson, these elements are applied at a middle school level, so we have not claimed them here. Students build off prior work in this unit and across other units related to idealized conditions and limited data to develop explanatory models of phenomena.

Slide U introduces statistics on vehicle crash fatalities, which may evoke trauma for students, teachers, and families. Please see the unit front matter for additional guidance in Lesson 1 about how to support social and emotional needs as you move through this unit. **Refrain from asking students to share their personal experiences unless they volunteer to do so.**

Students encounter the word *velocity* and the idea that it is a vector quantity when viewing a video about the cart/barrier collision system. In *Earth's Interior Unit*, they worked with the idea that rates of motion and forces can be represented as force vectors. This lesson is purposefully designed to motivate students to keep track of the mass times change in velocity and to notice that it is a conserved quantity in collisions before learning the scientific term for it. Only after seeing multiple lines of evidence that it is a conserved quantity is it named as *momentum* on day 3. It is recommended to delay adding any of the following words to students' Personal Glossaries until day 3: *velocity*, *elastic collisions, inelastic collisions, momentum.* **Do not** ask students to define or keep track of any words until after your class has developed a shared understanding of their meaning.

Where We Are NOT Going

This unit introduces thinking about velocity as a vector, but only in one dimension. Next, in *OpenSciEd Unit P.4: Meteors, Orbits, and Gravity (Meteors Unit)*, students will think about two-dimensional velocity vectors to understand and explain orbits.

If students completed *OpenSciEd Unit 8.1: Why do things sometimes get damaged when they hit each other? (Collisions Unit)* in a prior grade, they collected firsthand evidence that the forces in any collision are equal in strength and opposite in direction, even when the masses are not equal (Newton's third law). This is one reason why we are not collecting such data firsthand in this unit; the other is that Newton's third law is a learning target for middle school. **Slide Q** and **Slide R** and the related force graphs in the *Collisions D-F Forces* handout are designed to quickly re-establish that force symmetry relationship.

Some students may notice that the graphs for velocity shift from a linear region to a nonlinear region and back to a linear region on the *Collision A/B Predictions* handout and the *Motion: Collision B* handout. Though the shift from linear to nonlinear changes in velocity is evidence of changing force magnitudes in the system over the duration of the collision, this relationship is beyond what students need to notice or explain at this point in the unit.

Historically, scientists debated whether momentum or kinetic energy was more useful and/or sufficient for explaining many phenomena. This is referred to by historians as the *vis viva* controversy/debate. Eventually, they decided they needed both. No attempt is made to apply kinetic energy-related equations in this lesson.

LEARNING PLAN for LESSON 6

1 · NAVIGATE

4 min

MATERIALS: science notebook, *Student Mindfulness Resource* from Lesson 1, Initial Consensus Model poster from Lesson 1, Force and Motion Relationships poster

Navigate. Display **slide A**. Read the first statement of the Student Content Advisory aloud:

- *In this lesson, we will examine the physics of vehicle collisions in detail, as well as data on fatalities related to collisions.*

Give the class a moment to read the rest of the slide. Suggest that students find their *Student Mindfulness Resource* from Lesson 1 and move it to somewhere in their science notebook that is easy to reference. If there is time, take a moment to engage in one of the mindfulness activities on that handout.

Display **slide B**. Read the slide's text and question aloud:

- *We have looked at how to prevent a collision and at factors that affect the ability to do so. But sometimes a collision still occurs.*
    - *What factors did we identify in our initial consensus model that we said might affect the severity of the outcome of a collision?*

Cue students to refer to the Initial Consensus Model poster from Lesson 1 (displayed in the room) to individually review the factors in the model. Have a few students share what they see, then suggest that we start looking into the various factors.

Consider vehicle collisions with a fixed object. Display **slide C**. Read the slide's crash fatality statistic and follow-up question:

- *~20% of crash fatalities are from a vehicle running into a fixed object like a tree, utility pole, or barrier.*
- *Do we think the contact force (F) in these collisions has a similar relationship to the other 3 variables in our braking equations?*

Show students the newly added Force and Motion Relationships poster that shows the equations established in Lessons 4 and 5. Make note of the newly added label of "Newton's Second Law" and explain that this is a special name scientists have given this equation.

Give students half a minute to consider this question on their own. Then discuss the related questions below.

Suggested prompt	Sample student response
Will the vehicle come to a stop or change speed?	Accept all responses.

How will the magnitude of the contact force on the vehicle during the collision compare to typical braking forces? Will it be comparable, or will it be higher or lower than a typical braking force?	Higher
How will the duration that the force is applied, Δt, compare to the type of values we measured in our braking lab? Will it be between 1 and 4 seconds, or longer, or shorter?	Shorter

Suggest analyzing some data to see whether we can find evidence to support or refute our predictions.

ALTERNATE ACTIVITY	Some students may need more support in making the jump to thinking more abstractly about the motion and forces when using the smart cart. *Optional Collision Introduction* provides an additional activity that scaffolds students through semsaking about the data when the smart cart collides with a wall and stops as opposed to bouncing, which is more similar to the stopping they investigated in Lesson 4. The slides for this extra transitional activity are provided at the end of the slide deck on **slides C1-C4** and are used in place of **slide D**. This activity is meant to provide an alternative framing that is more concrete, more hands on, and clarifies the individual steps of the work in a coherent way. Be sure to review all safety protocols on *Optional Collision Introduction*. This additional, more scaffolded sequence will add about 20-25 minutes to the lesson.

2 · ANALYZE AND INTERPRET DATA

13 min

MATERIALS: science notebook, *Collision A/B Predictions*, *Motion: Collision B*, https://youtu.be/wmlCn22dTwk, dry erase marker (green), dry erase marker (red), dry erase marker (black)

Orient to the data collection system. Display **slide D**. Remind the class that we saw a cart like the one shown on the slide in Lesson 5 exit ticket.

Connect to considering scale. Say, *You predicted that the time period of the collision will be shorter. In our Earth's Interior Unit, we discovered that if we look at what's happening in a system at a much smaller or much larger scale than we can directly observe, it helps us see different patterns, which then helps us explain cause-and-effect relationships for what we observed.* ✱

✱ SUPPORTING STUDENTS IN DEVELOPING AND USING PATTERNS

Explicitly referencing work in the prior unit related to switching scales (down to the microscopic and up to all of Earth) can help students recognize the application of this

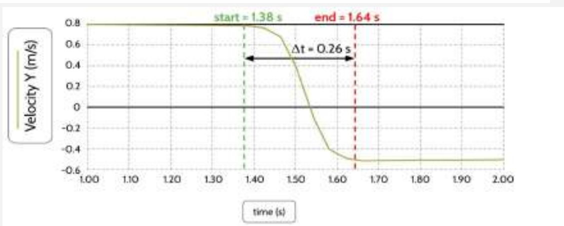
These carts collect data on things like force and speed many times per second. Let's get ready to analyze some data from before, during, and after a collision with a barrier. First, let's watch a short video about the system we'll use to collect our data. Show https://youtu.be/wmlCn22dTwk.

ADDITIONAL GUIDANCE	The video orient students to how the coordinate system is defined in the system and establish which direction is positive and negative for force vectors and velocity recorded by the cart. Students have previously developed the idea that vectors are often used in physics to represent any quantity that also has a direction; they have seen vectors used to represent the rate of motion of plates in particular directions, and they have used vectors to represent the magnitude and direction of contact forces. Students have not, however, previously encountered the word *velocity* as a way to refer to rate of motion in a particular direction. The video introduces this word in the context of what the cart's sensor will report. Help develop the idea that velocities can be negative in the context of the next data analyzed from the cart.

ADDITIONAL GUIDANCE	It is not important for students to know the specific sampling rate of the cart's different sensors. They may end up figuring out this difference when they see the related data tables in the *Motion: Collision B* handout. Those show that data on force was sampled 50 times per second (50 Hz) whereas speed (or velocity) was sampled at half this rate (25 times per second). The speed is derived from a position sensor that measures how much a wheel on the cart has turned.

Interpret a velocity graph of the cart. Display **slide E**. Distribute the *Collision A/B Predictions* handout to each student. Discuss the question on the slide:

- *What part of the graph is before the collision, and what part is after?*



As students agree upon the ideas below, annotate the graph projected directly onto a whiteboard, using different colored markers for these time points:

- The start of the collision is at about 1.38 seconds. Indicate this with a green marker.
- The end of the collision is at about 1.64 seconds). Indicate this with a red marker.
- The duration of the collision (Δt) is the difference between these two times (0.26 seconds). Indicate this with a black marker.

Have students record these annotations on their handout. They will need the value for Δt in a later step.

Use the prompts in the table below to support students in making sense of the flat versus curved parts of the graph and making sense of before and after the point in time when velocity goes to zero (about 1.53 seconds).

crosscutting concept (CCC) across various domains:

- Different patterns may be observed at each of the scales at which a system is studied and can provide evidence for causality in explanations of phenomena.

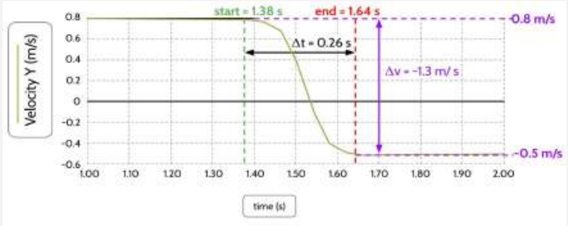
This will help them reuse this idea more fluently to make sense of subsequent phenomena they will encounter in other science classes.

✱ SUPPORTING STUDENTS IN ENGAGING IN USING MATHEMATICS AND COMPUTATIONAL THINKING

This sort of proportional reasoning, in which students consider what the outcome would be if one variable were doubled, reliably lends itself to making accurate mathematical predictions before seeing any related equations. Students previously engaged in this sort of thinking in Lesson 4. It continues to serve as a useful tool for making accurate predictions, particularly when the underlying relationship between the two variables is directly proportional.

Suggested prompt	Sample student response
What does the flat part of the graph tell us is happening to the velocity of the cart?	The speed is remaining (relatively) constant. The speed is barely decreasing.
In what part of the graph do we see a change in velocity due to contact force with the barrier?	Where it drops downward.
If we trace the velocity line as it drops, what would we say is happening to its speed here?	It is getting slower.
Notice that there is a point where it goes to zero for an instant (1.53 seconds) before going into the negative. What does it mean that the velocity becomes negative?	It is now moving in the opposite direction. The cart is moving to the left.
How would we describe what is happening to the cart's speed for that brief moment as the velocity becomes more and more negative, up to the point where it flattens out again?	It was getting faster backward. And it is speeding up in the direction opposite of its travel before the collision.

Determine the change in velocity. Display **slide F**. Ask students how much of a change in velocity there is from before the collision to after it.



- First establish agreement that this can be found by determining the difference between the *y*-values in the two flat regions of the graph.
- Use a purple marker to extend the flat regions with dotted lines and designate corresponding *y*-values (+0.8 m/s and -0.5 m/s).
- Use the purple marker to add a downward arrow on the graph from the top dotted to the bottom flat line, and label it "(Δv)", emphasizing that we are using delta again to refer to change and "v" to designate this variable as velocity rather than speed.
- Point out that the change from 0.8 to -0.5 crosses over 0, and that when we picture this on a number line, we should recall that we need to keep track of how far it is to zero **and** how far it is to the second value beyond it. Remind students that another way to think of it is how far apart these positive and negative values are. Give them about 10 seconds to consider this on their own, then have them share out. Establish consensus that the change in velocity is -1.3 m/s, and add this value with the purple marker.

- Give students half a minute to add corresponding annotations for Δv on their handout.

Emphasize that our annotation on the graph helps us see why velocity might be useful as a vector quantity, because it enables us to keep track of the total change in motion when an object's travel shifts from one direction to the opposite direction, whereas simply calculating the change in speed would not show this.

ADDITIONAL GUIDANCE

If students do not agree on Δv being negative, review that if the value is decreasing, we want to remind ourselves to represent that change using a negative symbol. If needed, a more familiar example is representing decreases in temperature with negative numbers.

This is not a productive time to introduce the convention for determining change over time to be the final value minus the starting value. That will happen on day 3, when students have calculators handy and motivation for solving for the starting or final velocity of an object in a system. For now, the graphical reasoning they are doing related to changes being either positive or negative should suffice.

Record predictions. Display **slide G**. Give students a minute to consider what we would see if we repeated the data collection, but for a cart moving at half as fast as the previous cart before hitting the barrier. Have them sketch their predicted shape of the line on the same graph as the prior collision and label the value for Δv that they predict for this new collision, using the same convention you previously demonstrated on the board.

ASSESSMENT OPPORTUNITY

What to look for/listen for in the moment:

- Students draw a predicted graph of velocity versus time for a collision that has two flat regions, one in the left part of the graph (velocity close to 0.4 m/s) and another in the right part (velocity close to -0.25 m/s). A region on their graph between these that is close to 0.25 second in duration. (SEP: 5.2)
- Students identify Δv on their graph with a downward arrow extending from the *y*-value of the left flat region to the *y*-value of the right flat region and a predicted value close to -0.65 m/s. (CCC: 1.4)

What to do: Collect *Collision A/B Predictions* at the end of the period. If students' predictions do not include the look-fors above, have them determine the Δv calculations for the six velocity graphs on *Collisions D-F Velocities* on day 2, and collect that handout to evaluate their progress in the lesson-level performance expectation (LLPE) by the end of class.

Building toward: 6.A.1 Analyze data collected from speed and force sensors and use multiple mathematical representations to describe and make claims about the patterns that show the relationship between different variables in a collision (force applied to a vehicle, mass, and change in velocity of a vehicle). (SEP: 5.2; CCC: 1.4; DCI: PS2.A.1)

Compare predictions to data. Display **slide H**. Instruct students to find a partner to work with. Distribute a copy of *Motion: Collision B* to each pair of students. After a minute, ask them how their final velocity and change in velocity compare to their predictions.

Whether students say yes or no does not matter. In either case, ask them how the pattern they see in the graph is related to the scale of the change in the system--decreasing the initial velocity by half. Listen for these ideas:

- The final velocity is approximately half the value of the final velocity in the prior collision.
- The change in velocity (Δv) is approximately half the prior change in velocity.

The agreed-upon Δv value should be approximately -0.62 m/s. ✱ Have students add this value to their predicted graphs, as they will need to reference it in the next couple of slides.

3 · USE AN EQUATION MODEL TO MAKE PREDICTIONS

5 min

MATERIALS: calculator, Force and Motion Relationships poster, chart paper markers, 3 sticky notes or index cards, tape

Revise our models. Say, *Many of us made some pretty reasonable predictions about velocity changes, using mathematical thinking. This seems like a good opportunity to extend our mathematical thinking back to the motion relationship equations we developed last time, to see whether we can apply the values for changes in velocity to make reasonable force predictions.*

Force and Motion Relationships

$$\Delta t = \frac{m * \Delta \text{velocity}}{F}$$

$$F = \frac{m * \Delta \text{velocity}}{\Delta t} = m * \left(\frac{\Delta \text{velocity}}{\Delta t}\right) = m * \text{acceleration}$$

$$\text{acceleration} = a = \frac{F}{m}$$

Display **slide I**. Point out that our Force and Motion Relationships poster had Δspeed in the equation, rather than Δvelocity, so we want to note our shift in terminology. Cover up the Δspeed part of each of the equations with a sticky note or an index card that has “Δvelocity” written on it.

Use a model to make predictions. Display **slide J**. Discuss the questions:

- *What are the known values for 3 of our variables in each collision?*
- *Which equation should we use to determine the value of the 4th variable?*

Collision A	
m (kg)	0.6
Δt (s)	0.26
Δv (m/s)	-1.3
F (N)	

Collision B	
m (kg)	0.6
Δt (s)	0.26
Δv (m/s)	-0.62
F (N)	

Listen for students to say that mass, change in time, and change in velocity are known and that the equation starting with “F =” is the most relevant (easiest) to use to calculate the unknown value for the variable of force. Have them use the values that are reported out to complete the data tables on *Collision A/B Predictions*. Fill out the first three variables for each table together.

Calculate the predicted force. Display **slide K**. Assign half the class to start with collision A and the other half to start with collision B, before solving for the other collision. Give students 2 minutes to record the related equation and use the space to the right of each table to plug in the known value and solve for force. Then have them share out answers to establish the following:

- Collision A: F (N) = -3
- Collision B: F (N) = -1.43

4 · ANALYZE AND INTERPRET DATA

8 min

MATERIALS: Force and Motion Relationships poster

Analyze and interpret data on forces. Display **slide L**. Distribute the *Collisions A/B Forces* handout to each student. Give them 3 minutes to compare the graphs and discuss the questions on the slide with a partner. Then briefly discuss the slide's questions as a class as shown in the table below.

Suggested prompts	Sample student responses	Follow-up questions
What was surprising?	The force got much bigger than we predicted. The force value changed over time.	*The force is registering as a negative value, so are we referring to changes in its magnitude?*
What was not?	The length of contact time (when there was a measurable force) was close to what we predicted (0.24 seconds). The duration of the collision was similar in both.	

ADDITIONAL GUIDANCE

Students may seem stuck on why the force increased in magnitude over one part of the graph and then dropped back down to zero rather than remaining constant over the length of the collision. If so, it may be helpful to remind them that previously in the *Earth's Interior Unit*, we learned that all solids exhibit elastic behavior up to a point, which could help us picture the the contact as an interaction between two relatively stiff materials that are still behaving elastically (like a spring) over a very short timescale that would be hard to study without specialized data collection equipment.

Exploring elastic versus inelastic behavior of materials in a collision will be emphasized later in this lesson and in Lesson 9, so it is fine to not raise this connection to elastic behavior yet.

Compare parts of the graph to predictions. Display **slide M**. Read the question aloud:

- *How is our predicted force value related to the data points above and below it?*

Point to the lowest point on the graph and ask whether this is the only point with a higher magnitude than we predicted. Give students half a minute to circle any force values in the data table that are below the predicted values for the collision they made predictions about. Then share out answers to establish the following:

- Collision A: There are four data points below our predicted values.
- Collision B: There are three data points below our predicted values.

Follow up by asking for ideas about how we can make mathematical comparisons to predicted values if some values in the data set are below the predicted value and some are above it. Accept all ideas.

- If one of the ideas is an average (the mean), suggest that we compare it to our predicted value.
- If no one suggests an average, ask for examples of when calculating an average from a set of values helped us make a relatively accurate prediction about a future or typical value.

Compare averages to predictions. Display **slide N**. Ask how the average force values on the slide compare to our predictions.

- If students are in agreement that the values are relatively close, move on.
- If students are not in agreement, remind them that in Lesson 5, we accepted a 10% error in a prediction versus a measurement as being relatively close. Calculate the percentage that the actual values deviate from our predicted values using the formula 100 * ((predicted force - actual average force) / actual average force). The result will be less than 1%.

Connect to the Force and Motion Relationships poster. Point out that we were able to extend our equation models to predict more phenomena than motion changes related to braking. Suggest checking the data to see whether our model can also predict motion changes in a collision. Emphasize that it is part of the nature of science to see whether we can apply model ideas to broader sets of phenomena. Ask for examples of another type of collision (besides one with a stationary barrier) that we should test our model against. Listen for students to suggest a two-vehicle collision.

5 · DESIGNATE VARIABLES

6 min

MATERIALS: Force and Motion Relationships poster

Brainstorm how to extend our model to a two-cart system. Display **slide O**. Read the text at the top aloud:

- *In some vehicle collisions, the mass of the two vehicles is not the same.*

Point out that we clearly need to make some changes or extensions to our equations on the poster to keep track of the carts involved in a two-vehicle collision. Give students 2 minutes to discuss those possibilities, then discuss the question on the slide as a class. Follow up as necessary to elicit the need to keep track of two separate masses, forces on two separate objects that make contact, and velocity changes of the two objects. Here are sample prompts:

- *How many different-mass es will we need to keep track of?*
- *How many different velocity changes will occur?*

Introduce the convention that scientists often differentiate between the same variable for different objects or systems by including a subscript (e.g., a number or letter) to indicate which variable goes with which object. Write the following example on the board:

- mass of vehicle 1: m_1
- mass of vehicle 2: m_2

Ask how we could use a similar convention to keep track of the velocity change, contact force, and time that the force is applied to each cart. Students will say we can add the same subscript to all of these variables. Because the contact time is the same for both carts, they will likely argue that time does not require a subscript, but if they want to include one, do so for now.

Write two versions of the F = equation on the board, one for each cart, using subscripts next to the corresponding variables. You do not need to add these to the Force and Motion Relationships poster.

$$F_1 = \frac{m_1 \times \Delta v_1}{\Delta t}$$

Contact force acting on cart 1

$$F_2 = \frac{m_2 \times \Delta v_2}{\Delta t}$$

Contact force acting on cart 2

Point out that we have now developed two equations, each referring to the unbalanced forces on a different part of the system during the time that they are in contact with each other.

Ask how many variables we now need to keep track of in the entire two-cart system when we take all the subscripts into account. Students will say either seven or eight, depending on whether they included a subscript on Δt.

6 · MAKE PREDICTIONS AND ANALYZE DATA

9 min

MATERIALS: *Collision A/B Predictions*, *Motion: Collision B*, *Collisions D-F Forces*, Force and Motion Relationships poster, https://youtu.be/DXFVadiFb_c?si=1v9mpU4L_gRx8CbT

Orient to the two-cart collisions. Display **slide P**. Explain that we are going to look at patterns in the data on all eight of these variables for three different collisions, but that before we do, it makes sense to get familiar with what those collisions look like. Preview that collision D will involve carts with the same mass and collisions E and F will involve carts with different-mass es, one double the other. Show https://youtu.be/DXFVadiFb_c?si=1v9mpU4L_gRx8CbT.

Make predictions. Display **slide Q**. Say, *Let's make some predictions about how the magnitude of the forces on each cart will compare during the brief period of contact in each collision.*

Read through the three choices to vote on:

- *There will be equal-magnitude forces on each cart in every collision.*
- *There will be unequal-magnitude forces on each cart in every collision.*
- *There will be equal-magnitude forces on each cart in some collision(s) and unequal-magnitude forces in other(s).*

Give students a moment to consider these, and then poll the class with a show of hands for each choice.

Use students' likely differences in predictions to emphasize that our different thinking on this is interesting, and that this seems like an important area to resolve to make progress on our Driving Question Board. Suggest that we need to analyze the data from these three collisions to resolve this.

In the less likely possibility that there is no controversy in students' predictions, frame the next step as needing to collect evidence to help support our thinking so we can build evidence-based arguments and design solutions related to what we want to figure out on our Driving Question Board.

Analyze data. Display **slide R**. Distribute a copy of the *Collisions D-F Forces* handout to each pair of students. ✱
Compare patterns in the force graphs. Guide students through making sense of the graphs on the handout with questions such as those in the table below.

Suggested prompt	Sample student response
What patterns did you notice in this part of the data? What do these graphs tell us?	These show us the forces on each cart over time. The same magnitudes of contact forces on both carts occur at the same points in time.

✱ ATTENDING TO EQUITY

Universal Design for Learning: Projecting **slide R** will help some students more easily keep track of which cart (yellow-green or blue) produced which data as they analyze the graphs on *Collisions D-F Forces*.

How do the strength or magnitude of those forces compare across the 3 collisions?	The force on one cart is in the opposite direction as the force on the other cart. There is a maximum force on carts A and B about midway through that time period. The magnitude of the contact forces on the carts varies over time in every collision. The maximum magnitude of the forces is higher in collision D than E or F. The maximum magnitude of the forces looks comparable in E and F.

ASSESSMENT OPPORTUNITY

What to look for/listen for in the moment:

- Students using features of the graphs to make claims (SEP: 5.2) such as:
    - The forces on each cart are opposite in value (direction) and equal in magnitude at every point in time. (CCC: 1.4; DCI: PS2.A.1)
    - The forces reach a higher magnitude in collision D, which has more total mass in the system than the others. (CCC: 1.4; DCI: PS2.A.1)
    - The forces reach a comparable magnitude in collisions E and F, which have the same total mass in the system. (CCC: 1.4; DCI: PS2.A.1)

What to do: Take note of the kinds of reasoning that students apply to analyze the trends in the collision scenarios, and be ready to provide support to those who are not yet fluent in the use of magnitude and direction of vectors to compare the forces on objects in a collision. If students don't know how to begin, provide sentence starters such as these:

- *A pattern I notice is ... I think this pattern might mean ...*
- *A difference I notice is ... I think this difference is because ...*

Building toward: 6.A.2 Analyze data collected from speed and force sensors and use multiple mathematical representations to describe and make claims about the patterns that show the relationship between different variables in a collision (force applied to a vehicle, mass, and change in velocity of a vehicle). (SEP: 5.2; CCC: 1.4; DCI: PS2.A.1)

Collect *Collision A/B Predictions* as an assessment opportunity.
Collect both of these to reuse across classes:

- *Motion: Collision B*
- *Collisions D-F Forces*

Remove the sticky notes you added to the Force and Motion Relationships poster, unless this is your last class for this lesson, in which case leave them up so the modified poster is ready for all classes to reference on day 2.

End of day 1

7 · NAVIGATE

3 min

MATERIALS: None

Connect what we figured out last time to vehicle safety. Display **slide S**. Discuss the questions as shown in the table below. Use the second question to elicit controversy, uncertainty, or examples from students' own experiences that suggest safety outcomes would not be equal. Students may even cite the equation they have been developing as something that predicts different outcomes (for change in velocity) for the same force and time but different-mass es. If they don't connect to that equation, that is fine, as this connection will be developed more fully later in the lesson.

Suggested prompt	Sample student response
What did we figure out last time?	Equal-magnitude contact forces on both carts occur at the same points in time. The force on one cart is opposite in direction as the force on the other cart, even when the masses are different.

Does what we figured out mean that different-mass vehicles should be equally safe if they collide with each other?	No. It feels like bigger vehicles are safer. But why would there be a different outcome if the forces are the same on each vehicle?
What evidence would we need to support or refute our arguments?	We need vehicle fatality or injury data for collision between different-mass vehicles. We should get additional data from the collision carts (like change in velocity).

8 · MAKE EVIDENCE-BASED ARGUMENTS CONNECTED TO VEHICLE SAFETY

9 min

MATERIALS: None

Consider related trauma. Propose that we need to analyze some vehicle fatality data to try to figure this out, but mention that before doing so, we must remind ourselves of something to be aware of when looking at such data with others. Display **slide T**. Give students a minute to review the text on their own.

ADDITIONAL GUIDANCE	**Social Emotional Learning (SEL):** This next part of the lesson addresses statistics on vehicle crashes and fatalities, which may evoke trauma for students, teachers, and families. Please see the unit front matter for additional guidance in Lesson 1 around how to support social and emotional needs as you move through this unit. **Refrain from asking students to share their personal experiences unless they volunteer to do so.**

Argue from evidence. Display **slide U**. Give students 2 minutes to analyze and interpret the data on the slide, then discuss the question as a class. Listen for these ideas:

- These data involve at least one truck in the collision.
- More people in smaller vehicles die in such collisions (multi-vehicle accidents) **and/or** every individual person in a smaller vehicle is at higher risk of fatality in a collision with a truck than every person in a truck.

Discuss possible reasons for the difference in fatalities. Display **slide V**. Use the first part of the slide's question to emphasize that the force magnitude equality on both vehicles that we discovered in our data is something that holds across all collisions. Then discuss the entire question as a class for a couple of minutes. Accept all answers.

Motivate the need for velocity data. Say, *Maybe our carts that we analyzed forces for can provide additional data to help make sense of this. Besides forces and time, what other measurements do we know they can give us?* Students will say velocity. Say, *Let's look at the velocity graphs from the same collisions (D-F) as last time and see what they tell us.*

9 · ANALYZE AND INTERPRET VELOCITY GRAPHS

7 min

MATERIALS: *Collisions D-F Velocities*

Analyze and interpret velocity graphs. Display **slide W**. Distribute a copy of *Collisions D-F Velocities* to each student. Ask them to look at the patterns in the graphs. The top row is the same force data they saw in *Collisions D-F Forces*.

Discuss the patterns as a class. Students should start to identify that there is something interesting in the magnitude of Δv in one or more of the collisions. Display **slide X**. Emphasize that you want everyone to try to find evidence for how Δv compares between the two carts in each collision, and to articulate that evidence.

ASSESSMENT OPPORTUNITY

What to look for/listen for in the moment: Students use features of the graphs to make claims (SEP: 5.2) such as:

- When masses are equal (in collision D), velocity changes are very nearly equal in magnitude but opposite in direction. (CCC: 1.4; DCI: PS2.A.1)
- When one mass is twice as big as the other (in collisions E and F), the magnitude of the velocity change of the smaller-mass cart is approximately twice as large as that of the other cart. (CCC: 1.4; DCI: PS2.A.1)

What to do: If students struggle with seeing relative differences in velocity changes for collisions E and F, encourage them to use two fingers to show the gap between the initial and final velocities to get a qualitative sense of how the changes in velocity of a cart compare to another, to help foreground the relative differences in magnitude of velocity change (one is about twice the other). Alternatively, you can ask volunteers to measure the gap on the projected images using a meter stick.

Building toward: 6.A.3 Analyze data collected from speed and force sensors and use multiple mathematical representations to describe and make claims about the patterns that show the relationship between different variables in a collision (force applied to a vehicle, mass, and change in velocity of a vehicle). (SEP: 5.2; CCC: 1.4; DCI: PS2.A.1)

10 · USE MATHEMATICAL THINKING

5 min

MATERIALS: calculator, *Collisions D-F Velocities*

Determine the exact values for Δv. Display **slide Y**. Suggest that we try to calculate the amount of velocity change for each cart, using the initial and final velocities of both. Instruct students to work with a partner to calculate Δv using a calculator for at least one collision, and more if time permits. Assign half of the classroom to start with each cart for collision D and the other half to start with each cart for collison E. Give them 2 minutes to do this.

Display **slide Z**. Give students half a minute to check their work. Then discuss what they notice about the values for Δv. Listen for the ideas outlined in the Key Ideas box as well as the following:

- The magnitude of Δv of one cart is twice the magnitude of Δv of the other cart in collisions E and F.
- The magnitude of Δv of each cart is the same in collision D.

Ask what is different about the carts in collisions E and F that could explain this 2:1 ratio in outcomes. Students will say the masses are different. Ask what the ratio of the masses is. They will say one cart is twice the mass of the other. Highlight the connection between the difference in mass and the difference in change in velocity.

11 · DEVELOP AND USE A MATHEMATICAL MODEL

6 min

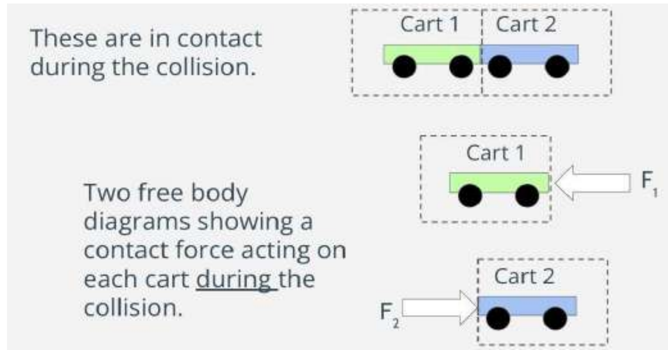
MATERIALS: dry erase marker (blue), dry erase marker (green)

Review force symmetry and combine a system of two equations. Display **slide AA**. Diagram the following on the whiteboard with student input.

✱ ATTENDING TO EQUITY

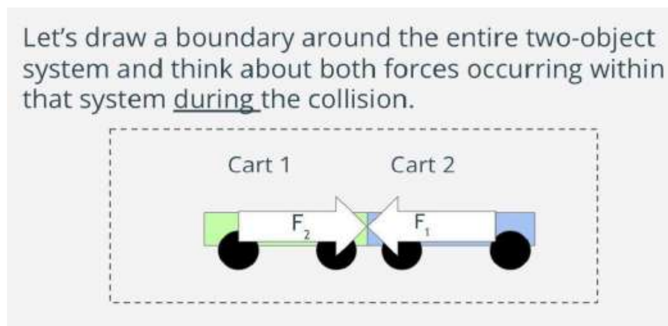
Supporting emergent multilingual students: English has many prefixes that mean "not" or "the opposite of," which can be confusing for students, particularly those who are multilingual. Point out that the term *inelastic* combines a word we have encountered before (elastic) with a prefix that means "not." Ask whether students can think of any other words that begin with "in" and mean "not" whatever comes after. A few examples are *incompatible, infinite*, and *inalienable*.

Draw the two carts (one in blue and one in green) and label them as "These are in contact during the collision."	

Explain that because it is hard to draw contact forces as vectors acting on both carts when they are drawn right next to each other, we can use a convention in physics called a free-body diagram. It is used to redraw the objects separated from the system they are in, to more easily show the forces acting on an individual object. Draw the representation shown and label it as "Two free-body diagrams showing a contact force acting on each cart during the collision".	

Ask how the size of the force vectors compares in this drawing. Students should say these are equal. Ask how the direction compares. They should say these are in opposite directions.

Ask whether this represents what we actually saw in the data, looking again at the graph on **slide AA**. Students will say it does. Point out that the data reported one force as positive and one as negative. Continue drawing on the whiteboard as shown below.

Ask what would happen if we changed the boundary of our system to include both carts. Draw the two-cart system boundary. Listen for students to say that both forces would be acting on the two-cart system.	
Write this equation on the board.	$F_1 + F_2 = ?$

Note: You can set up this equation with the question mark on the left side as an alternate representation.	

Ask students to consider the numerical values of each force, and what would be true at every point in time if we added the value of F1 to the value of F2. Listen for these ideas:

- They should cancel out.
- They will be opposite.
- They will add up to zero.

Continue on the whiteboard as shown.

Set the equation equal to zero and write “Equal and opposite unbalanced forces acting on each object” next to the equation. Note that this shows that when we consider the two-cart system, the net force on the system is zero.	$F_1 + F_2 = ?$ Equal and opposite unbalanced forces acting on each object $F_1 + F_2 = 0$

Then remind students that we have two other force equations relevant to this system. Add the two equations as shown. Point out that we can substitute what is in these equations into the equation above. Write out this substitution.	$F_1 + F_2 = ?$ Equal and opposite unbalanced forces acting on each object $F_1 + F_2 = 0$ $F_1 = \frac{m_1 \times \Delta v_1}{\Delta t}$ $F_2 = \frac{m_2 \times \Delta v_2}{\Delta t}$ If we substitute for F_1 and F_2 $\frac{m_1 \times \Delta v_1}{\Delta t} + \frac{m_2 \times \Delta v_2}{\Delta t} = 0$

Ask students what operations they use in math class to get rid of a common denominator in fractions or to cancel out a term in a denominator. They will say that they multiply by that term.

Document the steps of multiplying every term on both sides of the equation by Δt. Then cancel out this term from both sides of the equation. Rewrite what remains as shown.	By multiplying every term on both sides of the equation by Δt? $\Delta t \times \frac{(m_1 \times \Delta v_1)}{\Delta t} + \Delta t \times \frac{(m_1 \times \Delta v_1)}{\Delta t} = 0 \times \Delta t$ By multiplying every term on both sides of the equation by Δt? $\cancel{\Delta t \times} \frac{(m_1 \times \Delta v_1)}{\cancel{\Delta t}} + \cancel{\Delta t \times} \frac{(m_1 \times \Delta v_1)}{\cancel{\Delta t}} = \cancel{0 \times \Delta t}$ Which leaves only four variables: $m_1 \times \Delta v_1 + m_2 \times \Delta v_2 = 0$

Test our new mathematical model. Display **slide BB**. Suggest that we test our new model. Divide the class so different partners work together to plug in the values for collision D, E, or F. Give them 2 minutes to test the new equation. Then discuss the slide's question as a class:

- *Does our new equation predict these Δv values?*

Help students come to consensus that the model does hold for these collisions and that everything seems to cancel out in each case. Propose that if this is the case when one mass is double the other, maybe this holds for other cases.

Make individual predictions. Display **slide CC**. Give students a minute to think on their own and make a prediction, then ask what we would need in order to see whether our predictions are correct. They will say we need additional data from this situation.

Motivate the need to test more scenarios. Emphasize that testing only one other scenario like this might not be enough to discover how generalizable or limited our new model's application might be. Pose a few rhetorical questions to broaden the conditions we might want to consider, such as these:

- *And what about situations where we aren't just changing mass ratios?*
- *Our evidence so far is limited to cases in which one of the carts was stationary at first. What if both carts were moving before the collision?*
- *What if they don't bounce off each other, and instead stick together? This kind of outcome is called an inelastic collision.*

Then ask, *What are some examples of two things colliding and sticking together instead of bouncing off each other?* Have students share a few examples. ✱

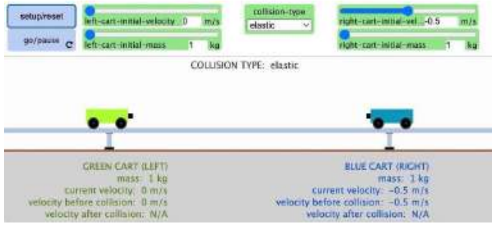
Introduce using a simulation to explore this question. Say, *I have a simulation that will allow us to explore whether this relationship holds for any of these other collision conditions we've been considering. It may also help us determine what limitations there might be to the application of our new equation.*

12 · TEST IDEAS WITH A SIMULATION AND CHECK THE RESULTS

15 min

MATERIALS: calculator, science notebook, whiteboard, dry erase markers, computer with access to https://openscied-static.s3.amazonaws.com/HTML+Files/Inelastic+and+Elastic+Collisions.html

Orient to the simulation. Display **slide DD**. Use the image on the slide to briefly point out the following interface elements that students can use in the simulation as described below.

Collision Cart Simulation:	Orient students to the simulation controls: ● two sliders that change the starting velocity of the carts ● two sliders that change the mass of the carts ● a chooser that selects the type of collision (elastic or inelastic) ● a "setup/reset" button that initializes a new model run using the values set in the three bulleted items above ● a "go/pause" button that runs the model to produce the outcomes of the simulated collision  If the image on the slides is not enough, consider opening the simulation and demonstrating these controls.

Carry out an investigation. Display **slide EE**. Review the directions on the slide. Share the URL for the simulation with the class: https://openscied-static.s3.amazonaws.com/HTML+Files/Inelastic+and+Elastic+Collisions.html.

Have students work with a partner to carry out this brief investigation and record the resulting data in their notebook. This should take 5 minutes or less. At 3 minutes in, you may want to have students raise their hands if they have collected data for at least one condition. This can be a helpful point to remind them that they only need one more condition and have 2 minutes left to collect that data.

Test our model. Display **slide FF**. Review the directions. Tell students to use their calculator and to document their calculations on a whiteboard, large enough to be easily visible for sharing with the class.

Argue from evidence. Display **slide GG**. Pose the slide's question:

- *Does the mathematical relationship we developed above hold for none, some, or all of the collisions we tested in the simulation?*

Explain that to answer this, we need to compare across all conditions tested. Compare results by having students hold up their whiteboards as evidence to support their claims and compare and discuss the results. The class should come to an agreement that the relationship holds for all the conditions.

ASSESSMENT OPPORTUNITY	**What to look for/listen for in the moment:** Documentation of testing the equation for collisions in a system of two objects, which should include each of the following elements: • writing a symbolic equation ($m_1 * \Delta v_1 + m_2 * \Delta v_2 = 0$) in a system of two objects (CCC: 4.2; DCI: PS2.A.2) • substituting known values into all the variables in the equation (SEP: 5.3) • one or more steps to keep track of products and/or sums of the resulting values on each side of the equation (SEP: 5.3) • both sides of the equation equaling zero (SEP: 5.3) • Interpreting this equality as showing the applicability of our equation to predicting the outcome of this collision (CCC: 4.2) **What to do:** If some students' results indicate the relationship doesn't hold, this is most likely due to an undetected error in data recordkeeping or in calculations. Make note of who those students are so you can give them additional practice opportunities, but do not yet provide feedback regarding how to correct their work. The guidance for the start of day 3 shows how to help uncover these errors and still honor the implications of students' initial conclusions. **Building toward: 6.B.1** Apply techniques of algebra to solve for an unknown initial condition or outcome of a collision of a two-object system using a version of Newton's second law, arranged to describe conservation of momentum. (SEP: 5.3; CCC: 4.2; DCI: PS2.A.2)

ADDITIONAL GUIDANCE	If some students' results indicate the relationship doesn't hold, it is most likely due to an undetected error in data recordkeeping or in calculations. To help uncover this error as a class and still honor the implications of these initial conclusions, suggest that we try to figure out what might be different about the collision conditions that led to this. You can then either (1) explore those collision settings in the simulation again as a class or (2) ask interested students to investigate these to figure out why they might be producing a different outcome. This can be done as home learning. Either option will uncover that there must have been a computational error in earlier work and that the relationship does hold for all the cases in the simulation.

End of day 2

13 · CONSIDER CONSERVED QUANTITIES AND USE GEOMETRIC REPRESENTATIONS

13 min

MATERIALS: dry erase marker (black), dry erase marker (red), Force and Motion Relationships poster

Review our mathematical model. Write this equation from the end of last class on the board:

- $m_1 * \Delta v_1 + m_2 * \Delta v_2 = 0$

Ask students to summarize what we did last time to test this equation. Listen for these ideas:

- We tested different cases using a simulation
- We plugged the results from the simulation into this equation to see if everything added up to zero.

Foreground a conserved quantity. Emphasize the following:

- *We have an equation here that fits our data and has an interesting structure, as it shows that when these two products are added together, they sum to zero.*
- *Adding up to zero is kind of weird, because it suggests that even though two parts of the system change, the combination of those changes results in no overall change in the system.*
- *When we uncover something we can measure about changes in the parts of a system that, when added together, shows that it doesn't change the system, we can say that what we're measuring is a conserved quantity.*
- *Something in this system is clearly conserved, because the measurements add up to zero in every case we tested.*

Turn and talk about conserved quantities. Present **slide HH**. Give students half a minute to talk with a partner to identify some other conserved physical quantities. Listen in and then say, *I heard ideas about conservation of energy and mass. We definitely used those ideas in earlier units. And now we have found a quantity that is being conserved.*

Develop a geometric representation. Say, *Whenever we think about a conserved quantity in a physical process, it can be useful to represent that quantity with alternate visualizations beyond just numbers or symbols. Let's try to visualize this conserved quantity in our collisions with a geometric model. A geometric model uses shapes to represent quantities visually. In our case, we need a shape to represent a quantity that is the product of two variables.*

Present **slide II**. Cue students to turn and talk to a partner about the question:

- *What shapes have we used in math class to represent a quantity that is the product of two variables (a * b)?*

After a minute, have a few students report out. They will suggest a rectangle. Ask, *Can someone tell us how the product of two variables can be represented by a rectangle?* Listen for ideas about how a rectangle has a base and a height and their product represents its area. If this doesn't

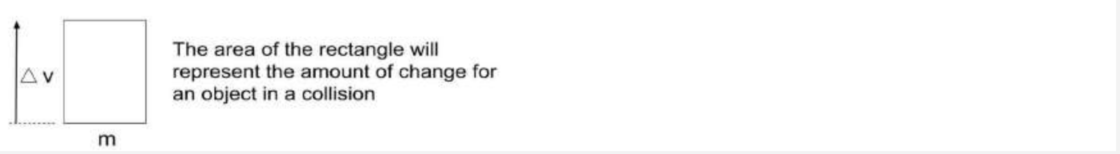
✱ ATTENDING TO EQUITY

Universal Design for Learning: Co-developing this geometric *representation* of the relationship provides an alternate way of *representing* the patterns students have seen that is very different from the bivariate graphs analyzed earlier. It is also very different from the symbolic/algebraic way of representing a conserved quantity like momentum, which is incrementally developed and refined later in this lesson.

Students have been reasoning about area constructs as conserved quantities in Common Core mathematics throughout much of grade school before working with symbolic representations or bivariate graphs in later grades. These geometric representations may therefore draw on additional intuitions about why a product of two variables (named as *momentum* later in this lesson) would either be conserved or transferred in a particular process or event (like a collision). Keep in mind that using such a representation in considering the idea of a "negative area" may be challenging for some students at first.

come out or needs clarification, say something like, *Another way to visualize the product of two values or variables is using the area of a rectangle, where the rectangle's base and height represent the values or variables.* ✱

Explain that we will try to represent a certain amount of change in this conserved quantity for one of the carts in a collision by using a rectangle for which one side represents its mass and the other side represents its change in velocity. Use a black marker to draw a rectangle with the variables on its sides and the corresponding description as shown.



Represent the amount of change for one object. Suggest that we consider a cart with a specific mass and velocity. Present **slide JJ**. Ask how to represent the amount of change using the rectangular representation we just introduced. Anticipated responses include these:

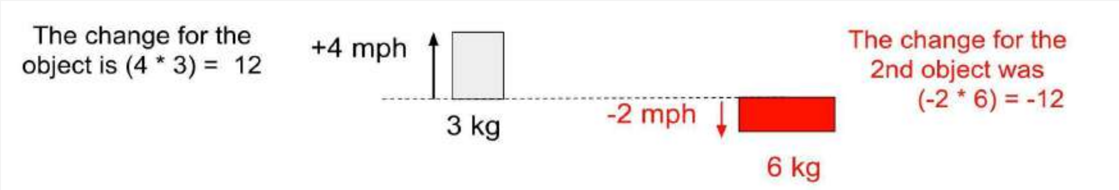
- Draw a rectangle with a base of 3 and a height of 4.
- It will have an area of 12, which represents the amount of change that the cart experiences.

Sketch and annotate the corresponding rectangle on the board as shown.

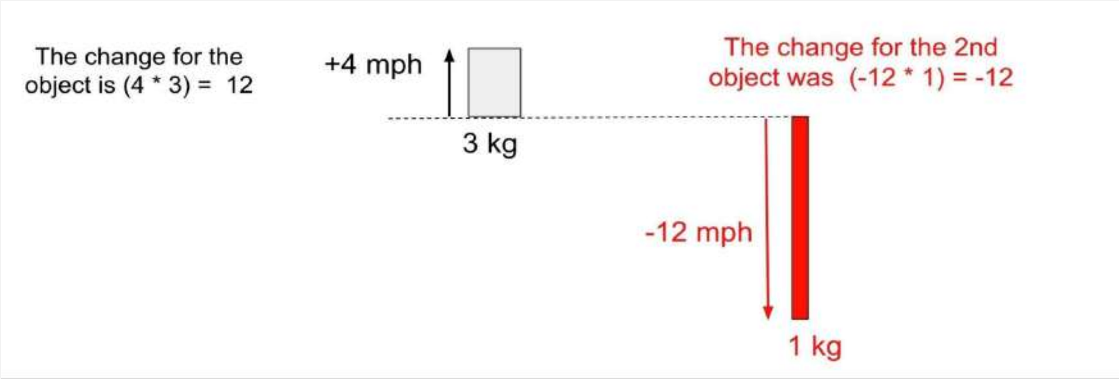
Represent the amount of change for a second object. Ask students to consider how much the velocity of the other cart in a collision should have changed if it had a mass of 6 kg. They should say 2 mph. Sketch and annotate the corresponding rectangle next to the first in red marker.

Ask whether the change is an increase or decrease for this cart if we know the sum total of the changes in the entire system must be zero. Students will say it should be a decrease.

Suggest that we show a decrease by drawing the arrow going in the other direction from a reference line drawn at the top of the new rectangle to the bottom of the first one, so we can better visualize this as a decrease in the amount of the conserved quantity for this second cart. Add negative signs to the annotations to represent the change in velocity as -2 mph and the change in the conserved quantity of the second object as -12.



Represent the amount of change for a different second object. Ask how we would draw this rectangle if the mass of the second cart were 1 kg instead of 6 kg. Students will say to draw a rectangle that is 1 kg wide and goes down 12 mph from the reference line. Erase everything in red and redraw the following:



Represent the change for another different second object. Ask how we would redraw this rectangle if the mass of the second cart were 24 kg. Students will say to draw a rectangle that is 24 kg wide and goes down 0.5 mph from the reference line. Erase everything in red and redraw the following:

Evaluate the geometric model. Display **slide KK**. Discuss the questions as a class as shown in the table below.

Suggested prompt	Sample student response
What did the geometric representation help us visualize?	It helped us see how two variables work together to produce the change in each object. It helped us see that if one variable is really small but the other is really big, it can result in the same amount of change. It helped us see how changes can cancel out. It helped us visualize the quantity represented by the product of the mass and the change in velocity.
What were some limitations of the geometric representation?	The overall change in the quantity when the velocity decreases is harder to visualize, because there isn't really anything like a negative area of a rectangle. Sketching the rectangles might take more time than just sticking with symbols and numbers.

14 · INTRODUCE MOMENTUM AND DEVELOP AN ALTERNATE FORM OF OUR EQUATION

11 min

MATERIALS: calculator, science notebook, whiteboard, dry erase markers, paper towel or whiteboard eraser, Momentum Relationships poster on whiteboard or chart paper, M-E-F triangle poster, dry erase markers or chart paper markers

Start the Momentum Relationships poster. Note: If this is your last class, capture this poster on chart paper. Otherwise, write these ideas and equations on a whiteboard. Say, *We developed an equation that shows how that quantity is conserved in a collision. I'll write a version of it with another type of subscript to refer to the two carts: A and B, instead of numbers. Notice that I also put the zero on the other side of the equation. This is a form of our equation we might encounter going forward.* Write the corresponding equation on the poster.	$m_A \times \Delta v_A + m_B \times \Delta v_B = 0$
Name the quantity we are visualizing. Display **slide LL**. Say, *As scientists realized that m * Δv is a conserved quantity within a two-object system when the objects exert force interactions on each other, they realized they needed a way to refer to that quantity more easily. So they gave it a name, m * Δv , a change in momentum.* Add the annotations and title to the poster.	Conservation of momentum relationships $\underbrace{m_A \times \Delta v_A}_{\text{Change in momentum of object A}} + \underbrace{m_B \times \Delta v_B}_{\text{Change in momentum of object A}} = 0$

Problematize what momentum is. *Say, We have a relationship represented here that describes a momentum change of a two-object collision system that can account for our results. This equation should raise a new question for us: If the left side of the equation represents change in momentum in each object, then what exactly is momentum itself?*

Read the third bulleted point on **slide LL**:

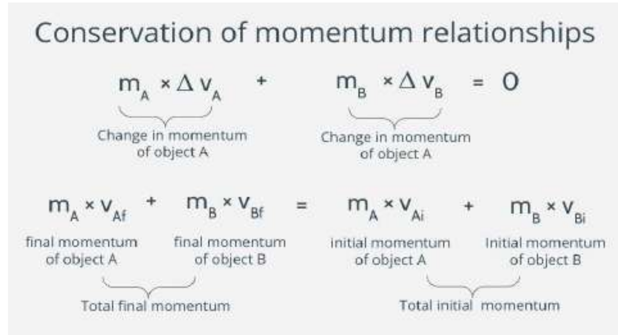
- *Let us s consider what momentum is by recalling how we determine the change (Δ) for a single variable*

Discuss the slide's question as shown below.

Suggested prompts	Sample student responses	Follow-up questions
How did we use data to determine the change in a single variable (i.e., Δt, Δv)?	We determined the difference between two points in time. We determined the difference between two velocities.	*So how many measurements do we need to determine change in the thing we are measuring?*

Substitute in final and two initial velocities for Δv. Demonstrate this step on the whiteboard but not on the chart paper poster. Only the final equation produced will be added to the poster.	$m_A \times (v_{Af} - v_{Ai}) + m_B \times (v_{Bf} - v_{Bi}) = 0$
Distribute the terms. Say, *Because Δv can be determined by subtracting the initial velocity from the final velocity, let's substitute variables for those velocities for Δv and rewrite our conservation side. Let's substitute symbols to represent the final and initial velocities for Δv.* Demonstrate this step on the whiteboard but not on the chart paper poster. Say, *If we distribute the terms, we see there are four separate terms: two related to final states and two related to initial states.*	$m_A \times (v_{Af} - v_{Ai}) + m_B \times (v_{Bf} - v_{Bi}) = 0$ $m_A \times v_{Af} - m_A \times v_{Ai} + m_B \times v_{Bf} - m_B \times v_{Bi} = 0$
Cancel out subtraction. Ask students how they cancel out subtraction of terms in an equation. They will say by adding the term to both sides of the equation. Demonstrate this step on the whiteboard but not on the chart paper poster.	$m_A \times (v_{Af} - v_{Ai}) + m_B \times (v_{Bf} - v_{Bi}) = 0$ $m_A \times v_{Af} - m_A \times v_{Ai} + m_B \times v_{Bf} - m_B \times v_{Bi} = 0$ $+ m_A \times v_{Ai} \qquad + m_A \times v_{Ai}$ $+ m_B \times v_{Bi} \qquad + m_B \times v_{Bi}$

Add a new equation to the Momentum Relationships poster. Ask what the terms on the left side of the equation are related to, compared to the right. Students will say the left side shows that what we end up with after the collision is equal to what we started with before the collision. Annotate this second equation to indicate this. Emphasize that this is another way to consider the two-object system, which is to keep track of all the relevant inputs and outputs.	Conservation of momentum relationships $m_A \times \Delta v_A + m_B \times \Delta v_B = 0$ ($m_A \times \Delta v_A$: Change in momentum of object A; $m_B \times \Delta v_B$: Change in momentum of object A) $m_A \times v_{Af} + m_B \times v_{Bf} = m_A \times v_{Ai} + m_B \times v_{Bi}$

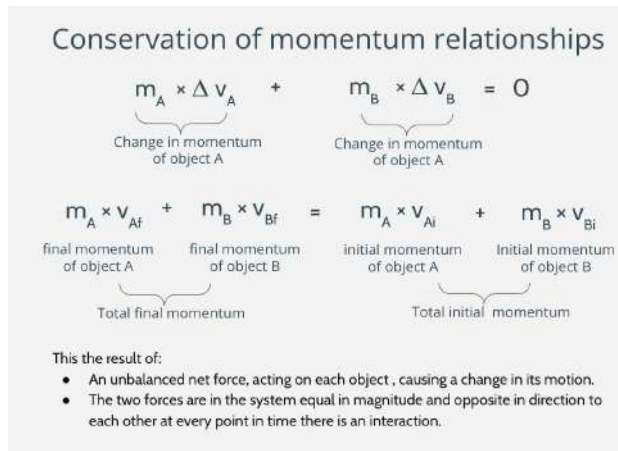


Compare equations. Ask students which of the two equations makes more sense in terms of something being conserved: the one set to zero that we started with, or the one in which the final quantities add up to the initial quantities. Accept all answers.

Emphasize that our new Momentum Relationships poster was a result of forces on each object: although the net force on each cart was unbalanced during the collision, the net force on each was equal in magnitude and opposite in direction at every point in time.

Add this text to the bottom of the poster: "This is the result of:

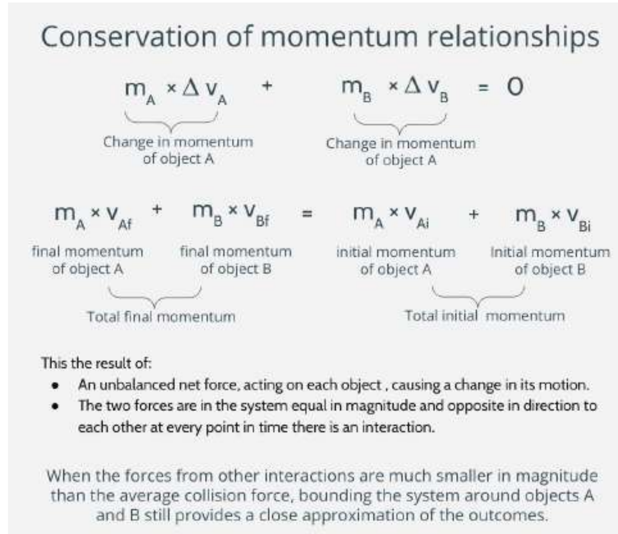
- An unbalanced net force, acting on each object, causing a change in its motion.
- The two forces in the system are equal in magnitude and opposite in direction to each other at every point in time that there is an interaction."



Introduce the limitations of these equations. Say, *Scientists have found that these relationships hold as long as no external forces are acting on two objects during a collision other than their contact forces on each other, as that might make the net forces on each object unequal. But they've also found it that provides a very useful approximation of the predicted outcome of any collision between two objects when relatively small external forces are acting on the objects in the system. This is the case for systems in which external forces like friction with the road or track or air resistance are relatively weak compared to the contact forces between the colliding objects, which was the case in three-cart collisions and is also the case in most vehicle collisions.*

Add a related qualification below the equations:

> "When the forces from other interactions are much smaller in magnitude than the average collision force, bounding the system around objects A and B still provides a close approximation of the outcomes."



Suggest adding the Momentum Relationships Poster to the left side of our M-E-F triangle poster because it describes connections between changes in matter and the forces acting on the matter. Point out that thinking of momentum as being different from kinetic energy is something that scientists debated for a long time before deciding that it was indeed something different--and that they needed a measure of this conserved quantity in systems to make better predictions and explanations about phenomena.

ADDITIONAL GUIDANCE

Do not introduce how kinetic energy is measured. Move on to the next activity, unless students specifically ask how kinetic energy is measured. If that happens, do the following:

- Point out that we clearly had evidence of energy transfer out of the system in the cart collision every time we heard a sound from the collision, which tells us the energy was not conserved in the cart system.
- Suggest that for now, we see how far we can apply the momentum equations alone to answer our current Driving Question Board questions. Then if the equations cannot answer them, we may want to try switching to an energy perspective.
- Encourage students to write down the questions that they want to make sure we revisit in the future. Have them add those questions on sticky notes to the side of the DQB.

Students do not need to quantify kinetic energy in this unit to explain any aspect of the phenomena we are studying. But in the next unit, *OpenSciEd Unit P.4: Meteors, Orbits, and Gravity (Meteors Unit)*, they will. The placement of the sticky notes to the side of the DQB can serve as a reminder that we anticipated that these questions might not be answered in this unit, so we want to carry these forward to the next unit.

15 · ADD TERMS TO PERSONAL GLOSSARIES

4 min

MATERIALS: science notebook

Add terms to the Personal Glossary. Present **slide MM**. Have students add terms developed in this lesson to their glossary. Symbols and words they may opt to include are *velocity*, *elastic collision*, *inelastic collision*, and *momentum*.

ADDITIONAL GUIDANCE	If students ask whether they should add the equations developed in this lesson to their Personal Glossary, tell them there is a reference sheet of these equations on the first two pages of the handout you are about to distribute. They can add those pages to their *Engineering Progress Tracker* at the start of Lesson 7 if they wish.

16 · PREDICTING OUTCOMES ACROSS A BROADER SET OF COLLISION PHENOMENA

9 min

MATERIALS: *Different Momentum Cases*, calculator, Momentum Relationships poster, chart paper markers

Motivate additional testing of the equation(s). Say, *Any one of these equations seems to be pretty useful in helping us make predictions and think about whether they are consistent with our other data. But we've only analyzed a few types of elastic and inelastic collisions, and all of them involved either vehicles or carts. So many other types of collisions occur in the world, as we've talked about.*

Present **slide NN**. Distribute the *Different Momentum Cases* handout to each student. Say, *Let's spend some time independently using this relationship to predict and explain things across a broader set of collision phenomena in our world.*

Emphasize the following:

- *Different forms of the momentum equation we developed in class are summarized on the first page as a reference for you.*
- *Let's see what other types of phenomena we can predict and explain with our momentum equations. The later pages of the handout have three scenarios to apply them to.*
- *You may only get through one scenario in class. That's OK, as the rest can serve as additional practice at home for becoming more fluent in using the model before our next summative assessment.*

Give students the remaining time in this activity to work independently on their handout.

17 · ASSIGN THE HOME LEARNING ON SELF-ASSESSMENT AND REFLECTION

5 min

MATERIALS: *Different Momentum Cases*, *Momentum Self-Assessment Key*, calculator, science notebook

Introduce a handout to self-assess understanding and confidence. Present **slide OO**. Distribute the *Momentum Self-Assessment Key* to each student. Say, *This is a key for the questions you started working on. At the end, you'll reflect on your confidence in using our equations to solve the practice problems. How could we use all of this to help us as learners so we're well prepared for doing this kind of thinking on our next summative assessment?* Accept all answers.

Display **slide PP**. Emphasize the following for the home learning:

- *Use the Momentum Self-Assessment Key to self-assess your work as you complete each question on your Different Momentum Cases handout.*
- *When you're done, complete Questions A-C to identify the parts of the process you're confident about and the parts you'd like additional practice or help with.*
- *Doing these assignments will enable your teacher to give you timely and targeted feedback, help, and additional practice.*

Say, *This is one of many opportunities we should take advantage of to communicate with each other in more detail and figure out ways to work together to address areas where we identify needs.* Set a future class period as a due date for students to turn in both the *Different Momentum Cases* handout and the *Momentum Self-Assessment Key*.

ASSESSMENT OPPORTUNITY

What to look for/listen for in the moment: For specific responses to individual answers, see the *Momentum Self-Assessment Key*. Look for the following across responses:

- Questions 1-4:
  a. defining the parts of the system that are changing motion using a diagram (CCC: 4.2)
- Questions 1-3:
  a. writing a symbolic equation for momentum conservation in a system of two objects (SEP: 5.3; CCC: 4.2; DCI: PS2.A.2)
  b. substitution of known values into all the variables in the equation (SEP: 5.3)
  c. one or more steps to keep track of products and/or sums of the resulting values on each side of the equation (SEP: 5.3)
  d. a correct predicted value (DCI: PS2.A.2)

- Question 4:
  a. describing how changes in velocity for the larger vehicle in all four graphs (4b, 4c, 4d, 4e) correspond to the outcome described for that vehicle
  b. describing how velocity changes predicted for the smaller vehicle are correctly represented in graphs 4b and 4d but not 4a and 4c (DCI: PS2.A.2)

What to do: Students complete the *Different Momentum Cases* handout as a key formative assessment to check their understanding, and they self-assess their understanding and confidence with the *Momentum Self-Assessment Key*. Make sure to collect both handouts on the agreed-upon due date. This is an opportunity to use the self-assessment feedback to target areas where some students are requesting additional help or may benefit from additional practice.

If the self-assessment indicates that students are not yet confident with the elastic/inelastic distinction from this lesson, review the home learning at the start of lesson 7 and use manipulatives (such as toy cars) to emphasize that in the grocery cart collision, the front cart bounced off the cart that hit it, propelling it forward; in the vehicle collision, both cars were moving, and the smaller car bounced backward off the larger car; but in the train collision, there was no bouncing. These terms will be important shared vocabulary in exploring collisions going forward.

Building toward: 6.B.2 Apply techniques of algebra to solve for an unknown initial condition or outcome of a collision of a two-object system using a version of Newton's second law, arranged to describe conservation of momentum. (SEP: 5.3; CCC: 4.2; DCI: PS2.A.2)

18 · NAVIGATE

3 min

MATERIALS: None

Connect to vehicle safety. Display **slide QQ**. Review the things that we know are constants in a collision between two different-mass vehicles at the top of the slide:

- *the magnitude of contact forces on both vehicles at the same points in time*
- *the total momentum in the system (before versus after)*

Discuss the slide's questions as a class:

- *What is changing in a collision between a large truck versus a small car that could affect passenger safety?*
- *What new questions does this raise for us?*

Listen for students to suggest differences in Δv in response to the first question. Listen for them to raise questions about how and why Δv would affect passenger safety in response to the second question.

If time permits, activate students' curiosity and motivate further investigation by asking something like, *Why would differences in the Δv of the vehicle that a passenger is in have an effect on their safety?* Accept all ideas.

Additional Lesson 6 Teacher Guidance

SUPPORTING STUDENTS IN MAKING CONNECTIONS IN MATH	**Number and Quantity** **CCSS.MATH.CONTENT.HS.N-VM.1 Vector and Matrix Quantities:** Represent and model with vector quantities. Recognize vector quantities as having both magnitude and direction. **CCSS.MATH.CONTENT.HS.N-VM.3 Vector and Matrix Quantities:** Represent and model with vector quantities. Solve problems involving velocity and other quantities that can be represented by vectors. **Algebra** **CCSS.MATH.CONTENT.HS.A-SSE.1b Seeing Structure in Expressions:** Interpret the structure of expressions. Interpret complicated expressions by viewing one or more of their parts as a single entity. **CCSS.MATH.CONTENT.HS.A-SSE.2 Seeing Structure in Expressions:** Interpret the structure of expressions. Use the structure of an expression to identify ways to rewrite it. **CCSS.MATH.CONTENT.HS.A-CED.2 Creating Equations:** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. **CCSS.MATH.CONTENT.HS.A-CED.4 Creating Equations:** Create equations that describe numbers or relationships. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.