Lesson 4: What affects the amount of time it takes a vehicle to stop after the driver presses the brakes?

Previous Lesson	*We used what we figured out from video data and mathematical modeling to identify design features that can decrease reaction distances to prevent collisions in the event of a sudden obstacle. We wondered about how speed at the moment of a collision affects the outcome of the collision, assuming that it cannot be avoided.*
This Lesson Investigation 3 days	 We use a speed versus time graph to predict how the initial speed, braking force, and mass of a moving vehicle impact the stopping time. We collect data to test our predictions in the Braking Investigation. We graph the data in CODAP and use curve fits to identify patterns that suggest a mathematical relationship connecting these variables. To further test this relationship, we use a simulation to gather additional data. We use these ideas to revise our initial predictions.
Next Lesson	*We will revisit our speed versus time graphs and name acceleration. We will rearrange our equations to show that unbalanced forces cause change in motion, and we will analyze graphs to explain why drivers react later in rainy conditions. We will revisit our Engineering Progress Tracker and complete an Electronic Exit Ticket.*

BUILDING TOWARD NGSS

HS-ETS1-3, HS-PS2-2, HS-PS2-3, HS-PS2-1

What students will do

4.A Use mathematical representations of the relationship between mass, initial speed, force, and stopping time and algebraic thinking to make a quantitative claim that predicts how much changing the braking force will affect the time it takes a vehicle to stop. (SEP: 5.2, 6.1; CCC: 3.5; DCI: PS2.A.1)

4.B Use simple limit cases and algebraic thinking to determine whether curve fits of data on the relationship between force, mass, initial speed, and stopping time make sense compared to what is known about the real world (SEP: 5.4; CCC: 3.5; DCI: PS2.A.1)

What students will figure out

- The more braking force that is applied to a moving cart, the less time it takes to stop.
- The more mass and/or initial speed of a moving cart, the more time it takes to come to a stop.
- Mathematical models can help make very good, but not perfect, predictions of the changes in motion of an real-world object.
- It is not possible to eliminate measurement errors, but steps can be taken to reduce them.

Lesson 4 • Learning Plan Snapshot

Part	Duration		Summary	Slide	Materials
1	2 min		**NAVIGATE** Discuss ideas in the exit tickets from Lesson 3 about how the speed of a car would impact the distance and time it takes to stop.	A	exit ticket from Lesson 3
2	4 min		**INTRODUCE BRAKING FORCE** Introduce braking force and make sense of it using a force diagram.	B	
3	14 min		**MAKE PREDICTIONS USING A MATHEMATICAL MODEL** Use the changes in motion of a vehicle represented on a speed versus time graph to make predictions about variables affecting stopping time.	C-E	*Braking Variables Predictions*, Speed versus Time Graph poster from Lesson 3
4	25 min		**COLLECT PRACTICE DATA WITH AN EXPERIMENTAL SETUP** Orient to the investigation materials. Collect data to test predictions in small groups.	F-I	https://codap.concord.org/app/static/dg/en/cert/index.html, Braking Investigation
					End of day 1
5	2 min		**NAVIGATE** Discuss data collection and accuracy. Suggest collecting additional data with these ideas in mind.	J	*Data Analysis Reading* (optional)
6	28 min		**COLLECT INVESTIGATION DATA** Collect additional data in small groups to serve as the basis for identifying a mathematical relationship between the variables.	K	Braking Investigation
7	15 min		**ANALYZE THE DATA AND NAVIGATE** Discuss the curve fit that matches the results. Discuss the role of force on stopping time. Use simple limit cases to compare curve fits. Make additional predictions.	L-N	*Braking Investigation*, *Data Analysis Reading* (optional)
					End of day 2
8	3 min		**NAVIGATE**	O	*Braking Investigation*

			Share ideas about the relationship between braking force, mass, initial speed, and stopping time in regard to vehicle safety.		
9	10 min		**PREDICT EFFECTS ON STOPPING TIME** Discuss the effects on stopping time when changing the mass, speed, and braking force of the vehicle. Have students carry out two practice exercises changing all three variables.	P-Q	
10	12 min		**USE A SIMULATION TO GATHER ADDITIONAL EVIDENCE TO TEST A MATHEMATICAL RELATIONSHIP AMONG VARIABLES** Collect additional data in small groups with a simulation as the basis for testing a mathematical relationship between braking force, mass, speed, and stopping time.	R	computer with access to https://tinyurl.com/3zwf4d9w, *Braking Simulation Instructions*, https://tinyurl.com/3zwf4d9w
11	10 min		**MAKE SENSE OF THE RESULTS FROM THE SIMULATION** Discuss the mathematical relationship among the variables, creating a table to keep a record of students' findings.	S-T	*Braking Simulation Instructions*
12	5 min		**TEST THE MATHEMATICAL MODEL** Use real-life examples to test the mathematical model that connects braking force, mass, speed, and stopping time.	U	whiteboard, dry erase markers
13	5 min		**NAVIGATE** Assign and collect the exit ticket handout.	V	*Braking Exit Ticket*

End of day 3

Lesson 4 • Materials List

	per student	per group	per class
Braking Investigation materials	• *Braking Investigation* • sanitized safety glasses with side shields	• variable carts (see *Cart Construction Instructions*) • computer with access to https://codap.concord.org/app/static/dg/en/cert/index.html • ramp setups (see *Braking Investigation Setup Instructions*) • pea gravel/sand • tumble buggy with spring scale (see *Braking Investigation Setup Instructions*) • 3-kg digital scale • stopwatch • 24-oz deli container • fender/braking washers • *Force Data Protocols* or *Mass Data Protocols* or *Speed Data Protocols* • *Force Data Protocols* • *Mass Data Protocols* • *Speed Data Protocols*	• smart cart • smart cart software • sticky tack
Lesson materials	• science notebook • exit ticket from Lesson 3 • *Braking Variables Predictions* • Speed versus Time Graph poster from Lesson 3 • *Data Analysis Reading* (optional) • *Braking Investigation* • whiteboard	• computer with access to https://tinyurl.com/3zwf4d9w • *Braking Simulation Instructions*	• https://codap.concord.org/app/static/dg/en/cert/index.html • https://tinyurl.com/3zwf4d9w

	• dry erase markers • *Braking Exit Ticket*		

Materials preparation (45 minutes)

Review teacher guide, slides, and teacher references or keys (if applicable).

Make copies of handouts and ensure sufficient copies of student references, readings, and procedures are available.

Make copies of the handouts for this lesson:

- *Braking Variables Predictions* (1 per student)
- *Data Analysis Reading* (1 per student; optional)
- *Braking Investigation* (1 per student)
- *Braking Exit Ticket* (1 per student)
- *Force Data Protocols* (1 per force group)
- *Mass Data Protocols* (1 per mass group)
- *Speed Data Protocols* (1 per speed group)
- *Braking Simulation Instructions* (1 per group of 2-3 students)

Day 3: Simulation Data Collection
Test the simulation: https://tinyurl.com/3zwf4d9w

Make sure you have the Speed versus Time Graph poster from Lesson 3 displayed on a visible area of the room.

Three-hole-punch all images and handouts so they can be added to students' science notebooks.

Day 1:
Make sure the laptops students are using can run CODAP: https://codap.concord.org/app/static/dg/en/cert/index.html

Days 1 and 2: Braking Investigation

- **Group size:** 4-5 students

- **Setup:** See the *Braking Investigation Setup Instructions* for setup that needs to be done each year and the *Cart Construction Instructions* for one-time setup.
- **Storage:** Store materials in a cabinet in the classroom.

Safety Protocols: When working with the lab materials, the following safety precautions are necessary.

- Students must conduct the experiment under the supervision of qualified personnel who can respond quickly to any unforeseen circumstances.
- Students involved in setting up the equipment and conducting the experiment must be properly trained in handling the sensor carts and understand the experimental procedures.
- Wear appropriate personal protective equipment (PPE) including sanitized safety glasses with side shields during the setup, experimentation, and takedown segments of the activities.
- Secure loose clothing, wear closed-toe shoes, and tie back long hair.
- Clear the workspace of any obstacles or hazards that could interfere with the experiment or cause accidents during the collision.
- Make sure that all parts of the carts are properly secured and stabilized before conducting the collision test. Follow manufacturer guidelines for setup and operation.
- Immediately clean up anything that falls on the floor, so it does not become a slip or fall hazard.
- Maintain a safe distance from the collision area during the activity to avoid injury from flying debris or malfunctioning equipment.
- Following the activity, inspect all equipment for any damage or wear and tear. Repair or replace any damaged components before further use.
- Wash hands with soap and water once all equipment is put in appropriate storage areas.

Lesson 4 • Where We Are Going and NOT Going

Where We Are Going

This lesson is designed to coherently build ideas related to the following disciplinary core idea (DCI):

- **PS2.A.1: Forces and Motion.** Newton's second law accurately predicts changes in the motion of macroscopic objects. (HS-PS2-1)

Day 1 is designed to motivate the need to investigate the relationships between the mass of a vehicle, its initial speed, its braking force, and the time it takes the vehicle to come to a stop. Students work in small groups to collect data for one of the three independent variables, and they use these results to support a discussion about issues of accuracy and data collection.

On day 2, students continue collecting data, and these results are used to derive the following mathematical relationship that connects these variables:
Time to stop = (mass of vehicle * initial speed) / braking force

Students use a simulation to gather additional data for additional evidence to test this relationship. In Lesson 5, they will transform this mathematical relationship into F = m * a.

This lesson is designed to support students toward the Next Generation Science Standards (NGSS) Nature of Science Understanding titled **Scientific Investigations Use a Variety of Methods** (Appendix H). The relevant understanding element reads:

- Science investigations use diverse methods and do not always use the same set of procedures to obtain data.

During the investigation on days 1 and 2, students discuss issues of accuracy and consider strategies that could be used to obtain more precise data. They also discuss the advantages and disadvantages of collecting data using the physical setup and the simulation.

Students encounter and/or co-construct ideas for the following terms during this lesson: *braking force, stopping time*. Be prepared for them to add a definition for these terms to their Personal Glossary. **Do not** ask students to define or keep track of any words until after the class has developed a shared understanding of their meaning.

Where We Are NOT Going

This is not a lesson about momentum. Although students investigate the role of mass and initial speed on stopping time, we do not explore how these variables relate to changes of speed over time. In Lesson 6, students will investigate the relationship between mass and velocity and add the word *momentum* to their Personal Glossary.

LEARNING PLAN for LESSON 4

1 · NAVIGATE

2 min

MATERIALS: science notebook, exit ticket from Lesson 3

Return to Lesson 3's exit ticket. Present **slide A**. Remind students that last class, they completed an exit ticket about their ideas of how the speed of a car would affect the distance and time it takes to stop. Ask them to turn and talk about their ideas using the slide's prompts:

- *How did you think speed would affect the distance and time it takes to stop after the driver hits the brakes?*
- *What other factors might affect how long it takes to stop after the driver hits the brakes?*

After a minute, invite students to share. Accept all ideas about the first prompt. For the second prompt, listen for suggestions of "how good the brakes are" and maybe mass.

If the discussion does not touch upon "better brakes" or factors associated with the quality of the brakes or the strength of the braking force, use the following prompts:

- *What about the brakes themselves?*
- *Are all brakes the same?*

Use this discussion to transition to the next slide by saying, *Let's pause for a moment and figure out what it means in physics terms for a car to brake.*

2 · INTRODUCE BRAKING FORCE

4 min

MATERIALS: None

Introduce *braking force*. Present **slide B**. Say, *When you press the brakes, a friction force is applied to the car that engineers call the braking force. Let's think about how the braking force is acting on the car for a moment.* Give students a minute to turn and talk with the slide's prompts:

- *In which direction is the braking force on the car in the image? How do you know?*
- *If someone "brakes harder" than another driver, what is different about the braking force?*

Develop a force diagram with the class. Invite volunteers to share their ideas about the first prompt. As they share, sketch a force diagram of the force on the car, either over the projection or by quickly drawing a car on the board at the front of the room. Position the arrow below the car to make it clear that this is a friction force with the ground, and label it "Braking force". You may want to remind the class that if the ground is exerting a force on the car, the car must also be exerting a force on the ground, but **do not** try to sketch how the brakes translate to a friction force, as the mechanics of these forces are very complicated.

Listen for students to say in response to the second prompt that the force will be greater. Then point to your diagram and ask, *How would I represent that in a model like this one we made of the braking force?* Listen for suggestions that the arrow might be bigger.

ADDITIONAL GUIDANCE	In physics, we often use the term *free-body diagram* for a simplified representation of the forces acting on an object. Though this term is used in OpenSciEd HS, it might be somewhat abstract for students. Simply calling this type of representation a *force diagram* more clearly conveys its purpose in this context. This shift in terminology can provide a clearer and more intuitive understanding for students when they are asked to model the forces acting on an object. It also provides more flexibility to develop and apply modeling conventions that make sense to the class, rather than adhering to the traditional conventions of free-body diagrams.

ADDITIONAL GUIDANCE	In Lesson 5, the class will add “unbalanced forces cause changes in motion”. If students have not completed *OpenSciEd Unit P.2: How forces in Earth’s interior determine what will happen to its surface? (Earth’s Interior Unit)* before this unit, you will need to include extra support in building understanding of balanced and unbalanced forces and the concept of net force. These concepts are coherently developed within the *Earth’s Interior Unit*. Although this lesson focuses on stopping time, it is the point in this unit when we begin diagramming forces, and it may be an appropriate time to insert additional instruction on net force.

3 · MAKE PREDICTIONS USING A MATHEMATICAL MODEL

14 min

MATERIALS: *Braking Variables Predictions*, science notebook, Speed versus Time Graph poster from Lesson 3

Motivate interpreting the Speed versus Time graph. Say, *We think the braking force is important. We also think speed is important, and we wrote about that in our exit tickets. I also heard some ideas about the size of the vehicle maybe being important. Let’s investigate these variables and make some predictions about how they might affect the outcome of a collision. In the last lesson, we used speed versus time representations to describe the way a car was moving.*

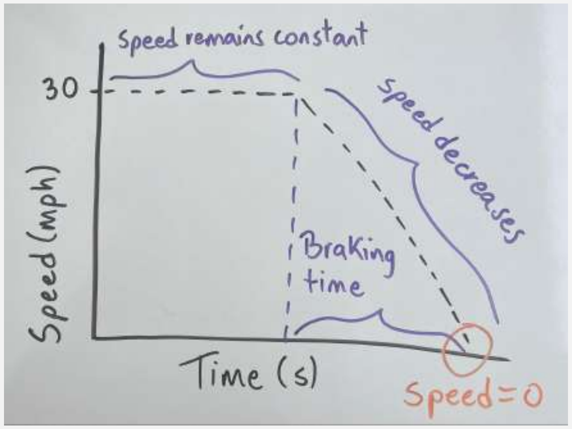
Direct students to refresh their memory with the graph in their notebook or on the Speed versus Time Graph poster for a few moments. Then ask, *How does a speed versus time graph like this one help us understand how long it takes a car to stop?* Elicit 1-2 ideas very quickly. Listen for ideas about how the graph shows the car slowing down, so we can see how long it takes to get from driving speed to zero.

Make predictions about speed versus time for speed, braking force, and mass. Present **slide C**. Mention that the graph on the slide shows the changes in a car's motion. Give students a minute of individual think time to consider how the graph would change if the car is moving faster, experiences more braking force, or has more mass.

Annotate a graph to show predictions. Distribute the *Braking Variables Predictions* handout to each student. Say, *Let's consider the example shown in the handout and the slide to make predictions about how the factors we discussed might affect the amount of time it would take a vehicle to stop*. Describe the example, and make sure students can understand the variables represented in the graph.

Present **slide D**. Give students a couple of minutes to discuss the prompts with a partner to analyze the baseline example:

- *Identify any specific points or regions on the graph where the speed:*
  - *remains constant*
  - *changes*
  - *reaches zero*
- *How would you represent in the graph a car that is traveling at the same speed and:*
  - *takes less time to stop?*
  - *takes more time to stop?*



Encourage students to mark the graph in their handout. Draw a similar graph on the board to capture their ideas. During this discussion, reinforce the idea that the slope of the speed versus time graph reflects a larger change in speed over a shorter period of time. The annotated graph might look like this example:

ALTERNATE ACTIVITY	If students feel comfortable reading and interpreting speed versus time graphs after Lesson 3, they may not require this much scaffolding. Feel free to move more quickly through **slide D** and the *Braking Variables Predictions*, or skip them, based on your students' needs.

Say, *Let's use our example to test our predictions about how the speed versus time graph will change if the braking force, initial speed, or mass of the car is changed.*

Make predictions about factors affecting stopping time. Present **slide E**. Give students a few minutes to complete the task on the slide individually, but encourage them to ask an elbow partner for support if they get stuck. If much of the class seems to be struggling, walk through the process together for one factor, such as mass, using the directions in the handout.

ASSESSMENT OPPORTUNITY	**What to look for/listen for in the moment:** Students should use the speed-time graph to predict how increasing the braking force or decreasing the mass and initial velocity of the vehicle will result in a steeper negative slope (faster decrease in speed), whereas increasing the mass and initial speed or decreasing the braking force will lead to a less steep slope (slower decrease in speed). (SEP: 5.2, 6.1; CCC: 3.5; DCI: PS2.A.1)
	
	See the *Braking Variables Predictions Key* for sample responses.
	
	What to do: If students struggle to graph the changes of speed over time, allow them to use their own words to describe these changes. For example, you can provide sentence stems such as:
	● *If I double (the mass of the car/the braking force/the initial speed of the car), the time it will take for the car to stop will be:*
	    ● *twice as long*
	    ● *half as long*
	    ● *longer*
	    ● *shorter*
	
	Do not force students to come up with the correct answer. At this point, the goal is to use their predictions to motivate further investigation of these variables.
	
	Building toward: 4.A.1 Use mathematical representations of the relationship between mass, initial speed, force, and stopping time and algebraic thinking to make a quantitative claim that predicts how much changing braking force will affect the time it takes a vehicle to stop. (SEP: 5.2, 6.1; CCC: 3.5; DCI: PS2.A.1)

Debrief predictions and motivate an investigation. Invite students to share their ideas, and keep a record of their predictions on the graph on the board. Follow up by asking about the possible mechanism behind these predictions, using prompts such as:

- *Why would doubling the braking force/initial speed/mass change the amount of time it takes for the car to break?*
- *Why would halving the braking force/initial speed/mass change the amount of time it takes for the car to break?*

Listen for students to mention the role of forces in the system. For example, they might suggest that a car moving at twice the speed would be harder to stop using the same braking force, which would result in a longer braking time. Say, *Let's investigate these predictions.*

4 · COLLECT PRACTICE DATA WITH AN EXPERIMENTAL SETUP

25 min

MATERIALS: Braking Investigation, https://codap.concord.org/app/static/dg/en/cert/index.html

Orient to the investigation materials. Present **slide F**. Gather the class around a workstation and present the various materials available to test their predictions. Use the slide's prompts to elicit ideas:

- *What is our dependent variable?*
- *What are our independent variables?*
  - *How can we change each independent variable?*
  - *How do we keep the other variables constant?*
- *How do we measure these variables?*

KEY IDEAS

Purpose of this discussion: Identify the independent and dependent variables that will be used during this investigation, and decide how they will be manipulated and measured with the available materials.

Listen for these ideas:

- The dependent variable is time. Time is always dependent because we cannot control it.
- The independent variables are the braking force, the speed of the cart, and the mass of the cart.
- The braking force can be manipulated by adding washers to/removing washers from the brake. When investigating braking force, the speed and mass need to be kept constant.
- The speed of the cart can be manipulated by changing the release point on the ramp. When investigating speed, the mass and braking force need to be kept constant.
- The mass of the cart can be manipulated by adding or removing gravel/sand. When investigating mass, the speed and braking force need to be kept constant.
- Time can be measured with a stopwatch, mass can be measured with a scale, and force can be measured with a spring scale.

Go through the safety protocols. Before students begin their data collection, display **slide G** and review the general safety protocols for the activity.

Organize the class in small groups. Divide the class into groups of 4-5 students. Assign each group a workstation and an independent variable to investigate. Distribute the *Braking Investigation* handout to each student and one copy of the *Force Data Protocols*, *Mass Data Protocols*, or *Speed Data Protocols* to each group based on the variable they were assigned. Additionally, distribute a laptop to each group with access to https://codap.concord.org/app/static/dg/en/cert/index.html. Display https://codap.concord.org/app/static/dg/en/cert/index.html and provide an example of how to record data in CODAP if necessary.

Present **slide H**. Read the instructions aloud:

✱ SUPPORTING STUDENTS IN ENGAGING IN PLANNING AND CARRYING OUT INVESTIGATIONS

Students will collect additional data next time. The goal of the first round of data collection is to discuss issues of error. During this discussion, push students to consider the accuracy of data, the number of trials needed to produce reliable measurements, and limitations on the data's precision due to the experimental setup. Encourage them to refine their data collection accordingly.

- *Read the directions on your group's Data Protocol reference.*
- *Follow the directions to collect practice data for 1 condition.*
- *Record your initial data on your Braking Investigation handout.*

Instruct the groups that are investigating speed to work together so they all experience measuring a speed value with the smart cart. The mass and force groups can work independently, though the mass groups need to take turns with the electronic scale.

ADDITIONAL GUIDANCE	When assigning variables to lab groups, consider differentiating based on the complexity of the data collection task. The mass data collection is the least complex, focusing on the use of an electronic scale. The force data collection uses the tumble buggy to read the braking force while the cart is dragged. The speed data collection can be either the most complex or most simple, depending on whether you have them do this using the smart cart or choose to do this for them ahead of time. Note that if you do pre-measure the speeds, it is possible that the ramps will shift and the measured values will be incorrect. Make sure students record their data on *Braking Investigation*. The physical copy serves as a backup in case of technical issues that prevent students from finding their digital results, such as forgetting their CODAP login password, having a faulty computer, or losing internet access.

Debrief observations using the materials. Present **slide I**. Have students return to their seats with their handouts. Mention that we will continue gathering additional data for our assigned variables next time. Use the slide's prompts to elicit ideas:

- *How did your group's results compare between trials?*
- *What factors could explain the difference in these results?*
- *What could we do to improve the accuracy of our results?*
- *What are the advantages and disadvantages of collecting data with this setup?* ✱

KEY IDEAS	**Purpose of this discussion:** Debrief preliminary results and consider the factors that can affect the accuracy of the results. During this discussion, make sure that students investigating each variable share their findings and ideas with the class. **Listen for these ideas:** • The results between trials are not the same, but they are similar. • Small changes during the experiment, such as to the launch point or the path the cart follows, might affect the results. • Our measurements might be slightly off every time we measure. • We can improve the accuracy by conducting several trials and following the calibration and data

collection protocols.

- We can isolate each variable to investigate how it affects the stopping time, but some variables might be hard to manipulate or measure.

End of day 1

5 · NAVIGATE

2 min

MATERIALS: *Data Analysis Reading* (optional)

Discuss the main ideas from the reading. Present **slide J**. Pose the prompt on the slide:

- *While collecting data, what are some steps we can follow to increase the accuracy of our results?*

Listen for the following ideas:

- We can collect additional trials. The more trials, the more confident we can be that our results are not due to errors.
- We should repeat measurements if we have doubts about how we carried them out.

Suggest that we collect additional data while keeping these ideas in mind.

ALTERNATE ACTIVITY	If students need extra support with data collection strategies, consider making time to read the *Data Analysis Reading* together or providing it as a resource.

6 · COLLECT INVESTIGATION DATA

28 min

MATERIALS: Braking Investigation

Collect additional data. Have students return to their work stations to continue collecting data for the variable they investigated last time. Present **slide K**. Orient the class by reading the prompts aloud:

- *Follow the directions to collect data for your independent variable on your Data Protocols reference.*
- *Record your data in the Data Collection section of your Braking Investigation handout and enter it into CODAP.*
- *Complete the Data Analysis section of your Braking Investigation handout.*

✱ SUPPORTING STUDENTS IN ENGAGING IN PLANNING AND CARRYING OUT INVESTIGATIONS

Give students 25 minutes for this task. ✱

The results obtained by the class may not yield the accuracy necessary to identify a clear pattern in the relationship between force and time for all three cases. Regardless of how similar or different the groups' obtained values are, discuss the challenges of measuring the time and forces, how this could have affected the accuracy of the results, and thus, the conclusions that can be drawn from them. The goal of this discussion is to recognize the importance of keeping sources of error in mind when conducting an investigation and measuring variables.

7 · ANALYZE THE DATA AND NAVIGATE

15 min

MATERIALS: *Braking Investigation*, *Data Analysis Reading* (optional)

Discuss the curve fit that matches the results. Present **slide L**. Say, *Scientists use mathematical tools to identify trends in their results. These graphs are an example of these tools, and they show us possible curve fits for our data. Curve fits are considered mathematical models because they use mathematical equations to describe the relationship between the independent and dependent variables.*

Use the slide's prompt and graphs to discuss the roles of mass and speed in stopping time:

- *Based on your results, how would you say the variable you investigated is related to time?*

Listen for the following ideas:

- The larger the mass, the longer the time it takes for the cart to stop.
- The higher the speed, the longer the time it takes for the cart to stop.

Discuss the role of force in stopping time. Mention that last time, we predicted that the greater the braking force, the less time it would take for the cart to stop. Ask, *Which of the graphs on the slide suggest the same force-time relationship?* Students should say that the linear (-) and inverse graphs reflect that relationship.

Then ask which of the graphs better matches our results. Accept all answers. It is OK if students do not mention that the force versus time relationship shows an inverse relationship. You will use the limit cases next to explore these relationships further.

Use simple limit cases to compare curve fits. Present **slide M**. Say, *One advantage of a curve fit is that it allows us to make predictions about conditions we haven't tested yet. Let's see if we can use this curve fit to predict what would happen to the time it takes the car to stop after changing our independent variables.*

Have students work with a partner to answer the Discussion Questions on their lab handout. Give them a couple of minutes to write their answers before inviting them to share their ideas with the class. In the discussion, be sure that the class comes to an agreement that the force data fits an inverse curve, if they haven't already. See the assessment guidance for details.

ASSESSMENT OPPORTUNITY

What to look for/listen for in the moment: Students' answers should include:

- When the force is too large, the stopping time will be very small. (SEP: 5.4; CCC: 3.5; DCI: PS2.A.1)
- When the force is too small, the stopping time will be very long. (SEP: 5.4; CCC: 3.5; DCI: PS2.A.1)

or

- When the mass/initial speed is too large, the stopping time will be very long. (SEP: 5.4; CCC: 3.5; DCI: PS2.A.1)
- When the mass/initial speed is too small, the stopping time will be very long. (SEP: 5.4; CCC: 3.5; DCI: PS2.A.1)

and

- reasoning connecting their answers to experiences of the real world (SEP: 5.4; CCC: 3.5)
- reasoning connecting their predictions to the curve fit they selected for their data

What to do: Consider providing the *Data Analysis Reading* as an additional resource on curve fitting. Discuss the simple limit cases for mass and speed first. When discussing the limit cases for force, highlight the following ideas:

- According to the linear (-) curve fit, if we continue increasing the force, there will be a point where the stopping time becomes negative. Encourage students to think about whether negative time makes sense. Encourage students to think about whether it is possible that there is no force acting on the vehicle. They should say no.
- The inverse curve fit suggests that as we continue increasing the force, the time will get really small, but will never be zero. This curve fit also shows that while the force cannot be zero, very small forces will result in very long stopping times.

Building toward: 4.B Use simple limit cases and algebraic thinking to determine whether curve fits of data on the relationship between force, mass, initial speed, and stopping time make sense compared to what is known about the real world. (SEP: 5.4; CCC: 3.5; DCI: PS2.A.1)

Say, *I think we're getting strong evidence to support our models. Let's use these models to make more predictions.*

Make additional predictions. Present **slide N**. Say, *Now that we have data and mathematical models that support the relationship between our independent variables and stopping time, let's see if we can make more accurate predictions.* Use the slide's prompt to elicit 1-2 ideas:

- *What would happen to the stopping time if we doubled the independent variable you investigated?*

Draw connections between the graphs on the slide and students' ideas about the effects on stopping time.

Navigate by forecasting connections to vehicle safety. Before wrapping up, tell students that next time, we will consider how these relationships we figured out might relate to vehicle safety.

End of day 2

8 · NAVIGATE

3 min

MATERIALS: science notebook, *Braking Investigation*

Consider connections between variables and safety. Present **slide O**. Say, *Last time, I asked you to consider how the results of your investigations might relate to vehicle safety. Let's refresh our memory.* Ask students to look back independently at their *Braking Investigation* handout to recall their results. Then have them turn and talk about the slide's prompt:

- *Does increasing the independent variable you investigated make things safer or riskier for people? Why?*

Listen for the following ideas:

- Increasing the speed and mass will make driving less safe because it will increase the stopping time, so the vehicle is more likely to get in a collision.
- Increasing the braking force will make driving safer because it will decrease the stopping time, so the driver might be able to stop in time.
- If the vehicle brakes too hard, it could be dangerous because that could hurt people.

9 · PREDICT EFFECTS ON STOPPING TIME

10 min

MATERIALS: science notebook

Use mathematical thinking. Present **slide P**. Say, *So far, we've considered each independent variable separately, but in real life, these are not separate. A vehicle's mass, speed, and braking force all influence the stopping time. Let's see if we can make predictions when two or more independent variables are changed.*

Start with the slide's first prompt and present a set of examples in which two independent variables are changed, as in the table below. Ask students to write their thinking in their notebook.

Suggested prompt	Sample student response
Consider a car that is 2 times the mass, and traveling at 2 times the speed, but using the same amount of braking force. What would happen to the stopping time?	Stopping time would be 4 times as much.
For those of you who mentioned a number, how did you get to that answer?	Each time something is doubled, the time would double, so first it would be twice as much and then 4 times as much.
Did anyone figure it out a different way?	I multiplied the 2 changes together.

Consider another example, in which the braking force and another variable are doubled. Follow the same routine as with the first example. Listen for students to say they used division. As they share, write the mathematical procedure being described on the board. Make sure everyone can make sense of the calculations being done.

Carry out two practice exercises changing all three variables. Display **slide Q**. Ask students to predict the stopping time if the braking force, mass, and speed are changed. Use the examples on the slide to have them calculate quickly:

- 4 times the mass, 3 times the speed, 2 times the braking force
- 8 times the mass, 3 times the speed, 6 times the braking force

Follow the same routine as above to debrief each answer. If students need more practice, pose this scenario:

- 2 times the mass, 2 times the speed, 8 times the braking force

Then transition to the next activity by using the slide's last prompt:

- *What data would make you more confident about our answers?*

Accept all answers. Say, *I have a simulation that will help us generate data to test our ideas.*

10 · USE A SIMULATION TO GATHER ADDITIONAL EVIDENCE TO TEST A MATHEMATICAL RELATIONSHIP AMONG VARIABLES

12 min

MATERIALS: science notebook, computer with access to https://tinyurl.com/3zwf4d9w, *Braking Simulation Instructions*, https://tinyurl.com/3zwf4d9w

Collect additional data with a simulation. Present **slide R**. Organize the class into groups of 2-3 students. Distribute a laptop computer and a copy of the *Braking Simulation Instructions* to each group.

Braking investigations simulation action:	This simulation is intended to replicate the cart investigations earlier in this lesson. Display https://tinyurl.com/3zwf4d9w and go through an example to orient the class, discussing each of the following points: ● each variable they can manipulate (braking force, mass, speed) with the simulation ● what this model is trying to represent (a system designed to study the role of braking force, mass, and speed on stopping time) ● what they can measure (speed over time, stopping time, represented by delta t)

Give the groups 8 minutes to collect data and 7 minutes to calculate delta t using their results.

11 · MAKE SENSE OF THE RESULTS FROM THE SIMULATION

10 min

MATERIALS: *Braking Simulation Instructions*

Discuss advantages and disadvantages of collecting data with a simulation. Present **slide S**. Use the prompts to elicit 1-2 ideas:

- *How is the data collected in the simulation different from the data collected using the physical carts?*
- *What are the advantages and disadvantages of collecting data with the simulation?* ✱

Listen for the following ideas:

- The simulation allowed us to manipulate the speed, mass, and force with a high level of precision.
- Simulations take place in a virtual environment, which oversimplifies the physical realities of the world.

✱ SUPPORTING STUDENTS IN ENGAGING IN PLANNING AND CARRYING OUT INVESTIGATIONS

Now is a good time to bring up the following Next Generation Science Standards (NGSS) Nature of Science Understanding titled

Discuss mathematical relationships among the variables. Present **slide T**. Draw a table on the board to record students' findings. Invite a group to share one of their results. Write the value for each variable in the table.

Once you have recorded at least five results, use the slide's first prompt to lead a discussion around the mathematical relationship connecting these variables:

- *What relationship have you discovered between mass, speed, and braking force that predicts the stopping time of a vehicle?* ✱

Invite volunteers to share. Listen for mentions of how the variables relate mathematically. Use the results in the table to test the proposed relationship. Ask whether other groups found the same or a different relationship.

Then pose the second prompt:

- *How can we represent the relationship we have identified as an equation?*

Guide the class through writing out the equation in words first. For example:
Stopping time = mass * initial speed / braking force

Say, *Equations are often made smaller by using variable symbols instead of words. Which words can we replace with symbols in our equation?*

The final equation should look something like this:

$$\Delta t = \frac{m * \Delta \text{ in speed}}{F}$$

Discuss the units of each variable, then ask: *Are we saying that for any combination of values for force, mass, and speed, we can actually predict the time in seconds that it will take a car to stop? This seems like it would be useful. But if this is true, does it hold for values that represent the actual mass and speed of a real vehicle?* Accept all answers. Suggest we try using our mathematical model to solve a couple of problems with real numbers.

Scientific Investigations Use a Variety of Methods (Appendix H). The relevant understanding element reads:

- Science investigations use diverse methods and do not always use the same set of procedures to obtain data.

While discussing this prompt, explore this idea by drawing comparisons between the data collected using the physical setup and using the simulation.

✱ **ATTENDING TO EQUITY**

Universal Design for Learning: Providing opportunities for students to employ mathematical thinking and create equations that represent relationships between variables they are investigating can promote equal participation among all students, fostering increased engagement.

12 · TEST THE MATHEMATICAL MODEL

5 min

MATERIALS: whiteboard, dry erase markers

Test the mathematical model with real-life examples. Present **slide U**. Say, *Now that we have a mathematical model to make sense of the relationship between speed, mass, force, and time, let's see which model best helps us predict the outcome of these scenarios.* Review the scenarios:

- ***Scenario A:** A car of 3,000 kg is traveling at 20 m/s and the driver applies a braking force of 15,000 N.*

- ***Scenario B:*** *A car of 2,500 kg is traveling at 30 m/s and the driver needs to stop in 1 second. What is the minimum braking force needed to do this?*

Give students a couple of minutes to calculate their answers on a whiteboard.

13 · NAVIGATE

5 min

MATERIALS: *Braking Exit Ticket*

Complete an exit ticket. Present **slide V**. Distribute the *Braking Exit Ticket* to each student. Use the slide's prompts to orient them to the exit ticket:

- *On your handout, make predictions about how the speed versus time graph will change if the braking force is tripled.*
- *Explain your reasoning using the mathematical model we developed.*

Give the class the remaining time to complete this exit ticket. Collect it at the end of the period.

Navigate by motivating putting the pieces together. Say, *We've figured out a lot, and we've been using a lot of new models to represent our ideas. Let's put the pieces together next time to see what progress we've made on answering our questions about vehicle collisions.*

ASSESSMENT OPPORTUNITY	**What to look for/listen for in the moment:** Students should use the speed-time graph to claim that tripling the braking force will reduce the stopping time to one third the original stopping time, and they explain this using the equation relating force, mass, initial speed, and stopping time. (SEP: 5.2, 6.1; CCC: 3.5; DCI: PS2.A.1) **What to do:** It may be difficult to distinguish between difficulty using the mathematical representation versus difficulty grasping the relationship between braking force and stopping time. To discern between these, encourage students to describe their predictions in their own words and see whether these align with their explanations for the last prompt on the handout. If students are curious to find the answer to the problem presented in the handout, suggest they use https://tinyurl.com/3zwf4d9w to test their predictions. If the answers to this exit ticket indicate that a significant portion of the class continues to struggle with using the mathematical representation, consider spending extra time working with the model and simulation to help

students get comfortable with the relationship, or review the connections between the equation and the graphs.

Building toward: 4.A.2 Use mathematical representations of the relationship between mass, initial speed, force, and stopping time and algebraic thinking to make a quantitative claim that predicts how much changing the braking force will affect the time it takes a vehicle to stop. (SEP: 5.2, 6.1; CCC: 3.5; DCI: PS2.A.1)

Additional Lesson 4 Teacher Guidance

SUPPORTING STUDENTS IN MAKING CONNECTIONS IN MATH	**Algebra** • **CCSS.MATH.CONTENT.HS.A-SSE.1b Seeing Structure in Expressions:** Interpret the structure of expressions. Interpret complicated expressions by viewing one or more of their parts as a single entity. • **CCSS.MATH.CONTENT.HS.A-CED.2 Creating Equations:** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. • **CCSS.MATH.CONTENT.HS.A-CED.4 Creating Equations:** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. **Functions** • **CCSS.MATH.CONTENT.HS.F-IF.4 Interpreting Functions:** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. • **CCSS.MATH.CONTENT.HS.F-LE.1a Linear, Quadratic, and Exponential Models:** Distinguish between situations that can be modeled with linear functions and with exponential functions. Students interpret the meaning of the expressions that are part of linear and nonlinear functions when they make sense of the results from the Braking Investigation. Along with this sensemaking, they discuss whether the results are better represented with an exponential or a linear model. Students develop a mathematical relationship by using results gathered with a simulation to represent the relationship between force, initial speed, mass, and time to come to a stop (t = m * speed / F).