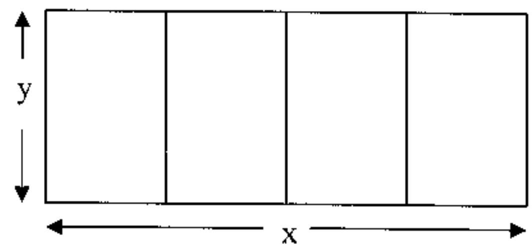
Optimization Area Problems

Name________________________________

1) A farmer wants to construct 4 equal sized rectangular corrals to house his chickens. He has a total of 1000 feet of fencing. Find the dimensions of the maximum area he can enclose. What is that area?



$5y + 2x = 1000$

$y = 200 - \frac{2}{5}x$

$A = xy$

$A = x(200 - \frac{2}{5}x)$

$A = 200x - \frac{2}{5}x^2$

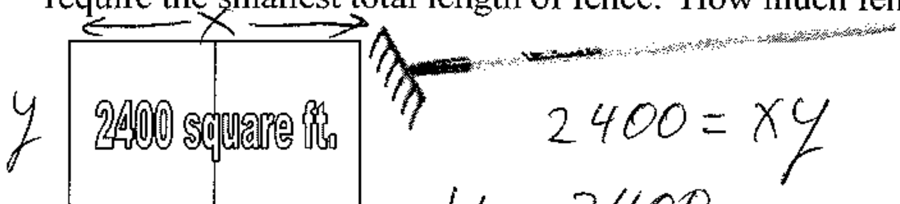
$A' = 200 - \frac{4}{5}x$

$\frac{4}{5}x = 200$

$\boxed{x = 250 \quad y = 100}$

$A'' = -$

2) A 2400 square foot area rectangular vegetable garden is to be enclosed by a fence and divided into two equal parts by another fence parallel to two sides. What dimensions will require the smallest total length of fence. How much fence is needed?



$2400 = xy$

$y = \frac{2400}{x}$

$F = 3y + 2x$

$F = 3\left(\frac{2400}{x}\right) + 2x$

$F = \frac{7200}{x} + 2x$

$F' = -\frac{7200}{x^2} + 2$

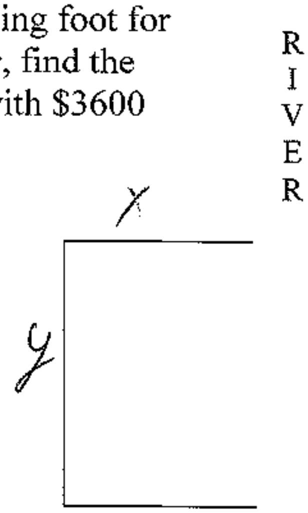
$2x^2 = 7200$

$\boxed{x = 60 \quad y = 40}$

$F'' = \frac{14400}{x^3} = +$

60

3) A rectangular field is to be fenced off along the bank of a river; no fence is required along the river. If the material for the fence cost $8 per running foot for the two ends and $12 per running foot for the side parallel to the river, find the dimensions of the field of largest possible area that can be enclosed with $3600 worth of fence.



$$3600 = 12y + 8(2x) \qquad A = xy$$

$$\frac{3600 - 12y}{16} = x$$

$$225 - .75y = x$$

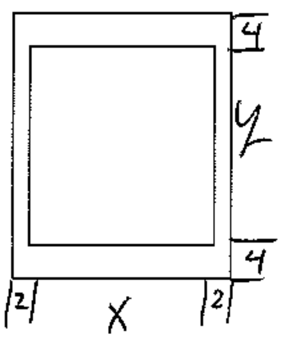
$$A = (225 - .75y)y$$

$$A = 225y - .75y^2$$

$$A' = 225 - 1.5y$$

$$\boxed{\begin{aligned} y &= 150 \\ x &= 112.5 \end{aligned}}$$

4) Determine the minimum area of a poster that will contain 50 square inches of printed material with 4 inch margins on the top and bottom and 2 inch margins on the right and left sides.



$$A = (x+4)(y+8)$$

$$50 = xy \qquad \frac{50}{x} = y$$

$$A = (x+4)\left(\frac{50}{x} + 8\right)$$

$$A = 50 + 8x + \frac{200}{x} + 32$$

$$A' = 8 - \frac{200}{x^2}$$

$$\frac{200}{x^2} = 8$$

$$\boxed{\begin{aligned} x &= 5 \\ y &= 10 \end{aligned}}$$

$$A'' = \frac{400}{x^3}$$