Chapter 6

Impulse and Linear Momentum

6.1 Mass accounting

OALG 6.1.1 Observe and explain

Equipment: Plastic bottle with water, a scale

Take a plastic bottle full of water and put it on a scale. Record its mass and take a photo of the bottle. Then, place the bottle in the freezer until the water inside is completely frozen solid. Next, take the bottle out and place it on the scale again.

a. What happened to the mass of the bottle? What happened to its volume (use the photo you took before to compare)?

b. Can you explain the change in mass (if the mass did change?

c. How can you test your explanation?

OALG 6.1.2 Represent and reason

a. Imagine that you have 10 oranges in a bag. You give 3 to your friend Darla and 2 to your friend Paolo. What happened to the mass of the oranges when considering just the oranges in the bag as the system? Then, repeat the same analysis for the system that contains the bag, oranges, and your friends. Can you say that the mass of the system is constant in both cases? Can the mass disappear?

b. Now, read and interrogate Section 6.1 of the textbook (pay attention to the difference between "conserved" and "constant") and draw two bar charts for the process described above. Bar chart 1 considers the bag + oranges as the system, and bar chart 2 considers the bag + oranges + your friends as the system. Are the two bar charts consistent with each other?

c. What is the difference between the terms "constant" and "conserved"? Can you say that the mass of the oranges is constant in all experiments above? Can you say that it is conserved? Explain.

OALG 6.1.3 Reason

Imagine, you have a few briquettes of charcoal. You burn them.

a. Is the smaller mass of burned charcoal embers compared to the mass of the starting briquettes a violation of mass conservation? Explain.

b. Describe a testing experiment you could perform to test whether mass is conserved (but not necessarily constant) or whether it is not conserved in a combustion process. Make sure you describe both predicted outcomes: 1. What should happen in your experiment if mass is conserved? 2. What should happen in your experiment if mass is not conserved?

OALG 6.1.4 Read and interrogate

Read and interrogate Section 6.1 in the textbook and answer Review Question 6.1.

6.2 Linear momentum

OALG 6.2.1. Observe and find a pattern

Observe the following four videos of the experiments with a two-object system. Two carts (the system consists of *both* carts) move on a dynamics track:

a. Cart A (500 g), moving right at constant 0.37-m/s speed, hits identical cart B (500 g) that is stationary. Cart A stops and cart B starts moving at the speed of 0.37 m/s to the right. [https://youtu.be/Ikc03NBA11U]

b. Cart A loaded with a block (total mass of the cart with the block is 1470 g), and moving right at 0.31 m/s hits stationary cart B (mass 500 g). After the collision, both carts move right: cart B at the speed of 0.47 m/s and cart A at the speed of 0.15 m/s. [https://youtu.be/HbMBplGL3Zo]

c. Cart A (500 g), with Velcro attached to its front, moves right at 0.31 m/s. Identical cart B (500 g) moves left at constant speed 0.31 m/s. The carts collide, stick together, and stop [https://youtu.be/m9JO6LrZ1Mk]

d. Repeat experiment **c.** but this time cart A is loaded (with a block, total mass is 1470 g) moves right at constant speed of 0.35 m/s. Cart B (500 g) moves left at constant speed of 0.35 m/s. After the collision, both carts stick together and travel right at the speed of 0.17 m/s. [https://youtu.be/tciBA4w4ZiU]

For *each* experiment, sketch the process before the collision and after the collision. Create a table like the one that follows to help you determine if anything is the same before and after the collision for the two-cart system. (Use the table below to look for quantities.)

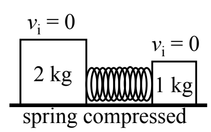
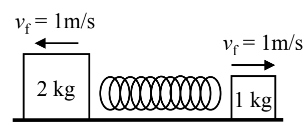
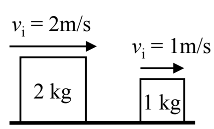
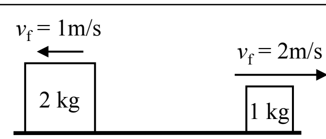
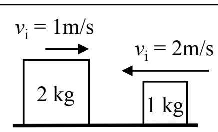
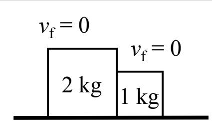
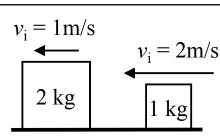
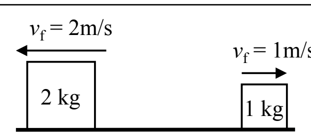
Possible physical quantity→	**Mass** m	**Speed** v	**Velocity** $\vec{v}$	**Mass times speed** mv	**Mass times x-velocity component** mv_x
Cart A (*before* collision)					
Cart B (*before* collision)					
Combined physical quantity for Cart A & Cart B (add line 1 and line 2 for each quantity)					
Cart A (*after* collision)					
Cart B (*after* collision)					
Combined physical quantity for Cart A & Cart B (add line					

1 and line 2 for each quantity)					

After you come up with a physical quantity that is the same before and after each collision, decide whether this quantity remains constant in all four experiments. Then, read and interrogate Observational Experiment Table 6.1 on page 149 in the textbook. Did you come up with the same quantity?

OALG 6.2.2 Reason

The initial and final situations for four processes are shown below. Which ones *are* possible according to your knowledge of the physical quantity you came up with in OALG 6.2.1? Explain your reasons. The numbers indicate the masses and the speeds of the blocks.

	Initial state	Final state
a.		
b.		
c.		
d.		

6.2.3 Test your idea

a. Observe the following experiment [https://youtu.be/EapnaC8zfLI].

b. Use the data and from the experiment and your knowledge of the physical quantity you invented in Activity 6.2.1 to do the following: predict the speed of the carts and the shape of the velocity-*vs*-time graph after they collide and stick together.

c. Watch the video of the carts after the collision [https://youtu.be/x57a8td-W6E] and compare your prediction to the outcome. Do you need to revise your idea or the additional assumptions you made?

d. Read and interrogate Section 6.2 in the textbook and answer Review Question 6.2. Explain in your own words, what it means when we say "The linear momentum of an isolated system is constant". Clarify the terms linear momentum, system and constancy.

OALG 6.2.4 Test your idea

The photo below shows Bor and Eugenia on rollerblades. Your goal is to use the physical quantity you've come up with in the previous experiment to make a prediction of the ratio of Bor's and Eugenia's speeds after Eugenia pushes Bor. Bor's mass is 70 kg and Eugenia's mass is 54 kg.

a. What is the physical quantity (that stays the same for a system before and after a collision) that you are using to make this prediction?

b. Use this quantity and information provided to make a numerical prediction of the ratio of Bor's and Eugenia's speeds after Eugenia pushes Bor.

c. Run the video [https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-phys-egv2e-alg-6-2-3] and take appropriate measurements by stepping the video frame by frame. Record and tabulate your data appropriately. Include estimates of the uncertainties of your measured quantities.

d. Make a judgment about whether the experimental measurement agrees with your prediction for the ratio of the two speeds. What can you say about your confidence in the new physical quantity that you used to make the prediction?

OALG 6.2.5 Practice

Answer Questions 4, 5, 6, and 7 on page 169 and solve Problems 3-5, 7 and 8.

6.3 Impulse and momentum

OALG 6.3.1 Derive

a. Suppose you push a 1 kg cart with your hand, exerting a constant force of 5 N for a time of 5 seconds. If the cart starts from rest and there is no friction, what is the final speed of the cart (after the 5 seconds have elapsed)?

b. Now do the same thing in symbols: Your hand pushes horizontally on a cart of mass *m*, exerting a force $\vec{F}_{\text{H on C}}$ for a time interval ($t_f - t_i$). The forces exerted downward on the cart by Earth and upward on the cart by the track are balanced. The track is very smooth, so we assume that $\vec{F}_{\text{H on C}}$ is the only force exerted in the *horizontal* direction on the cart. The cart is initially moving at velocity $\vec{v}_{\text{Ci}}$ at the clock reading t_i and, after the pushing, is moving at velocity $\vec{v}_{\text{Cf}}$ at the clock reading t_f. Write an expression for $\vec{v}_{\text{Cf}}$ in terms of $\vec{v}_{\text{Ci}}$, the unbalanced force $\vec{F}_{\text{H on C}}$, and the time interval ($t_f - t_i$).

c. Rearrange your expression from **b.** to show that:

$$\vec{F}_{\text{H on C}}\left(t_{\text{f}} - t_{\text{i}}\right) = m_{\text{C}}\vec{v}_{\text{Cf}} - m_{\text{C}}\vec{v}_{\text{Ci}}$$

The term on the left $\vec{F}_{\text{H on C}}\left(t_{\text{f}} - t_{\text{i}}\right)$ is called the *impulse* $\vec{J}$ due to the external force $\vec{F}_{\text{H on C}}$ during that time interval. The term on the right $m_{\text{C}}\vec{v}_{\text{Cf}} - m_{\text{C}}\vec{v}_{\text{Ci}}$ is called the *change in the momentum* of the cart.

d. Suppose that friction is not negligible. How would you modify the expression for the impulse on the cart to include both the effect of the hand and of friction?

e. Consider a *new* situation. Suppose a cart is already moving (with no friction present); the cart's forward velocity is decreased due to the force exerted by a hand pushing lightly back on it. How would you write the expression for the impulse due to that force?

f. Read and interrogate section 6.3 in the textbook and answer Review Question 6.3. Which of Newton's laws help us understand momentum constancy in an isolated system? Explain.

OALG 6.3.2 Test your ideas

a. Use your knowledge of impulse to explain the motion of the carts (specifically, identify the object that exerts the impulse on the carts) and to predict the shape of the velocity-vs-time graphs in the following experiments [https://youtu.be/LEHPPXRwLX4]

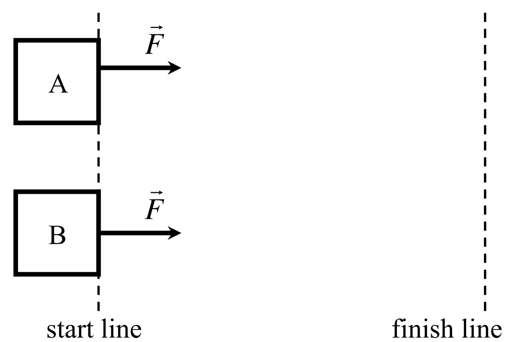
b. Watch the video at [https://youtu.be/AB2F3yukAkE] to compare your prediction to the outcome. Discuss the differences if any occurred.

c. Use the graphs and other data from the video above to estimate the impulse and force that the air exerts on the carts. Check if this force is the same in all experiments. Should this force be the same or different?

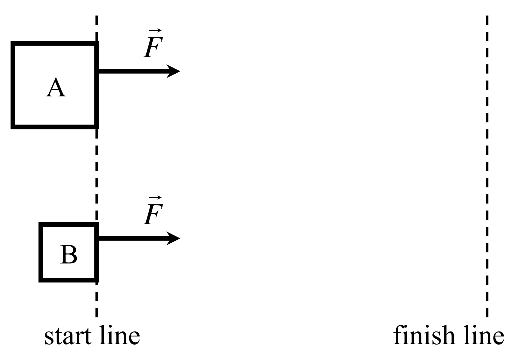
OALG 6.3.3. Reason

Answer the following questions.

a. You continuously push equally hard on identical blocks A and B from the start line to the finish line. Block B is initially at rest whereas block A is initially moving right. Which block has the greater change in momentum in moving from the start to the finish line? Explain your answer.



b. Suppose that both blocks in the previous problem start at rest, but that block A has four times as much mass as block B. Which block has the greater change in momentum in going from the start to the finish line? The blocks are pushed with equal-magnitude forces.



OALG 6.3.4 Represent and reason

Consider the following scenario: A ball falls toward the floor and rebounds upward.

a. Draw an arrow representing its velocity just before touching the floor; another arrow for its velocity just after leaving contact with the floor on the rebound; and a velocity change vector.

b. What is the direction of the net average impulse (net average force times the time interval during which this force is exerted) that the floor exerts on the ball?

c. Construct a force diagram on your whiteboard for the ball when it is in contact with the floor. Make the force arrows correct relative magnitudes (consistent with the velocity change).

OALG 6.3.5 Practice

Answer Questions 1, 2, 12, 13, 14 and 26 on pages 169-170 and solve Problems 16 and 21 – 26.

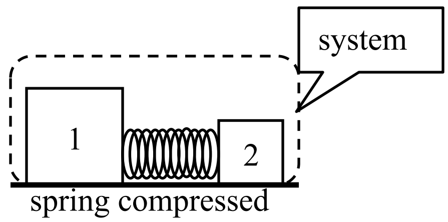
6.4 The generalized impulse-momentum principle

OALG 6.4.1 Read and interrogate

Section 6.4 in the textbook (pages 156 – 159) summarizes what you have learned so far about momentum and impulse. This section also introduces a new tool to analyze processes involving momentum and impulse. Read and interrogate this section and attempt all worked examples before you read the solutions. Notice that in order to use a bar chart to write down the mathematical description of a particular process, you first need to (1) identify the system; (2) identify initial and final states of the process you wish to analyze, and finally (3) draw the bar chart to represent the process. Remember that momentum is a vector quantity and has direction. Thus, the direction needs to be reflected in the bar chart.

OALG 6.4.2 Represent and Reason

Blocks 1 and 2 rest on a horizontal frictionless surface. A compressed spring of negligible mass separates the blocks. Block 1 has twice the mass of block 2. When the spring is released, the blocks are pushed apart. Answer the questions below concerning this process.



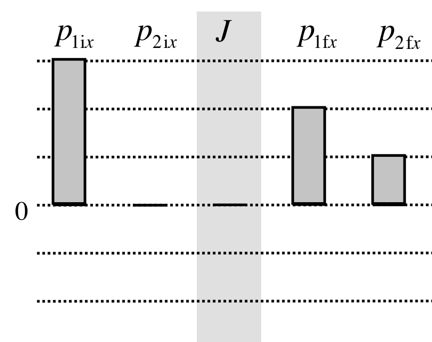
a. Compare the momentum magnitudes of block 1 and 2 after the spring is released.

b. Compare the combined momentum of both blocks before the spring is released and the combined momentum after the spring is released.

c. Compare the speed of block 1 to that of block 2 after the spring is released.

d. Draw a qualitative impulse-momentum bar chart to represent the process for a system consisting of the spring and two blocks. What are the initial and final states? Do any other objects exert external forces on the objects included in the system—especially in the horizontal direction?

OALG 6.4.3 Bar chart Jeopardy



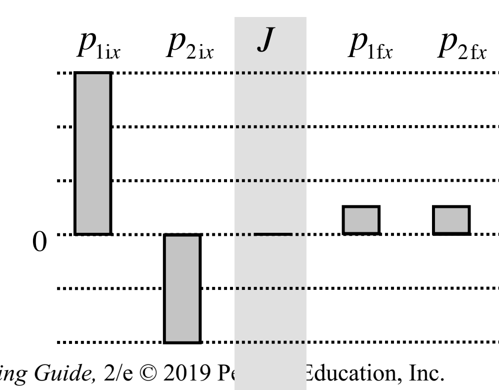
The bar chart on the right represents an impulse-momentum process.

a. Describe in words and sketch a physical process that the bar chart might describe. Be specific.

b. Describe what would happen to p_{1fx} if p_{2fx} was positive three units instead of positive one unit.

c. Describe what would happen to p_{1fx} if p_{2fx} was positive four units instead of positive one unit.

OALG 6.4.4 Bar chart Jeopardy



The bar chart on the right represents an impulse-momentum process.

a. Describe in words and sketch a real-life physical process that the bar chart might describe. Be specific.

b. Describe what would happen to p_{1fx} and p_{2fx} if p_{2ix} was negative three units instead of negative two units. Assume that the final momenta of the two objects are equal but different from before.

c. Describe what would happen to p_{1fx} and p_{2fx} if p_{2ix} was positive one unit instead of negative two units. Assume that the final momenta of the two objects are equal but different from before.

OALG 6.4.5 Observe and represent

a. Watch the video [https://mediaplayer.pearsoncmg.com/assets/_frames.true/secs-egv2e-knocking-a-plank-with-a-ball]. Two balls have approximately the same mass.

b. Describe what you observe. What is the difference between the two balls? What is the difference in the two experiments with the plank?

c. Draw a bar chart for each of the collisions of the balls with the plank. First, choose the plank and the ball as the system and then repeat when the plank is the system. The initial state is right before the ball hits the plank and the final state is right after the ball hits the plank.

d. Use the bar charts to explain the difference in the outcomes of the collisions. If you are having trouble, consult Example 6.3 in the textbook on page 158.

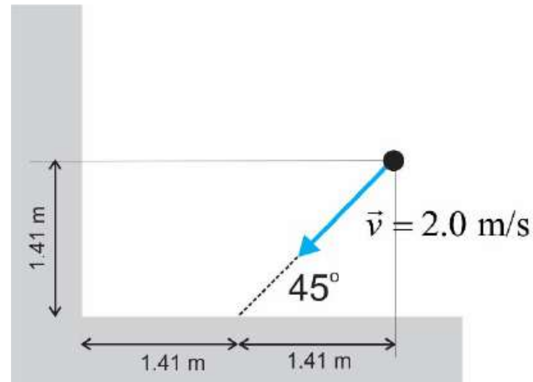
OALG 6.4.6 Represent and reason

a. Watch Eugenia wearing rollerblades and holding a ball [https://youtu.be/fx4XxAh8izc]. She pushes off the floor once and continues rolling at constant speed across the floor. Then she drops the ball. Describe everything you can about momentum in this process using words, bar charts, and mathematics. Choose Eugenia and the ball as the system and initial state is when Eugenia is rolling while holding the ball and the final state is right after she lets go of the ball. Does dropping the ball change Eugenia's speed? What if, instead of the basketball, she dropped object that is 1/10 of her mass in the same manner as before? Would her speed change? To answer these questions, remember that momentum is a vector quantity.

b. Watch Eugenia wearing rollerblades and holding a heavy medicine ball [https://youtu.be/kwWTb0yWIlE]. She pushes the ball away from her. Describe everything you can about momentum in this process using words, bar charts, and mathematics. Decide what your system is and what the initial and final states are.

OALG 6.4.7 Represent and reason

A puck is moving on horizontal ice toward a corner made of two concrete walls. The figure shows the top view of the puck at time $t = 0$. The speed of the puck is 2 m/s. Assume that the puck rebounds from the wall with the same speed as before the collision.

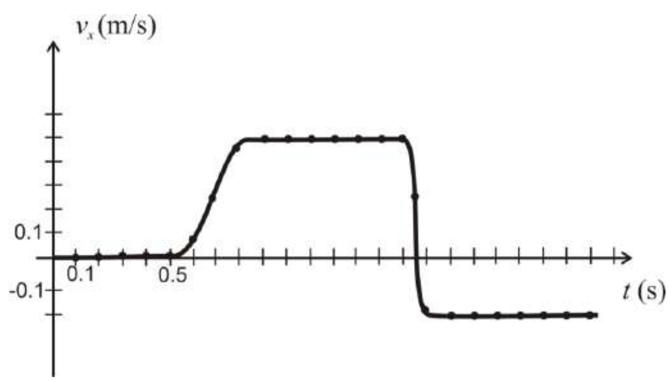


a. Draw v_x-versus-time and v_y-versus-time graphs for the motion of the puck during the time interval $0 \le t \le 3.0$ s.

b. For each collision with each wall, draw the momentum bar charts (before/after the collision, separate for the x-direction and y-direction), taking the puck as a system.

OALG 6.4.8 Reason

Elizabeth pushes cart A, which is initially at rest, along a horizontal track toward cart B, which is also initially at rest. Before Elizabeth started pushing the cart, Daniel had started recording the motion of cart A using a motion detector. He obtained the following velocity-versus-time graph for cart A:



a. Estimate the clock reading when the carts collided.

b. Based on the data from the graph, can you estimate the ratio of the average force exerted by Elizabeth on cart A and the force exerted by cart A on cart B during the collision? If you think you can, determine this ratio. If you think you cannot do so, what additional data would you need to be able to determine this ratio?

c. Elizabeth says that the frictional forces exerted on cart A are negligible. Describe which features of the graph support the validity of her assumption.

OALG 6.4.9 Read and interrogate

Use the material in Sections 6.1-6.4 of the textbook to explain the difference between the terms *conserved* and *constant*. Describe an example of when a quantity is conserved but not constant in a particular process. Think of money – can it be treated as a conserved quantity?

OALG 6.4.10 Practice

Solve problems 27- 32 and 34 on page 172 in the textbook.

6.5 Skills for analyzing problems using impulse and momentum

OALG 6.5.2 Problem solving strategy

Solve the following problem using the steps of the problem solving strategy outlined below. Then compare your work to the solution in Worked Example 6.4. After you correct certain parts if necessary, solve the *Try It Yourself* problem at the end of the example.

A 0.020-kg bullet traveling horizontally at 250 m/s embeds in a 1.0-kg block of wood resting on a table. Determine the speed of the bullet and wood block together immediately after the bullet embeds in the block.

Sketch and translate • Sketch the initial and final states and include appropriate coordinate axes. Label the sketches with the known information and identify the unknowns. Decide on an object of reference. • Choose a system based on the quantity you are interested in. For example, a multi-object isolated system to determine the velocity of an object, or a single-object non-isolated system to determine an impulse or force. Sometimes it is better to decide what is	

in your system and then identify initial and final states.	
Simplify and diagram • Determine if there are any external impulses exerted on the system. Drawing a force diagram could help determine the external forces and their directions. • Draw an impulse-momentum bar chart for the system for the chosen direction(s) to help you understand the situation, formulate a mathematical representation of the process, and evaluate your results.	
Represent mathematically • Use the bar chart to apply the generalized impulse-momentum principle along the chosen axis. Each nonzero bar becomes a nonzero term in the equation. The orientation of the bar determines the sign in front of the corresponding term in the equation. • Remember that momentum and impulse are vector quantities, so include the plus or minus signs of the components based on the chosen coordinate system.	
Solve and evaluate • Insert the known information to determine the unknown quantity. • Check if your answer is reasonable with respect to sign, unit (s), and magnitude. Also make sure your answer is valid for limiting cases, such as objects of very small or very large mass.	

OALG 6.5.2 Equation Jeopardy 1

Devise a problem that is consistent with the mathematical description of a process shown below. Multiple processes may be described by the equation.

$$(24\ \text{kg})(-2.0\ \text{m/s}) + (30\ \text{kg})(+3.0\ \text{m/s}) = (24\ \text{kg} + 30\ \text{kg})v$$

OALG 6.5.3 Equation Jeopardy 2

Devise a problem that is consistent with the mathematical descriptions of a process shown below. Multiple processes may be described by the equations.

$$F\left(t_{\mathrm{f}}-t_{\mathrm{i}}\right)=(0.010\ \mathrm{kg})\left[(+100\ \mathrm{m/s})-(+300\ \mathrm{m/s})\right]$$

where

$$\left(t_{\mathrm{f}}-t_{\mathrm{i}}\right)=\frac{(0.040\mathrm{m}-0)}{\dfrac{(+100\mathrm{m/s})+(+300\mathrm{m/s})}{2}}$$

OALG 6.5.4 Evaluate the solution

The problem: A 0.40-kg bundle of unexploded fireworks moves horizontally to the right at speed 2.0 m/s. It creates a small explosion, which breaks the bundle into two pieces. The first piece with 10% of the mass moves right at 40 m/s. What is the velocity of the remaining piece?

Proposed solution:

$$(0.40\mathrm{kg})(2.0\mathrm{m/s})=(0.04\mathrm{kg})(40\mathrm{m/s})+(0.40\mathrm{kg})v$$

or

$$v=\frac{(0.80\mathrm{kg}\cdot\mathrm{m/s}-1.6\mathrm{kg}\cdot\mathrm{m/s})}{(0.40\ \mathrm{kg})}=2\mathrm{m/s}$$

a. Identify any missing elements or errors in the solution.

b. Provide a corrected solution if you find missing elements or errors.

OALG 6.5.5 Evaluate the solution

The problem: A 2000-kg pickup truck traveling at 20 m/s collides with a stationary 1000-kg compact car. The vehicles lock together and skid on a level surface for 16 m until stopping. Determine the coefficient of kinetic friction between the tires and the road.

Proposed solution:

Part I: The collision

Momentum conservation: $(2000\text{kg})(20\text{m/s}) = (3000\text{kg})v$

$$v = 13.3\text{m/s}$$

Part II: Skidding to a stop

Stopping time: $(t_\text{f} - t_\text{i}) = \dfrac{(x_\text{f} - x_\text{i})}{v} = \dfrac{16\text{ m}}{13.3\text{m/s}} = 1.2\text{s}$

Stopping force: $f_\text{k} = \dfrac{(mv_\text{f} - mv_\text{i})}{(t_\text{f} - t_\text{i})} = \dfrac{(2000\text{ kg})(20\text{ m/s})}{(1.2\text{ s})} = 33{,}300\text{ N}$

Coefficient of kinetic friction: $\mu_\text{k} = \dfrac{f_\text{k}}{N} = \dfrac{f_\text{k}}{mg} = \dfrac{(33{,}300\text{ N})}{(3000\text{ kg})(9.8\text{ N/kg})} = 1.1$

a. Identify any missing elements or errors in the solution.

b. Provide a corrected solution if you find missing elements or errors.

OALG 6.5.6 Reason

a. Describe a situation where the sum of the forces exerted on a system by other objects in the horizontal direction is not zero but momentum is constant in the vertical direction.

b. Describe a situation where the sum of the forces in the vertical direction is not zero but momentum is constant in the horizontal direction.

OALG 6.5.7 Reason

For each situation described below, indicate if it is more appropriate to use i. the generalized impulse-momentum principle or ii. the momentum constancy equation to solve the problem. Explain your choices, but do not solve the problems.

a. Estimate the speed of an initially at rest 800-kg rocket sled on a horizontal frictionless surface after its engine fires with a thrust of 2.0×10^4 N for 10 s.

b. An 1800-kg rocket sled with a 20-kg cannon ball initially travels on a horizontal surface at speed 80 m/s relative to the ground. What is the sled's speed after it ejects the cannon ball in the backward direction at a speed of –40 m/s relative to the ground?

c. A 6-kg bowling ball moving at 3 m/s hits a single 2-kg pin, knocking it forward at 6 m/s. What is the speed of the bowling ball after the collision?

d. A 6-kg bowling ball moving at 3 m/s hits a single 2-kg pin, knocking it forward at 6 m/s. If the collision took 0.050 s, what average force was exerted on the pin?

OALG 6.5.8 Argue

How would you convince your friend that an air bag in a car can save a passenger's life during a head-on collision? Try making some sort of side-by-side comparison (a collision with or without an air bag) on your whiteboard. Use the Internet to obtain data and make as many representations as you need (equations, bar charts, words, etc.).

OALG 6.5.9 Practice

Solve problems 44, 47 and 49 on pages 172-173 and problems 68, 72, 73, and 74 on page 174 in the textbook.

6.6 Jet propulsion

OALG 6.6.1 Observe and analyze

Observe the video of a balloon cart [https://youtu.be/4u0hWnmOYuU]. Draw separate bar charts that describe the process when you take the cart and the balloon (with air in it) as the system and when you take only the cart as the system. Choose initial and final states and discuss your assumptions.

OALG 6.6.2 Read and interrogate

Read and interrogate Section 6.6 in the textbook and answer Review Question 6.6.

OALG 6.6.3 Real-life application

A 3000-kg spaceship travels toward the Moon at a speed of 1.5×10^4 m/s. The captain of the ship wishes to change direction by 5°. Rockets eject fuel at a speed of 3.0×10^6 m/s perpendicular to the ship's initial direction. Determine the mass of fuel that must be ejected to change course as the captain wishes. What assumptions did you make?

6.7 Collisions in two dimensions

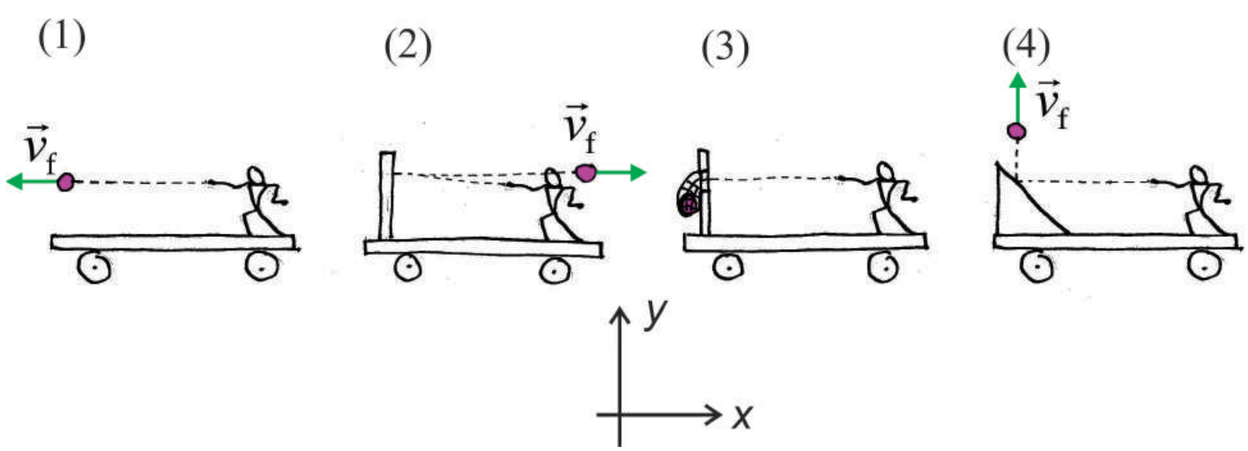
OALG 6.7.1 Real-life application

Decide who is at fault in this car accident: You are in an automobile accident and believe the other car was traveling faster than indicated by its driver. Before the collision, your 1400-kg car was traveling south at 35 mph. The other 1800-kg car was traveling west at an unknown speed (the driver claims 30 mph). After the collision, the two cars became locked together and traveled in a direction 25° south of west. Was the other car exceeding the 35-mph speed limit?

If you are having trouble, study Example 6.7 on page 165 in the textbook.

OALG 6.7.2 Apply

Shawn (mass M) performs four experiments on four carts. All carts have equal masses $2M$. The carts are on a frictionless track and initially at rest. In all experiments, Shawn is standing on the cart and throws a ball of mass *m* horizontally in the negative *x*-direction, always with the same initial velocity $\vec{v}_{\mathrm{i}}$ (initial state). The final states are shown in the figures below. Assume $|v_{\mathrm{i}}| = |v_{\mathrm{f}}|$ in experiments 1, 2, and 4, and the final velocity of the ball in experiment 3 is zero.



a. Draw an impulse-momentum bar chart for each experiment. The system is Shawn, the cart, and the ball. Show separate bars for each object of the system.

b. For each experiment, derive an expression for the final speed of the cart in terms of relevant parameters. Discuss the direction of the final velocity.

OALG 6.7.3 Practice

Solve Problems 58, 61, and 62 in the textbook on page 173. If you are having trouble, read and interrogate Section 6.7 in the textbook.