Chapter 9

Rotational Motion

9.1 Rotational kinematics

OALG 9.1.1 Observe and find a pattern

Equipment: 2 1-meter sticks or 1 2-meter stick.

For this activity, you need to collect all of your housemates and go outside (you will need 5 people in total for this experiment). Use two meter sticks (or any other sticks about 1-m long) connected together to form a 2-m long stick, or use a 2-m stick. Have one group member hold one end of the meter stick and stand at the same position at all times. The second group member holds it at the 50-cm mark, the third one at the 1-m mark, the fourth one at the 150-cm mark, and the last one at the end of the second stick (the 2-m mark). The four non-fixed group members need to move so that the one member holding the 2-m end of the stick runs in a circle at a comfortable constant speed.

a. Observe the motions of these four moving group members. What do you notice? Discuss with your group.

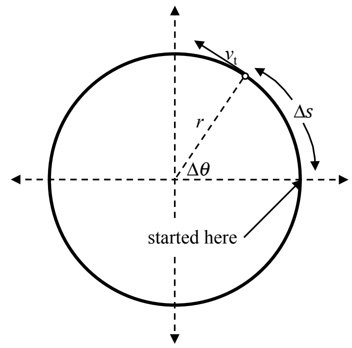
b. What physical quantities characterizing motion are different for the four group members? Discuss with your group.

c. What quantities are the same?

d. Compare your answers to Fgure 9.7 on page 255 in the textbook. How does it help you with the answers?

OALG 9.1.2 Describe

Equipment: Rotating platform with objects fixed to it; the platform needs to move with acceleration. You can use a record player if you have one or a bicycle wheel.

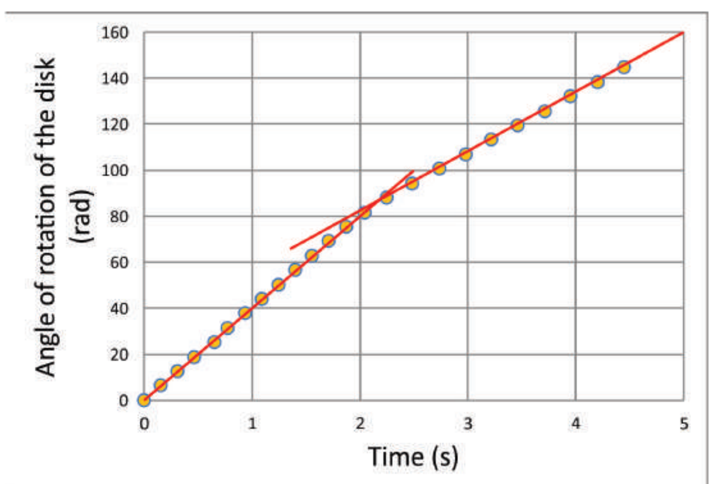


The goal of this activity is to learn to relate variables for linear motion with variables for rotational motion. Imagine we have a disk that spins in a horizontal circle, as shown at right (you can also imagine a bicycle wheel). The rate of rotation may increase or decrease. We will follow the motion of a point on the disk marked with the small open circle. How is the angle $\Delta\theta$ swept out by this point related to the arclength Δs and radius r? How is the rotational velocity ω of this point related to the tangential velocity v_t of the point and the radius r? How is the rotational acceleration α of this point related to the tangential acceleration a_t of the point and radius r? (For help, read and interrogate Section 9.1 in the textbook.)

Angular variable	**Define in words**	**Mathematical definition**	**Relation to variables describing linear motion**
θ	Amount of angle swept out starting from the x-axis, rotating counterclockwise.		$\theta = \dfrac{\Delta s}{r}$
ω		$\omega = \dfrac{\Delta\theta}{\Delta t}$	
α			

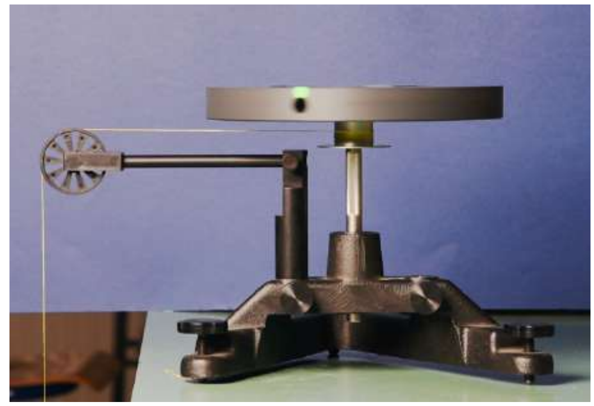
OALG 9.1.3 Represent and reason

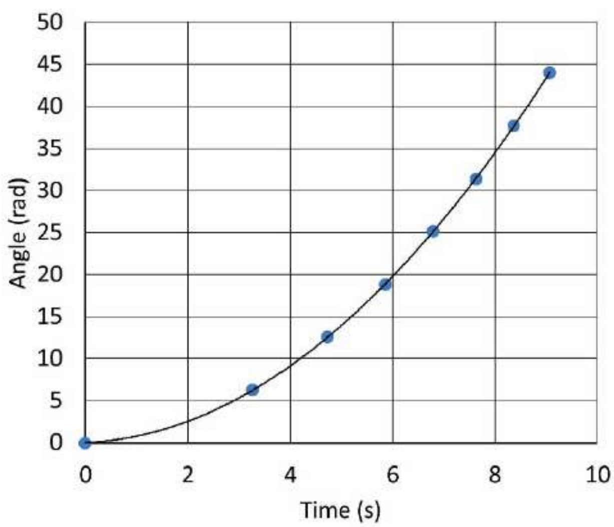
On the right, you see data collected from observing a rotating cylinder. Tell all you can about the graph. What happened to the cylinder? How do you know?

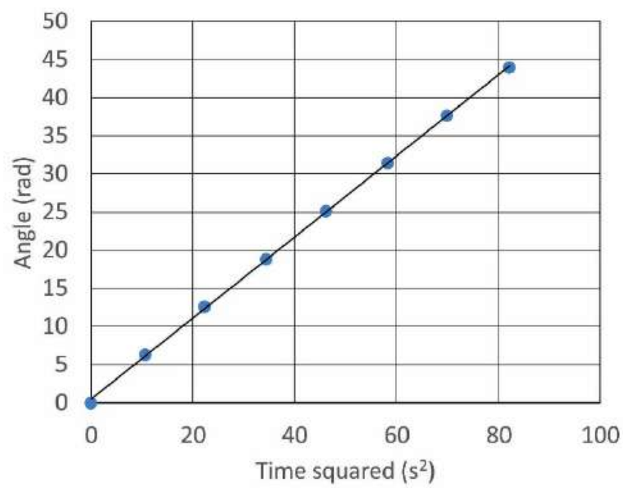


OALG 9.1.4 Observe and analyze

A disk can rotate with almost no friction around a vertical axis. You wind a string around the axis. While the string exerts a constant force on the axis, the disk rotates (see the photo on the right). The graphs below show angular displacement of the disk-versus-time and angular displacement of the disk-versus-time squared for the same experiment.







a. Which features of the graphs support the hypothesis that the disk was rotating at a constant angular acceleration? Comment on the features of both graphs.

b. Determine the angular acceleration of the disk by using data from the graphs.

OALG 9.1.5 Reason

The goal of this activity is to develop mathematical representations to describe rotational motion at a constant rate and rotational motion at a constantly changing rate. We will do this by making an analogy to similar mathematical representations for linear motion which we've already established. Fill out the table below. We use the symbol ω to denote rotational velocity and α to denote rotational acceleration.

Description	**Mathematical representations describing linear motion**	**Description**	**Mathematical representations describing rotational motion**
Change in the position or *displacement* of an object moving in a straight line.	Δx	The change in the angle in the counterclockwise direction between a reference line (usually the positive *x*-axis) and a line drawn from the axis of rotation to the *point* on a rotating object.	$\Delta\theta$
Velocity is the change in position per unit time.	$v_x = \dfrac{\Delta x}{\Delta t}$		
	$a_x = \dfrac{\Delta v_x}{\Delta t}$		
	$v_{\mathrm{f},x} = v_{\mathrm{i},x} + a_x \Delta t$		
	$\Delta x = \dfrac{1}{2}(v_{\mathrm{i},x} + v_{\mathrm{f},x})\Delta t$		
	$\Delta x = v_{\mathrm{i},x}\Delta t + \dfrac{1}{2}a_x \Delta t^2$		

Now, read and interrogate all of the Section 9.1 in the textbook and compare your answers with the answers in Table 9.1 on page 256.

OALG 9.1.6 Practice

Solve Problems 8 and 13-15 on page 278 in the textbook.

9.2 Physical quantities affecting rotational acceleration

OALG 9.2.1 Observe and find a pattern

Equipment: access to a door that rotates freely.

Find a door and conduct the following experiment where you push perpendicularly to the surface of the door at different distances from the hinge, and with varying amounts of force. Compare (qualitatively) the angular acceleration of the door for each case and fill out the table below:

Point at which you push	**Magnitude of force exerted by you on the door**	**Torque**	**Resulting angular acceleration**
About 5 cm from the hinge	F		
Middle of the door	F		
As far as you can be from the hinge	F		
As far as you can be from the hinge	$2F$		

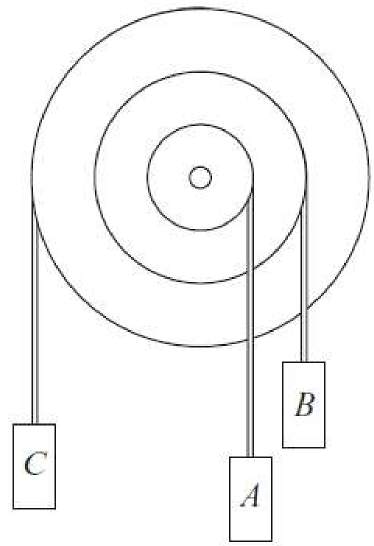
a. What does the angular acceleration of the door depend on?

b. How do you think the angular acceleration of the door would be affected if you doubled the mass of the door?

c. Work through Observational Experiment Table 9.2 (watch the videos!) in the textbook and compare the patterns you found with the patterns in the table.

OALG 9.2.2 Observe and find a pattern

Three blocks hang from strings wrapped around a multi-radius pulley (see the figure at right) that is initially stationary. The radii are r, $2r$, and $3r$. By changing the masses of the hanging blocks, we can get the pulley to rotate clockwise (cw) faster and faster, to rotate counterclockwise (ccw) faster and faster, or to remain stationary. The table below indicates the results of five such experiments.



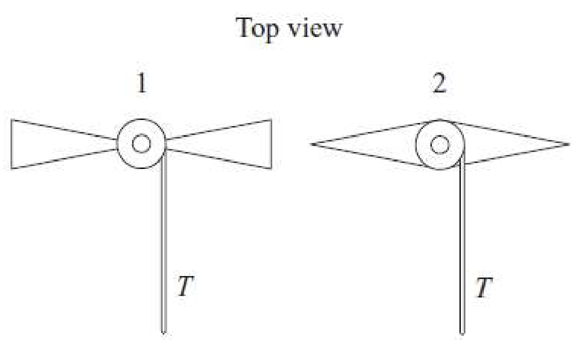
Experiment	**Block *A***	**Block *B***	**Block *C***	**Pulley behavior**
Experiment 1	2 kg at r	1 kg at $2r$	2 kg at $3r$	Starts rotating ccw faster and faster.
Experiment 2	2 kg at r	2 kg at $2r$	2 kg at $3r$	Remains stationary.
Experiment 3	2 kg at r	3 kg at $2r$	2 kg at $3r$	Starts rotating cw faster and faster.
Experiment 4	2 kg at r	1 kg at $2r$	3 kg at $3r$	Starts rotating ccw faster and faster, with the rotational acceleration greater than that in experiment 1.
Experiment 5	3 kg at r	2 kg at $2r$	2 kg at $3r$	Starts rotating cw faster and faster, with the rotational acceleration greater than in experiment 3.

a. Find a pattern in the data from experiments 1 through 5. Formulate a general rule that will allow you to predict when the pulley does not start rotating and when it does start rotating.

b. Analyze the outcomes of experiments 1 and 4 and of experiments 3 and 5 to find a pattern in the magnitude of the acceleration. Are they consistent with the patterns you found in Activity 9.2.1?

OALG 9.2.3 Observe and explain

Two objects of equal mass but different mass distribution that are attached to a pulley system are initially at rest, as shown in the illustration below. Strings pull on the objects, exerting a force of the same magnitude on the axles. The rate of rotation of object 2 increases more rapidly than that of object 1.



a. Discuss with your group members how you can explain this phenomenon. Why would the same force cause different rotational accelerations for these objects of the same mass?

b. Read and interrogate Testing Experiment Table 9.3 in the textbook on page 258 (do not forget to watch the video!). Is the result of the testing experiment in the table consistent with your answer in part **a**?

c. Describe other phenomena from real life where the mass distribution makes it easier or more difficult to cause the object to start rotating.

OALG 9.2.4 Test an idea

Equipment per group: a broom.

Use the ideas about the role of mass distribution for rotational motion to predict the outcome of the following experiment (do not perform the experiment yet): Imagine that you first try to increase the broom's rotational speed about a vertical axis by spinning it with one hand holding the broomstick near the end opposite the broom head. Then imagine doing it again while holding the broom in the middle, nearer the broom head. Which turning method should be easier if the hypothesis that the farther the mass is from the axis of rotation, the slower the rotational acceleration for the same torque is correct? Write your prediction here. Then perform the experiment and record the outcome. Did it match the prediction? What is your judgment about the hypothesis?

OALG 9.2.5 Read and interrogate

Read and interrogate Section 9.2 in the textbook and answer Review Question 9.2.

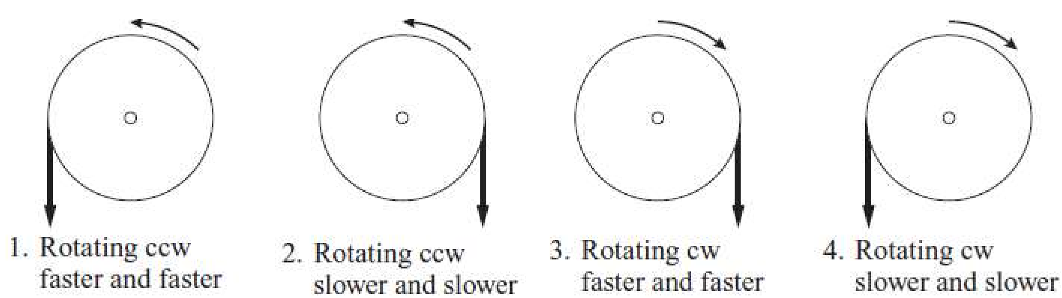
OALG 9.2.6 Practice

Answer Questions 1, 3, and 5 on page 277 in the textbook.

9.3 Newton's second law for rotational motion

OALG 9.3.1 Observe and explain

For each situation shown in the figure below, determine the signs of the net torque and the initial velocity (see the curved arrows). The signs are + for counterclockwise (ccw), – for clockwise (cw), and 0 for zero torque or for a stationary disk.



The sign of the rotational acceleration is the same as the sign of the rotational velocity if its magnitude is increasing, opposite the sign of the rotational velocity if its magnitude is decreasing, and zero if the rotational velocity magnitude is not changing. Note that the curved arrows on the illustrations represent the initial direction of rotation, and the straight arrows represent the direction of a force exerted by the rope on the pulley.

a. Fill in the results in the table that follows. Note that $\sum\tau$ is the sum of the torques – the net torque, ω is the initial rotational velocity, and α is the rotational acceleration.

Experiment	Sign of $\sum\tau$	Sign of ω	Sign of α
Situation 1			
Situation 2			
Situation 3			
Situation 4			

b. Is there a relationship between the sign of the sum of the torques and the sign of the rotational velocity? If so, what is the relationship? Explain.

c. Is there a relationship between the sign of the sum of the torques and the sign of the rotational acceleration? If so, what is the relationship? Explain.

OALG 9.3.2 Test your idea

Equipment: A bicycle turned upside down, and any other equipment of choice.

Use a bicycle for this experiment. Turn it upside down and place the saddle on the floor. Design and carry out an experiment to test your answers to parts b. and c. in Activity 9.3.

a. Describe the experiment in words and with a sketch.

b. Describe what physical quantities you will analyze.

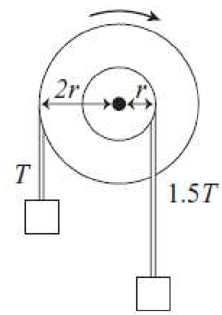
c. Make predictions of the outcome using the ideas under test.

d. Conduct the experiment, record the outcome, and compare it to the prediction.

e. What is your judgment concerning the answers to parts b. and c. in Activity 9.3.1?

OALG 9.3.3 Represent and reason

Answer the following questions for the situation shown in the figure. Note that the multi-radius pulley is initially turning clockwise.



a. Write an expression for the net torque exerted on the pulley in terms of r and T.

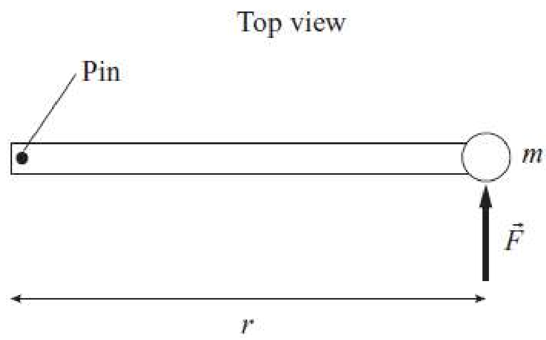
b. In words, describe the behavior of the pulley as time progresses.

c. How do we need to change the force exerted by the left rope on the pulley so that the angular velocity of the pulley remains constant? Give result in terms of r and T. Explain.

d. How do we need to change the force exerted by the right rope on the pulley so that the angular velocity of the pulley remains constant? Give result in terms of r and T. Explain.

OALG 9.3.4 Represent and reason

Derive a rotational form of Newton's second law: A rod of negligible mass has a disk of mass m attached at one end. The other end has a pin through it, as shown in the illustration below. The rod and disk start at rest and rotate in a horizontal circle about the pin on a frictionless air table. A person exerts a constant force $\vec{F}$ on the disk (shown by the arrow in the sketch), always pointing tangent to the circle.



a. Apply Newton's second law to the disk in a direction tangent to the circle.

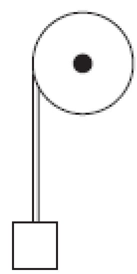
b. Write an expression for the torque τ that the force exerts about the pin.

c. Multiply both sides of the equation in part **a.** by the same quantity so that the left force side of the equation becomes the torque determined in part **b**.

d. Multiply the right side of the equation by *r/r* and use one of these *r*'s to convert the acceleration *a* to a rotational acceleration α. What is the result?

e. Read and interrogate page 260 in the textbook. Compare your result to Equation 9.8.

OALG 9.3.5 Test your idea



Equipment: [https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-phys-egv2e-alg-9-3-5].

The goal of this activity is to test Newton's second law for rotational motion. You have a block that hangs from a string. The other end of the string wraps around a flywheel. The information about the flywheel and the block can be obtained from the video.

a. Use this information and Newton's second law for rotational motion that you constructed in earlier activities to answer the following questions to predict the time interval needed for the block, initially at rest, to touch the floor. Then, compare this time interval to the reading of the clock in the video.

b. List the assumptions you made.

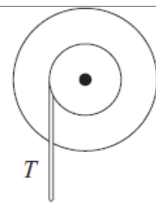
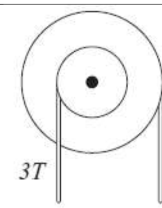
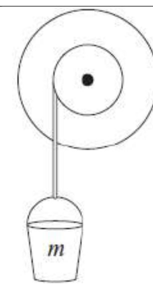
c. Compare the actual time to your prediction.
[https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-phys-egv2e-alg-9-3-5]

d. Explain whether the experiment supported or disproved Newton's second law for rotational motion.

e. Read and interrogate pages 261 and 262 in the textbook. What do they say about Newton's second law for rotational motion?

OALG 9.3.6 Represent and reason

Fill in the table that follows. Remember to use symbols when writing the rotational form of Newton's second law for each situation (and the translational form for the pail). The rotational inertia of the multi-radius pulley shown below is I, the inner radius is r, and the outer radius is $2r$.

Sketch and translate			
Simplify and diagram Choose a system.			
Draw a force diagram.			Pail: Pulley:
Represent mathematically Write Newton's second law for rotational motion (or translational motion for the pail).			Pail: Pulley:

OALG 9.3.7 Equation Jeopardy

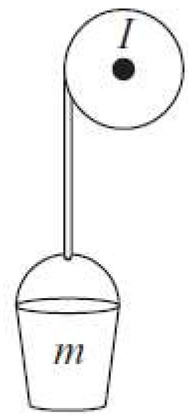
The equations that follow describe many possible physical situations. Think of one situation that might be described by each set of equations. For each mathematical description, create a sketch of the situation that the equation might describe and write a word problem for which the given equation is a solution.

a. $-(100\ \text{N})(0.40\ \text{m}) = (80\ \text{kg}\cdot\text{m}^2)\,\alpha$ and $\omega = 0 + \alpha(3.0s)$

b. $+(100\ \text{N})(0.40\ \text{m})-(160\ \text{N})(0.20\ \text{m})=(20\ \text{kg}\cdot\text{m}^2)\alpha$ and $\theta=0-(4.0\ \text{s}^{-1})t+(1/2)\alpha\,(2.0\ \text{s})^2$

OALG 9.3.8 Evaluate the solution

The problem: A 2.0-kg pail hangs from a rope that wraps around a 0.20-m radius disk of rotational inertia $0.80\ \text{kg}\cdot\text{m}^2$. Determine the rotational speed of the disk 2.0 s after the pail is released from rest. Assume that $g=10$ N/kg.



Proposed solution: The situation is pictured at the right. We choose the disk as the object of interest. The rope holding the pail exerts a downward force on the disk of magnitude $mg=(2.0\ \text{kg})(10\ \text{N}/\text{kg})=20\ \text{N}.$ The rotational acceleration of the disk is:

$$\alpha=\sum\tau/I=(20\ \text{N})(0.20\ \text{m})/(0.80\ \text{kg}\cdot\text{m}^2)=5.0\ \text{s}^{-2}$$

The rotational velocity after 2.0 s will be:

$$\omega=0+\alpha t=(5.0\ \text{s}^{-2})(2.0\ \text{s})=10\ \text{rad/s}$$

a. Identify any errors in the solution.

b. Provide a corrected solution.

OALG 9.3.10 Observe and analyze

Watch the video "Tuna can and the magnets" [https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-phys-egv2e-alg-9-3-10] (advance the movie frame by frame). The video consists of three parts. The first part shows the rotation of a tuna can on a carpet, the second part shows the rotation of the same tuna can with two magnets added close to the axis of rotation, and the third part shows the rotation of the tuna can with the magnets further away from the axis.

a. Using the data from the video, determine the rotational acceleration of the tuna can in the first experiment and the coefficient of friction between the tuna can and the carpet.

b. Also, determine the rotational accelerations of the can in the second and third experiment. Compare the magnitudes of the rotational accelerations in the three parts and explain (qualitatively) their relative values (why did the rotational acceleration increase/decrease/not change in a particular experiment?).

OALG 9.3.11 Read and interrogate

Read and interrogate Section 9.3 in the textbook and answer Review Question 9.3.

OALG 9.3.12 Practice

Answer Questions 4, 6, and 11 on page 277 in the textbook and solve Problems 21, 25, 26-28, and 40 on pages 278-280 in the textbook.

9.4 Rotational momentum

OALG 9.4.1 Analyze and find a pattern

Below is a table describing several observational experiments. Read through the experiments and describe the patterns you find. Put those patterns on a whiteboard.

Observational experiments	Analysis
a. A figure skater initially spins slowly with one leg and two arms extended. Then she pulls her leg and arms close to her body and her spinning rate increases dramatically.	*Initial situation:* Large rotational inertia I and small rotational speed ω. *Final situation:* Smaller rotational inertia I and larger rotational speed ω.
b. Watch video OET 9.6 on page 267 in the textbook. A man sitting on a chair that can spin with little friction initially holds barbells far from his body and spins slowly. When he pulls the barbells close to his body, the spinning rate increases dramatically.	*Initial situation:* Large rotational inertia I and small rotational speed ω. *Final situation:* Smaller rotational inertia I and larger rotational speed ω.

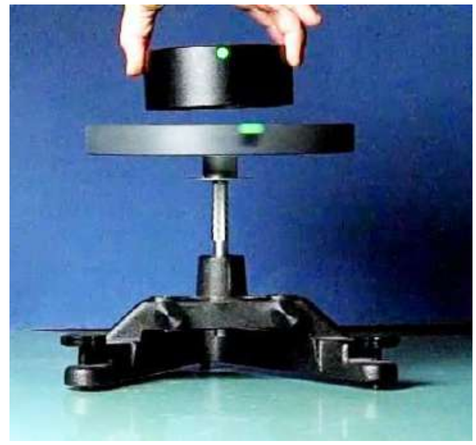
OALG 9.4.2 Reason by analogy

Discuss with your group and put your ideas on a whiteboard:

a. Use the analogy between translational and rotational motion to write a mathematical expression for rotational momentum (do not forget to check the units) and one expression for rotational momentum constancy in an isolated system. After you write the expressions, read and interrogate Section 9.4 in the textbook. Did you arrive to the same expression as Equation 9.11?

b. If the system is not isolated, what physical quantity would account for the change in rotational momentum (think of an analogous quantity for impulse in translational motion)?

OALG 9.4.3 Analyze



A solid disk (radius 114 mm, thickness 25 mm, mass 1418 g) can rotate with almost no friction around a vertical axis (see photo at right). You make the disk rotate with a constant speed and hold a hollow cylinder (outer radius 64 mm, inner radius 56 mm, thickness 50 mm, mass 1428 g) directly above the disk so that the axes of both objects coincide. When you release the cylinder, it falls onto the disk and slides on it for a few seconds until both objects rotate with the same constant speed. A high-speed video of the experiment (recorded at 300 frames per second) is given here [https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-phys-egv2e-alg-9-4-3]. Answer the questions below:

a. Draw a rotational momentum bar chart for the experiment (for help, check the bar charts in Example 9.6 in the textbook on page 269). Indicate your choice of system and the initial/final states.

b. Estimate the ratio between the rotational inertia of the cylinder and the rotational inertia of the solid disk, $\frac{I_{\text{cylinder}}}{I_{\text{disk}}}$, using data from the video. Calculate the same ratio from the dimensions and masses of the objects given above. Compare both results and explain any differences between them.

OALG 9.4.4 Practice

Answer Questions 7 and 8 on page 277 in the textbook and solve Problems 45, 46, 48, 49, 68, and 69 on pages 280 - 282.

9.5 Rotational kinetic energy

OALG 9.5.1 Reason by analogy

Discuss with your group and put your ideas on a whiteboard:

Use the analogy between translational and rotational motion to write a mathematical expression for rotational kinetic energy. Check whether the units of rotational kinetic energy are as expected. After you finish, read and interrogate Sction 9.5 in the textbook and compare your expression to Equation 9.14.

OALG 9.5.2 Analyze

Use the video from Activity 9.4.3 and work with your group to answer the following questions:

a. Draw a work-energy bar chart for the experiment. Indicate your choice of system and the initial/final states.

b. Estimate the change in the internal energy of the disk and the cylinder during the experiment.

OALG 9.5.3 Observe and analyze

The experiment in the video shows a ball rolling up a ramp [https://youtu.be/MV_2BKINOx4]. Use the data that you can collect from the video to decide whether the ball can be modeled as a point-like object, a solid sphere, or a hollow sphere.

a. Draw an energy bar chart to represent the process for three different models of the ball.

b. Using the speed of the ball on the horizontal surface, predict how high each model predicts the ball to climb.

c. Compare the actual height of the climb to the height that each model predicts. What additional assumptions do you need to make to explain the outcome of the experiment?

OALG 9.5.4 Observe and analyze

Observe the video of a solid and a hollow cylinder of equal masses rolling down an incline [https://mediaplayer.pearsoncmg.com/assets/_frames.true/sci-phys-egv2e-alg-9-5-3]. Draw energy bar charts to explain why the solid cylinder wins the race.

OALG 9.5.5 Practice

Answer Question 9 on page 277 in the textbook and solve Problems 51, 52, 55, 59, and 61 on page 281.

9.6 Tides and Earth's day

OALG 9.6.1 Regular problem

Suppose that two imaginary beings (Superpeople) each exert a force of magnitude F on opposite sides of Earth's equator, tangent to its surface, for a period of 1 year. How large must the forces be to stop Earth's rotation? List the assumptions that you made.

OALG 9.6.2 Read and interrogate

Read and interrogate Section 9.6 in the textbook and answer Review Question 9.6.

OALG 9.6.3 Regular problem

As the tides slide across Earth, they exert a friction force that opposes Earth's rotation. Because of this, the time required for one Earth rotation increases by 0.0016 s every 100 years.

a. Determine the rotational deceleration of Earth.

b. Assuming that Earth is a uniform solid sphere, what torque do the tides exert on Earth?

c. If the friction force caused by the tides were all concentrated at the equator, how large would this force be?

d. How long will it take to stop Earth's rotation, assuming constant decceleration? Compare this time with the 4.5 billion years that the Sun is expected to remain the same.

