# Chapter 22

# **Reflection and refraction**

# 22.1 Light sources, light propagation, and shadows

### OALG 22.1.1 Observe and explain

Go into a room that is completely isolated from all external light sources—natural and artificial. Turn off the internal lights and wait in the dark room for several minutes. Record your observations and propose an explanation.

### OALG 22.1.2 Observe and explain

*Equipment:* laser pointer and a spray bottle or chalk dust. Take a laser pointer and point it at a wall. Can you see the beam of light it sends or only the shiny spot on the wall?

**a.** What path did the light follow to reach the wall? You can find it by trial and error - by trying to block the light with a small piece of paper at several locations along its path to the wall, or by using the water spray bottle.

**b.** What can you say about the path of the light from the laser to the wall? Represent that light path by a long line with an arrow, called a ray. A ray is not real; it is just a way to show the direction that light is traveling.

**c.** Explain why the water droplets (or chalk dust) makes it possible to see the light beam that was previously not visible.

d. Discuss the conditions needed for us to see something.

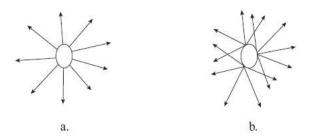
Video alternate if you can't find the equipment: https://mediaplayer.pearsoncmg.com/assets/ frames.true/secs-experiment-video-51

### OALG 22.1.3 Test an idea

Equipment: frosted, relatively large lightbulbs, and other materials if needed.

Place a powered, **frosted** lightbulb on a table in the center of a dark room and observe that the walls are almost uniformly lit. Draw a ray diagram in your notes that illustrates how the frosted bulb lights the room.

Tovi (a) and Jaeline (b) draw two ray diagrams to try to model how a lightbulb's light can reach the walls and the ceiling in the room.



**a.** Compare and contrast the two diagrams. Consider how each point of the bulb emits light according to each diagram.

**b.** Design an experiment to test which of the diagrams represents the way a lightbulb emits light —does each point emit one ray, or does each point emit rays in all directions? Describe the experiment with a picture and write a prediction of the outcome of the experiment based on each diagram.

**c.** Perform the experiment and decide which diagram led to the prediction that did not match the outcome. Which diagram will you use to represent how each point of the bulb emits light?

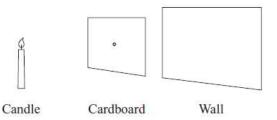
**d.** Watch two experiments at <u>https://mediaplayer.pearsoncmg.com/assets/\_frames.true/secs-</u> <u>experiment-video-52</u>. Which model predicts their outcomes? Draw ray diagrams to support your answer.

#### OALG 22.1.4 Test your ideas

*Equipment:* a candle or another asymmetrical very bright light source, a piece of thin cardboard with an opening as shown below.

Imagine that you put a candle (or bulb) on a table and place a piece of thin cardboard between the candle and a nearby wall. Use the figure to draw a ray diagram to predict what you will see on the wall if you make a tiny hole in the cardboard. Then predict what you will see if you poke two holes.

Then light the candle (or turn on the bulb) and turn off the room lights to observe the outcome of the experiment(s). Then revise your diagram if necessary.



# OALG 22.1.7 Read and interrogate

Read and interrogate Section 22.1 in the textbook and answer Review Question 22.1.

# OALG 22.1.8 Practice

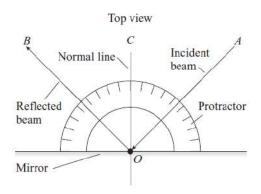
Answer Questions 1-6 on page 707 and solve Problems 2-4 and 8 on page 708.

# 22.2 Reflection of light

# OALG 22.2.1 Observe and find a pattern

Equipment: laser pointer, white paper, protractor, ruler.

Place a protractor on a tabletop and put the flat edge of the protractor against a mirror that is held upright and perpendicular to the tabletop and protractor. Shine a laser pointer across the protractor so that the beam hits the mirror slightly above the center zero point on the protractor. The reflected light returns across the protractor (see the figure below). Then change the angle at which the laser ray hits the mirror and see if there is any pattern in the direction of the reflected light. Record your results in the table. Notice the normal line *CO* in the figure—this is a line perpendicular to the surface of the mirror at a point where the incident beam hits the mirror.



Angle AOB	Angle between	Angle between the	Angle between the	Angle between the
between the	the incident beam	reflected beam BO	incident beam AO	reflected beam BO
incident and	AO and the	and the mirror	and the normal line	and the normal line
reflected beams	mirror		СО	СО
	90°			
	70°			

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60°		
45°		
30°		
10°		

**a.** Find a pattern in the data and express the pattern in words and mathematically. If you do not have a laser pointer, use the protractor to collect data from the following video: <a href="http://islephysics.net/pt3/experiment.php?topicid=12&exptid=175">http://islephysics.net/pt3/experiment.php?topicid=12&exptid=175</a> and fill out the table above.

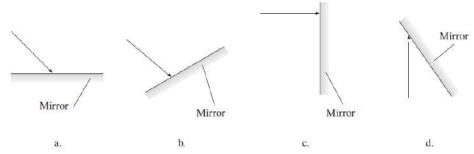
**b.** Read and interrogate text on page 690 above the new subsection "Specular and diffuse reflection and absorption" and compare your findings to the findings in the text.

# OALG 22.2.2 Test your idea (1)

Imagine that you have two vertical mirrors on a flat surface so that their faces make a right angle. Use any of the relationships that you found in Activity 22.2.1to draw a ray diagram to predict what will happen to a laser beam incident on one of the mirrors. Once you made the prediction, watch the following video <u>http://islephysics.net/pt3/experiment.php?topicid=12&exptid=176</u> and decide whether your prediction matched the outcome. If it did not, how do you need to revise the ray diagram?

# OALG 22.2.3 Represent and reason

Rays in the figures below represent beams of light from a laser pointer.



Draw on the figures reflected rays for each arrangement.

# OALG 22.2.4 Explain

**a.** In Activity 22.1.2 you learned that you only see an object if emitted or reflected light travels from the object to your eye. In Activity 22.2.1 you learned that when a laser beam is reflected off a smooth surface, the incident beam and the reflected beam are at the same angles relative to a line perpendicular to the surface. Imagine that a laser beam hits a wall. If you stand at any place in the room, you see a bright spot on the wall where the laser beam hits it. How can you

reconcile these two phenomena? *Hint:* Examine the surface of the wall and compare it to the mirror.

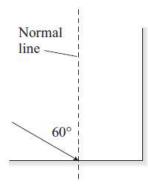
**b.** Read and interrogate the subsection "Specular and diffuse reflection, and absorption" and compare your answer in part **a** to the text.

### OALG 22.2.5 Represent and reason

A beam of light hits a plane mirror perpendicular to the mirror's surface. Determine the angle between the incident and reflected beams if you tilt the mirror 30°. Include a sketch of the initial situation before tilting the mirror and the final situation after tilting it. Draw in labeled rays representing the incident and reflected beams for both orientations.

### OALG 22.2.6 Represent and reason

Two mirrors are placed together at a right angle, with one mirror oriented vertically and the other oriented horizontally. A ray strikes the horizontal mirror at an incident angle of  $60^{\circ}$  relative to the normal line, reflects from it, and then hits the vertical mirror.



a. Determine the angle of incidence relative to the vertical mirror.

**b.** Use the law of reflection and the drawing to show that the ray leaves the vertical mirror parallel to its original direction.

#### OALG 22.2.7 Practice

Answer Questions 14 and 24 on page 707 and solve Problems 13-15 on page 708 in the textbook.

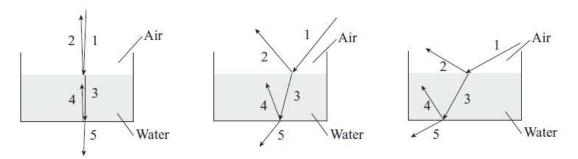
# 22.3 Refraction of light

#### OALG 22.3.1 Observe and find a pattern

Equipment: a laser pointer, a clear plastic container filled with water.

Shine the light from a laser beam on to the surface of a clear plastic container filled with water. If you cannot do the experiment, watch the video at

https://mediaplayer.pearsoncmg.com/assets/ frames.true/secs-experiment-video-53 and use the figures below.



**a.** What happens to the beam of light that is incident on an interface between two different media —for example, ray 1 reaching the top surface of the water or ray 3 reaching the bottom? Traditionally physicists use a line perpendicular to the air–water interface at the point at which a ray strikes it to record the changes in the direction of the beam.

**b.** For the three situations illustrated, describe any pattern(s) you observe when looking at rays 1 and 2 or rays 3 and 4.

**c.** For the three situations illustrated, describe any pattern(s) you observe when looking at rays 1 and 3 or rays 3 and 5.

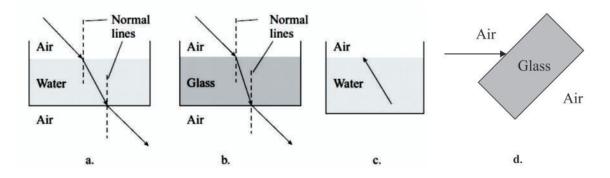
**d.** For the three situations illustrated, describe any pattern(s) you observe when looking at rays 1 and 5. Is this pattern consistent with the patterns you discussed in part **c**.? Explain.

e. Compare and contrast your patterns to the patterns listed on page 693 in the textbook.

#### OALG 22.3.2 Represent and reason

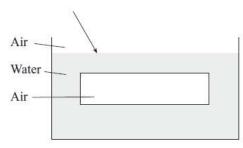
Parts **a.** and **b.** of the figure below depict the paths of light beams moving from air into water or glass and then out into air again. (The container holding the water is made of glass with very thin walls so that the boundaries air/glass and water/glass can be disregarded). The reflected beams are not shown. Note the bending of the light at the interfaces between the two media. In

particular, keep track of the light path relative to the normal lines that are perpendicular to the interfaces. Based on the patterns you observe in parts **a**. and **b**., draw onto the figure for part **c**. a normal line and a ray indicating the light path after it moves from the water to the air. For part **d**., draw normal lines and rays as the light moves from the air into the glass, through the glass, and out into the air on the other side. Then draw reflected rays on all interfaces on all figures.



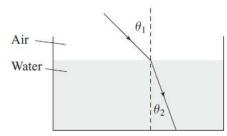
### OALG 22.3.3 Represent and reason

Imagine that you place a closed, empty glass box (with thin glass walls) filled with air underwater and hold it there. A beam of light shines on the top surface of the water, as represented by the ray in the figure. Draw arrows that indicate the beam's path from the top of the water out of the bottom of the container (the walls of the container are made of infinitely thin glass and can be disregarded). After you draw the refracted beams, go back and draw reflected beams.



### OALG 22.3.4 Observe and find a pattern

We filled a fish tank with water and shone the light from a laser pointer at different angles on the top surface of the water. The angle  $\theta_1$  of the incident beam relative to the normal line and the angle of the beam that propagates into the water (the so-called refracted ray  $\theta_2$ ) are shown in the diagram and recorded in the table. Various trigonometric functions of those angles are recorded as well.



Incident angle	cos	sin	Refracted angle	cos	sin
$\boldsymbol{\theta}_{1}$	$\theta_1$	$\boldsymbol{\theta}_{1}$	$\boldsymbol{\theta}_{2}$	$\theta_2$	$\theta_2$
20°	0.94	0.34	15°	0.97	0.26
30°	0.87	0.5	22°	0.93	0.37
40°	0.77	0.64	29°	0.87	0.48
50°	0.64	0.77	35°	0.82	0.57
60°	0.50	0.87	41°	0.76	0.65

**a.** Use any or all of the values given in the table to devise a rule that relates the angle of incidence and the angle of refraction. *Hint:* You might see if the ratio of two quantities has the same value for all angles. If so, use this to help devise a rule.

**b.** Compare your rule to Equations 22.2 on page 693 and 22.3 on page 22.3 in the textbook.

c. While performing the experiment, you see a dot of light on the ceiling. Explain.

### OALG 22.3.5 Observe and find a pattern

We repeated Activity 22.3.4 only this time we shone the light through air onto a block of glass with a smooth surface. The table records the angle between the incident light and the normal line and the angle between the refracted light and the normal line.

Incident angle	Refracted angle	
$\boldsymbol{\theta}_{1}$	$\theta_2$	
20°	13°	
30°	19°	
40°	25°	

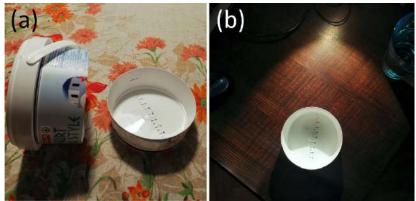
50°	30°
60°	35°

**a.** Use the rule relating the angle of incidence and the angle of refraction that you devised in the previous activity, only this time apply it for light propagation from air into glass. Compare and contrast the air–glass refraction with the air–water refraction.

# OALG 22.3.6 Apply

### Equipment: A plastic container, permanent marker, ruler.

Obtain a large white plastic container (such as 2-pound yogurt container) and cut the bottom part (about 4 cm high) to obtain a shallow tray (see figure a). Mark a scale with 5 mm steps along the diameter of the tray, using a permanent marker. Put the tray on a horizontal surface that is illuminated by a sunlight or by a small distant lamp. Turn the tray so, that the marked scale points toward the light source.



**a.** Observe the shadow of the edge of the tray that forms at the bottom of the tray (figure b). Record at which mark is the edge of the shadow.

**b.** Without moving the tray, slowly add water into the tray and observe what happens to the shadow. Describe your observations.

c. Try to explain what you observed using your knowledge of geometrical optics.

**d.** Propose an experimental procedure based on the experiment that you just analyzed, that will allow you to determine the index of refraction for water. (*Hints*: to avoid complicated calculations, let the water level in the container be half the height of the tray. Also, if the wall of your container is not vertical but tilted, you should consider this in your calculations in order to obtain a more accurate result.)

## OALG 22.3.7 Apply

Watch the following experiment

<u>http://islephysics.net/pt3/experiment.php?topicid=12&exptid=181</u> and explain why we cannot see the beaker.

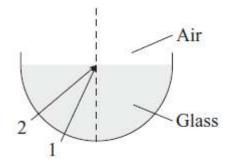
## OALG 22.3.8 Practice

Answer Questions 8-10, and 27 on page 707 and solve Problems 20, 21 and 26 on pages 708-709.

# 22.4 Total internal reflection

# OALG 22.4.1 Test your ideas

Use your knowledge of refraction to predict qualitatively and quantitatively what will happen in the described experiment. Note that the index of refraction of air is 1.0 and of water is 1.33.



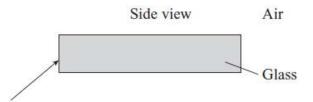
Experiment: Shine a laser beam so that it passes through glass and then refracts into the air above. Vary the angle of incidence on the glass–air interface (e.g., see rays 1 and 2 in the figure at right).

**a.** Predict what will happen if you gradually increase the angle of incidence. Identify a special angle of incidence where there is no longer a refracted ray. Explain your prediction.

**b.** After making the prediction, perform the experiment—did you correctly identify the special angle? What happens at incident angles greater than this special "critical" angle?

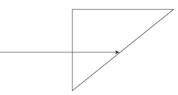
### OALG 22.4.2 Represent and reason

Imagine that you shine a laser beam at a glass plate, as shown in the figure below. Use your knowledge of reflection and refraction to predict the path of the beam. Draw rays to indicate the path of the beam. (*Hint:* Before you draw the rays, decide on the direction of the normal line at the point at which the laser light first hits the glass.)



#### OALG 22.4.3 Represent and reason

Light enters a right-angle prism as shown in the figure below and experiences total internal reflection. Draw the path of the light on the figure. What is the minimum refractive index for the prism for total internal reflection to occur?



#### OALG 22.4.4 Read and interrogate

Read and interrogate Section 22.4 in the textbook and answer Review Question 22.4.

# 22.5 Skills for analyzing reflective and refractive processes

#### OALG 22.5.1 Explain

At sunset, you are walking along a river on the west bank. When you are looking toward the east bank of the river, you notice a bright rectangular spot on the river surface (see the photo on the right). Explain how the spot forms using a ray diagram (*Hint*: think about where the Sun is and note a tall building on the other side of the river).



#### OALG 22.5.4 Equation Jeopardy

Mathematical representations of two situations are shown below.

- 1. 1.00 sin  $53^\circ = n_2 \sin 41^\circ$
- 2.  $1.00\sin 53^\circ = 1.56\sin \theta_2$  and  $1.56\sin \theta_2 = 1.00\sin \theta_3$ .
- **a.** Solve for the unknown(s).
- **b.** Sketch the situation that the equation describes.

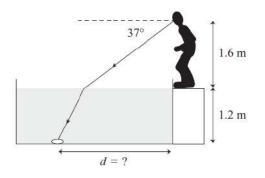
c. Write a word description of the process.

#### OALG 22.5.5 Evaluate the solution

*The problem:* The eyes of a person standing at the edge of a 1.2-m-deep swimming pool are 1.6 m above the surface of the water. The person sees a silver dollar at the bottom of the pool at a  $37^{\circ}$  angle below the horizontal. Determine the horizontal distance *d* from the person to the dollar.

#### Proposed solution:

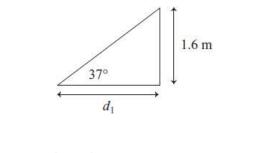
Sketch and translate The situation is sketched below. Simplify and diagram The water surface is smooth, and the index of refraction of water is 1.33.



A ray from the eye to the coin is shown in the figure above.

Represent mathematically, solve, and evaluate

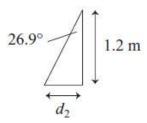
The horizontal distance  $d_1$  from the eye to the water surface where the ray enters is determined using trigonometry (see the triangle in the figure below).



$$\sin 37^\circ = (1.6 \text{ m})/d_1$$
 or  $d_1 = 2.67 \text{ m}$ 

Apply Snell's law to find the angle of the ray while in the water:

1.00 sin 37° = 1.33 sin 
$$\theta_{\text{in water}}$$
 or  $\theta_{\text{in water}} = 26.9^{\circ}$ 



Finally, we can use the triangle in the water (shown above) to determine the extra horizontal distance  $d_2$  in the water:

$$\sin 26.9^\circ = d_2 / (1.2 \text{ m})$$
 or  $d_2 = 0.54 \text{ m}$ 

The total horizontal distance from the edge of the pool under the person's feet is:  $d = d_1 + d_2 = 2.67 \text{ m} + 0.54 \text{ m} = 3.2 \text{ m}$ 

**a.** Identify any errors in the student solution.

**b.** Provide a corrected solution if you find errors.

# OALG 22.5.6 Explain

Watch the following video <u>https://mediaplayer.pearsoncmg.com/assets/\_frames.true/secs-egv2e-pinhole-camera-movie</u> and explain how it is possible to have movie of the world "upside-down" having a box and a video camera.

# OALG 22.5.7 Apply

**a.** Watch the following experiment <u>https://youtu.be/QTOggdH1NQY</u> and explain using a ray diagram the change in location of the spot on the black screen after the container is filled with water.

**b.** Explain how minor modifications of the set-up in the previous experiment <u>https://youtu.be/dDuwBLkNdvY</u> can result in such different outcome. Use a ray diagram in your explanation.

# OALG 22.5.8 Read and interrogate

Read and interrogate Section 22.5 in the textbook and answer Review Question 22.5.

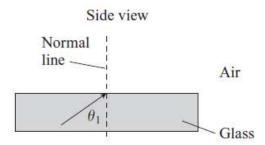
# OALG 22.5.9 Practice

Answer Questions 11 and 12 on page 707 and solve Problems 45, 47, and 52 on pages709 - 710.

## 22.6 Fiber optics, prisms and the color of the sky

#### OALG 22.6.1 Represent and reason

A laser beam shines up through a piece of glass of refractive index 1.56 and reaches the glass–air interface at the top, as shown in the figure below.



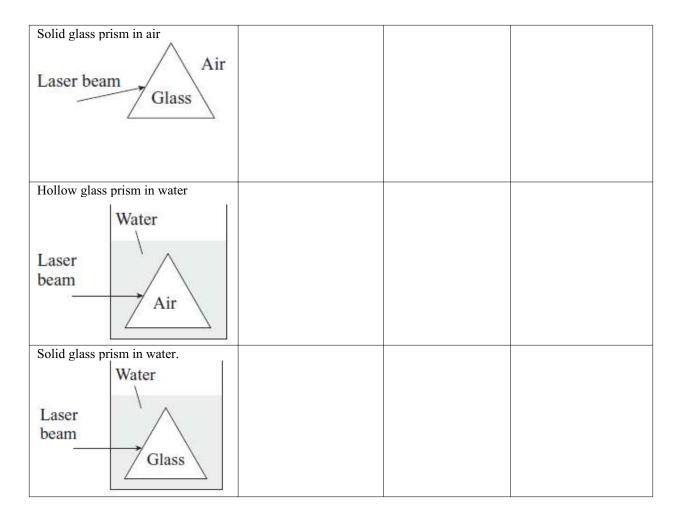
**a.** For what range of incident angles  $\theta_1$  will the laser beam not pass out of the glass and into the air? Explain your prediction.

**b.** Use your prediction in part **a**. to explain how a glass rod (actually a thin glass fiber) can become a pipe that transmits light signals without light losses out of the side walls of the fiber. Indicate any assumptions you made.

#### OALG 22.6.2 Test your idea

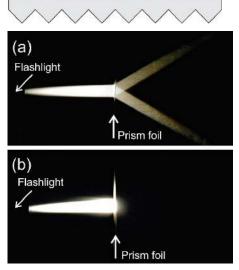
We observed earlier in this chapter that light moving from air to glass or to a liquid bends (refracts) toward a normal line that is perpendicular to that surface. Also, light moving in a glass or liquid into air bends away from a line that is perpendicular to that surface. Use these ideas to predict qualitatively what happens to a laser beam in each of the experiments below. *Hint:* Do not forget to draw a normal line at the location at which the light beam hits the border of the two media. Use a solid glass prism and a hollow glass prism to complete the table that follows.

Illustration of the experiment	Use your knowledge of	Perform the	Discuss whether your
	refraction to predict	experiment and	prediction was
	qualitatively the path	record the results	successful or if the
	of the beam.	(i.e., the path of the	relationship needs to
		beam).	be modified.



### OALG 22.6.3 Explain

Transparent prism foil is part of a backlight system in LCD monitors that are widely used today. The upper figure on the right shows a magnified cross section of such a foil (the foil is about 0.15 mm thick). The foil consists of prismatic ridges with 90° angles at their peaks. The index of refraction of the foil is 1.58. This foil has very interesting optical properties. When you shine a light beam perpendicular to one side of the foil, the beam splits in two symmetrical beams (see photo (a)). When you turn the foil around and repeat the experiment, the light beam reflects from the foil and no light passes through (see figure (b)).



**a.** Using ray diagrams, determine which side of the foil (the flat or prism side) was facing the flashlight in experiment (a) and which in experiment (b).

**b.** How will the performance of the foil change if you cover the prism side of the foil with a layer of water (thick enough to fill and flatten the grooves).

### OALG 22.6.4 Explain

Examine two photos of the same street lamp shown below. The photo on the left, taken on a rainy fall night, shows the view of the lamp through the wet branches of a tree. The photo on the right was taken at the exactly same spot a few days later, during the day. Notice how tiny reflections in the first photo form kind of circles around the lamp, although it is clear from the second photo that the branches have no circular orientation. Propose an explanation for this pattern. If time and equipment permit you, propose testing experiments, make predictions based on the explanation under test, perform the testing experiments and decide if the outcomes agree with predictions. (*Hint*: wet branches can be modeled as reflecting rods that are randomly distributed in the space; the street lamp can be modeled as a point-like light source.)



### OALG 22.6.5 Reason and explain

Different colors of light have different indexes of refraction when passing though water droplets (see the table below).

Color	Index of refraction
Red	1.613
Yellow	1.621
Green	1.628
Blue	1.636
Violet	1.661

How can this information qualitatively explain why we see a rainbow while it rains? Think about where the Sun is located with respect to an observer when the observer sees a rainbow. What assumption about the shape of the water droplets in the air can we make? If you need help, consult textbook Section 22.6 and the Reading Passage at the end of the Chapter 22.

### 22.6.6 Read and interrogate

Read and interrogate Section 22.6 in the textbook and answer Review Question 22.6.

# 22.7 Explanation of light phenomena: two models of light

### OALG 22.7.1 Explain

In the previous activities you found that light reflects and refracts (bends) as it travels between different media. One explanation that scientists formulated hundreds of years ago to explain this observed phenomenon was a "particle-bullet model" of light. They thought that an object that radiates light emits tiny particles, like little bullets, that travel in all directions. Use this model to explain the following.

**a.** How does light travel in straight lines in the same medium?

**b.** How does light form shadows if it encounters obstacles?

c. How does the angle of incidence equal the angle of reflection?

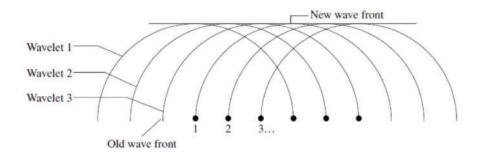
d. How does light bend when passing from one medium into another, different medium?

**e.** Light bends toward the normal line when it travels from air into any other medium. What do you need to assume about the components of velocity parallel to the surface and perpendicular to the surface of a light particle as it passes from air to the second medium?

# OALG 22.7.2 Represent and reason

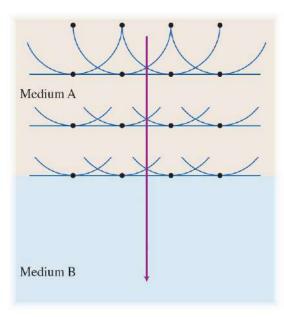
Christiaan Huygens, a contemporary of Newton, developed a wave model of light that competed with Newton's (at the time) more popular particle-bullet model. Huygens wondered what would happen if several waves simultaneously traveled through a medium. To answer this question, let's try a paper-and-pencil experiment similar to that used by Huygens. We mark six dots across a page, each dot separated by 1 cm from the adjacent dot (see the figure below). *The dots represent points on the crest of a wave moving toward the top of the page*. According to Huygens, each dot is the source of a small wave disturbance that moves up the page in the direction the wave is

traveling. In the figure the 3-cm-radius half circles, called *wavelets*, represent these disturbances. On the figure provided, note places above the dots where the net disturbance from the six wavelets is two or more times bigger than the disturbance caused by any one wavelet—places where the wavelets add together to form bigger waves. This is the new crest of the wave. Draw a line on the sketch indicating the location of the new wave crest that was formerly at the position of the dots. Also, draw a ray indicating the direction the wave is traveling. The pattern is even clearer if you make many more dots and wavelets in the same space.



#### OALG 22.7.3 Represent and reason

In Activity 22.7.2, semicircular wavelets all traveling at the same speed make up a new wave front. Waves often travel at different speeds in different places. For example, water waves travel more slowly in shallow water than in deeper water. Sound travels more slowly in cold air than in warm air. The difference in speed in different regions causes the wave to bend—to change direction. Huygens' principle can be used to understand this better. In the sketch below, three wave crests (wave fronts) of waves moving vertically down with a speed  $v_A$  are constructed using Huygens ' principle. The distance between the two successive wave fronts is the wavelength in medium A. The speed of waves in medium A is three times the speed of waves in medium B ( $v_A = 3v_B$ ).

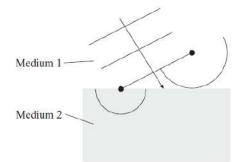


**a.** On the sketch, draw a continuation of the waves in medium B. Draw at least four new wave fronts.

**b.** Determine the ratio between the wavelengths  $\lambda_A / \lambda_B$  and between the frequencies  $f_A / f_B$  of the waves in the two media. Explain your answer.

### OALG 22.7.4 Represent and reason

Imagine a wave whose wave fronts moving in one medium are incident on a boundary with another medium. The wave ray is not perpendicular to the boundary of the two media (see the figure below). The wave travels faster in medium 1 than in medium 2. During a certain time interval, the wavelet that earlier left the right edge of the wave front is just reaching the boundary between medium 1 and medium 2. The wavelet that left the lower left edge of the wave front at the same time travels shorter distance (i.e., moves more slowly) in medium 2.



**a.** The wavelets leaving the middle of the wave front travel part of the time in faster medium 1 and part of the time in slower medium 2. Note that their radii in medium 2 should be shorter than

the radius of the wavelet on the lower-left edge, because they reach the boundary later. What is the orientation of the new wave front formed from these wavelets—now completely in medium 2? Draw a ray indicating the direction of the wave in medium 2.

**b.** Compare your sketch with the figure in the Activity 22.3.4. Based on your analysis here and on that sketch, decide whether light travels faster or slower in water than in air. Explain your answer.

### OALG 22.7.5 Read and interrogate

Read and interrogate Section 22.7 in the textbook and answer Review Question 22.7.

### OALG 22.7.6 Practice

Answer questions 32, 33, and 43 on page 707 in the textbook.