Lesson 9: Problem Solving When the Percent Changes

Student Outcomes

- Students solve percent problems where quantities and percents change.
- Students use a variety of methods to solve problems where quantities and percents change, including double number lines, visual models, and equations.

Lesson Notes

In this lesson, students solve multi-step word problems related to percents that change. They identify the quantities that represent the *part* and the *whole* and recognize when the whole changes based on the context of a word problem. They will build on their understanding of the relationship between the part, whole, and percent. All of the problems can be solved with a visual model. Students may solve some of the problems with an equation, but often the equation will require eighth grade methods for a variable on both sides of the equation. If students generalize and solve such equations, they should be given full credit.

Classwork

Example 1 (5 minutes)

MP.1 Begin class by displaying Example 1. Have the students work in groups or pairs to try to start the problem on their own.

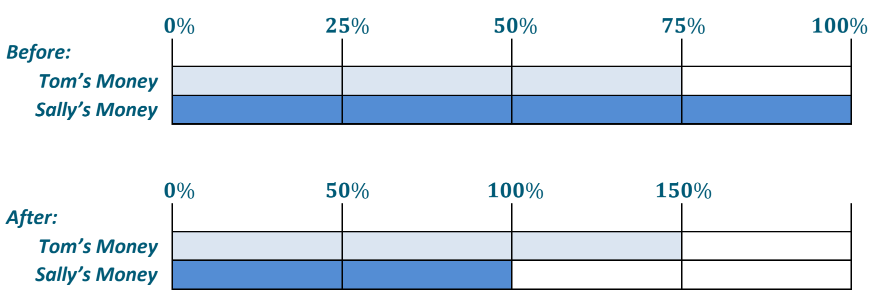
- Based on the words in the example, which person's money should represent the whole?
  - *The first whole is Sally's beginning money. The second whole is Sally's ending money.*

Scaffolding:

- Students that had difficulty solving equations in earlier modules may need additional practice working with these one-step equations. Students should continue to use calculators where appropriate throughout the lesson.
- Where appropriate, provide visual models with equations to show an alternative problem-solving strategy for visual learners.

Example 1

The amount of money Tom has is 75% of Sally's amount of money. After Sally spent $\$120$ and Tom saved all his money, Tom's amount of money is 50% more than Sally's. How much money did each have at the beginning? Use a visual model and a percent line to solve the problem.



Each bar is $\$60$. Tom started with $\$180$, and Sally started with $\$240$.

Example 2 (10 minutes)

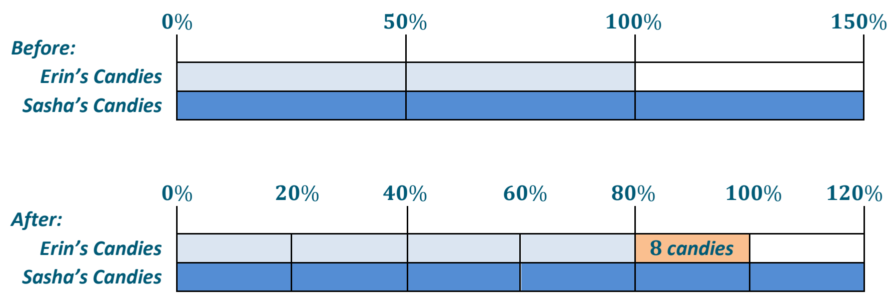
Following the discussion of Example 1, have students try to start Example 2 without modeling. Students will solve the example using a visual model and an equation to show the change in percent. Pose possible discussion questions to the class as you solve the problem.

- Which person's candy represents the whole?
  - *Erin's candy represents the whole.*

Example 2

Erin and Sasha went to a candy shop. Sasha bought 50% more candies than Erin. After Erin bought 8 more candies, Sasha had 20% more. How many candies did Erin and Sasha have at first?

a. **Model the situation using a visual model.**



b. **How many candies did Erin have at first? Explain.**

Each bar in the "after" tape diagram is 8 candies. Sasha has 48 candies. Each bar in the "before" tape diagram is 16 candies. Erin started with 32 candies.

Example 3 (7 minutes)

MP.4 The previous example presented a visual model approach. In this example, allow students to choose their preferred method to solve the problem. It is important for students to first write an algebraic expression that represents each person's money before they can form an equation.

Point out that since Kimberly and Mike have an equal amount of money in the beginning, the same variable can be used to represent the amount.

Scaffolding:

- Consider having some groups solve the problem using a visual model and other groups using an equation.
- Have students explain their models to other groups and look for comparisons for problem solving.
- For the exercises, the teacher could select specific individuals to solve problems using an assigned method to allow students to get comfortable with choosing and utilizing problem-solving methods of choice and efficiency.

Example 3

Kimberly and Mike have an equal amount of money. After Kimberly spent $\$50$ and Mike spent $\$25$, Mike's money is 50% more than Kimberly's. How much did Kimberly and Mike have at first?

a. **Use an equation to solve the problem.**

Equation Method:

Let x be the amount of Kimberly's money, in dollars, after she spent $\$50$. After Mike spent $\$25$, his money is 50% more than Kimberly's. Mike's money is also $\$25$ more than Kimberly's.

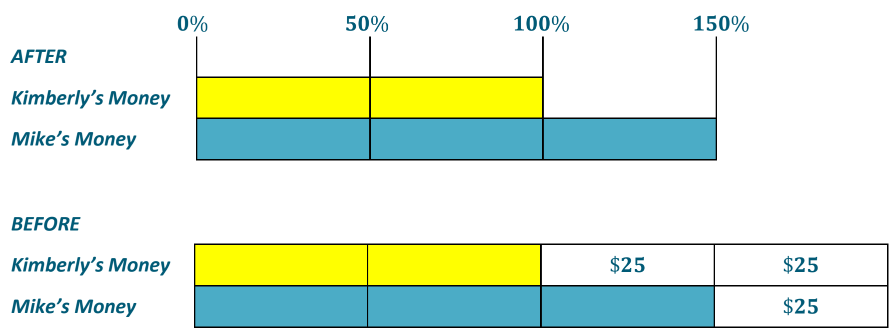
$$0.5x = 25$$
$$x = 50$$

Kimberly started with $\$100$ because $\$100 - \$50 = \$50$. Mike has $\$75$ because $(1.5)50 = 75$.

They each started with $\$100$.

Lead the class through constructing a visual model for part (b). Since we are subtracting money, first create the *after* picture, then, add the money to get the *before* picture.

b. **Use a visual model to solve the problem.**



Each bar is $\$25$. They both started with $\$100$.

c. **Which method do you prefer and why?**

Answers will vary. I prefer the visual method because it is easier for me to draw the problem out instead of using the algebraic properties.

Exercise (13 minutes)

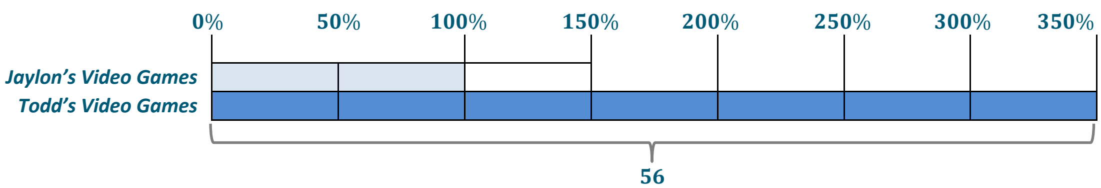
This exercise allows students to choose any method they would like to solve the problem. Then, they must justify their answers by using a different method. After about 10 minutes, ask students to present their solutions to the class. Compare and contrast different methods, and emphasize how the algebraic, numeric, and visual models are related.

Exercise

Todd has 250% more video games than Jaylon. Todd has 56 video games in his collection. He gives Jaylon 8 of his games. How many video games did Todd and Jaylon have in the beginning? How many do they have now?

Answers may vary. Sample answer is provided below.

Visual Model:



Each bar in the dark box is 7 games.

Equation Method:

Let z be the number of video games that Jaylon had at the beginning. Then, Todd started with $3.5z$ video games.

$$3.5z = 56$$
$$z = 16$$

In the beginning, Jaylon had 16, and Todd had 56. After Todd gave Jaylon 8 of his games, Jaylon had 24, and Todd had 48.

Closing (3 minutes)

- What formula can we use to relate the part, whole, and percent?
  - $\text{Quantity} = \text{Percent} \times \text{Whole}$
- Describe at least two strategies for solving a changing percent problem using an equation.
  - *You must identify the first whole and then identify what would represent the second whole.*
  - *You must use algebraic properties such as the distributive property to solve the problem.*

Lesson Summary

- **To solve a changing percent problem, identify the first whole and then the second whole. To relate the part, whole, and percent, use the formula**

$$\textbf{Quantity} = \textbf{Percent} \times \textbf{Whole}.$$

- **Models, such as double number lines, can help visually show the change in quantities and percents.**

Exit Ticket (7 minutes)

Name ____ Date ____________________

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Exit Ticket

Terrence and Lee were selling magazines for a charity. In the first week, Terrance sold 30% more than Lee. In the second week, Terrance sold 8 magazines, but Lee did not sell any. If Terrance sold 50% more than Lee by the end of the second week, how many magazines did Lee sell?

Choose any model to solve the problem. Show your work to justify your answer.

Exit Ticket Sample Solutions

Terrence and Lee were selling magazines for a charity. In the first week, Terrence sold 30% more than Lee. In the second week, Terrence sold 8 magazines, but Lee did not sell any. If Terrence sold 50% more than Lee by the end of the second week, how many magazines did Lee sell?

Choose any model to solve the problem. Show your work to justify your answer.

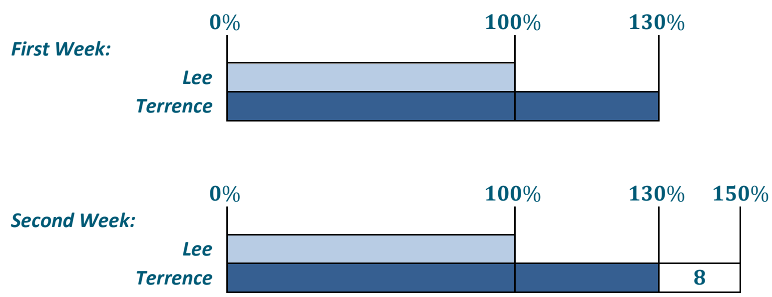
Answers may vary.

Equation Model:

Let m be the number of magazines Lee sold.

$150\% - 130\% = 20\%$, *so* $0.2m = 8$ *and* $m = 40$

Visual Model:



By subtracting 150% and 130%, I determined that 8 magazines represents 20% of the magazines Lee sold in the first week. Then, I multiplied both the percent and the number of magazines by 5 to find that 40 magazines represents 100% of the magazines Lee sold.

Problem Set Sample Solutions

1. **Solve each problem using an equation.**
   a. **What is 150% of 625?**

      $n = 1.5(625)$
      $n = 937.5$

   b. **90 is 40% of what number?**

      $90 = 0.4(n)$
      $n = 225$

   c. **What percent of 520 is 40? Round to the nearest hundredth of a percent.**

      $40 = p(520)$
      $p \approx 0.0769 = 7.69\%$

2. The actual length of a machine is 12.25 cm. The measured length is 12.2 cm. Round the answer to part (b) to the nearest hundredth of a percent.

   a. Find the absolute error.

   $|12.2 - 12.25| = 0.05$ cm

   b. Find the percent error.

   $\frac{0.05}{|12.25|} \times 100\% = 0.4082$

   percent error $\approx 0.41\%$

3. A rowing club has 600 members. 60% of them are women. After 200 new members joined the club, the percentage of women was reduced to 50%. How many of the new members are women?

   40 of the new members are women.

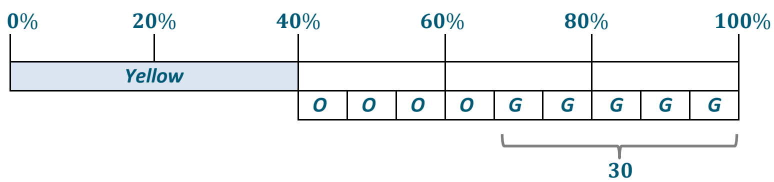
4. 40% of the marbles in a bag are yellow. The rest are orange and green. The ratio of the number of orange to the number of green is $4:5$. If there are 30 green marbles, how many yellow marbles are there? Use a visual model to show your answer.

   5 units $= 30$ marbles

   1 unit $= 30 \div 5$ marbles $= 6$ marbles *4 units $= 4 \times 6$ marbles $= 24$ marbles*

   $30 + 24 = 54$ marbles represents 60% of the total. If I divide both 54 and 60% by 3, I can determine that 20% would be 18 marbles. Then, I can double both of these to find that 36 marbles make up 40% of the total.

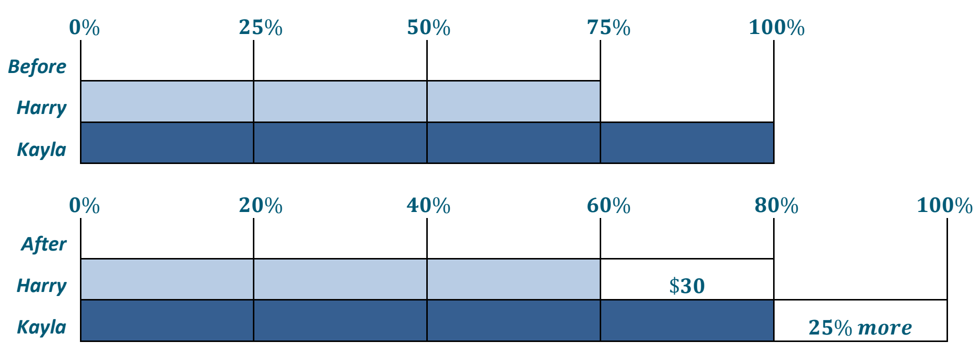
   There are 36 yellow marbles because 40% of the marbles are yellow.



5. Susan has 50% more books than Michael. Michael has 40 books. If Michael buys 8 more books, will Susan have more or less books than Michael? What percent more or less will Susan's books be? Use any method to solve the problem.

   Susan has 25% more.

6. Harry's amount of money is 75% of Kayla's amount of money. After Harry earned $\$30$ and Kayla earned 25% more of her money, Harry's amount of money is 80% of Kayla's money. How much money did each have at the beginning? Use a visual model to solve the problem.



   Each bar is $\$30$. Harry started with $\$90$, and Kayla started with $\$120$.