Trigonometry & Radicals

Topics of Study:

Pythagorean Theorem
Trigonometric Ratios
Simplifying Radicals
Operations with Radicals

Video Tutorials:

- Basic Trigonometry
- Trigonometry Lesson Part I
- Sine, Cosine and Tangent
- Trigonometric Ratios Tutorial
- Pythagorean Theorem
- Donald in Mathemagic Land on Pythagorean Theorem
- Pythagoras in 2 min
- Simplifying Radical Expressions
- Adding and Subtracting Radicals
- Multiplying Radicals

"In any right triangle, the square of the length of the hypotenuse is equal to the sum of the squares of the lengths of the legs."

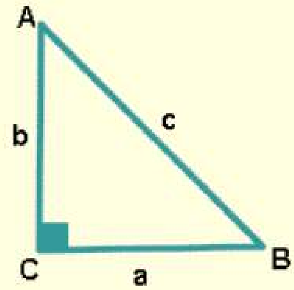
This relationship can be stated as:

$$c^2 = a^2 + b^2$$

for any right triangle

and is known as the

Pythagorean Theorem.



a, b are legs.
c is the hypotenuse
(c is across from the right angle).

REMEMBER: The Pythagorean Theorem **ONLY** works in **Right Triangles**!

Pythagorean Triples.

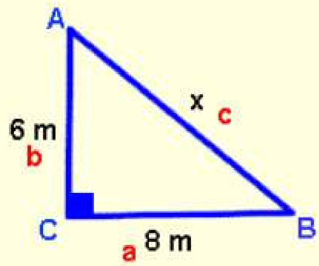
The most common Pythagorean Triples are:

3, 4, 5

5, 12, 13

8, 15, 17

Example 1:



Find x.

$$c^2 = a^2 + b^2$$

$$x^2 = 64 + 36$$

$$x^2 = 100$$

$$\sqrt{x^2} = \sqrt{100}$$

$$x = 10$$

Answer: 10 m

This problem could also be solved using the Pythagorean Triple 3, 4, 5. Since 6 is 2 times 3, and 8 is 2 times 4, then x must be 2 times 5.

Example 2:

A triangle has sides 6, 7 and 10.
Is it a right triangle?

Let $a = 6$, $b = 7$ and $c = 10$. The longest side MUST be the hypotenuse, so $c = 10$. Now, check to see if the Pythagorean Theorem is true.

$$c^2 = a^2 + b^2$$

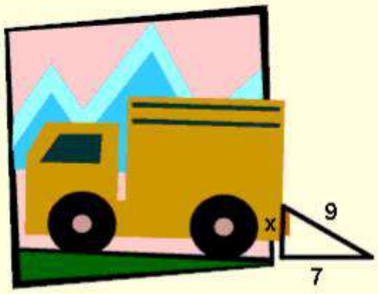
$$10^2 \mathrel{?} 6^2 + 7^2$$

$$100 \mathrel{?} 36 + 49$$

$$100 \neq 85$$

Since the Pythagorean Theorem is NOT true, this triangle is NOT a right triangle.

Example 3:



A ramp was constructed to load a truck. If the ramp is 9 feet long and the horizontal distance from the bottom of the ramp to the truck is 7 feet, what is the vertical height of the ramp to the nearest tenth of a foot?

Since the ramp is described as having horizontal and vertical measurements, a right angle is implied. Solve using the Pythagorean Theorem:

$$c^2 = a^2 + b^2$$

$$9^2 = x^2 + 7^2$$

$$81 = x^2 + 49$$

$$32 = x^2$$

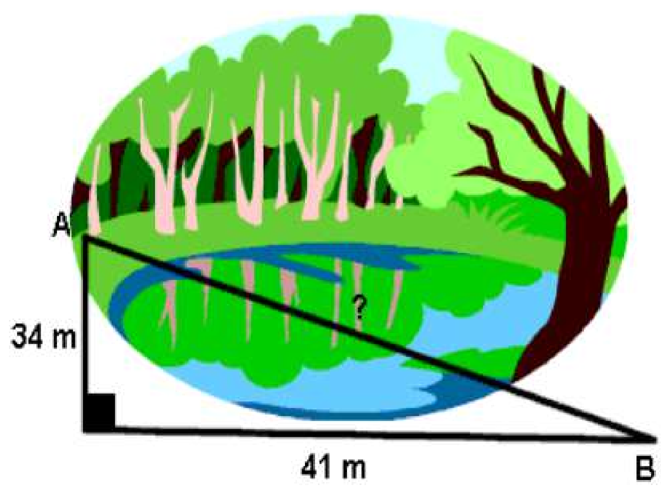
$$\sqrt{32} = \sqrt{x^2}$$

$$5.66 = x$$

The height of the ramp is 5.7 feet. The ramp will allow packages to be loaded into an area of the truck that is too high to be reached from the ground.

Practice Examples

1.



Choose:

To get from point A to point B you must avoid walking through a pond. To avoid the pond, you must walk 34 meters south and 41 meters east. To the *nearest meter*, how many meters would be saved if it were possible to walk through the pond?

- 22
- 34
- 53
- 75

2.

A baseball diamond is a square with sides of 90 feet. What is the shortest distance, to the *nearest tenth* of a foot, between first base and third base?

Choose:

- 90.0
- 127.3
- 180.0
- 180.7

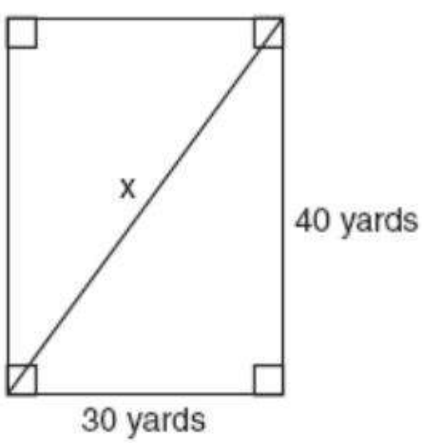
3.

A suitcase measures 24 inches long and 18 inches high. What is the diagonal length of the suitcase to the *nearest tenth* of a foot?

Choose:

- 2.5
- 2.9
- 26.5
- 30.0

4.



Tanya runs diagonally across a rectangular field that has a length of 40 yards and a width of 30 yards, as shown in the diagram.

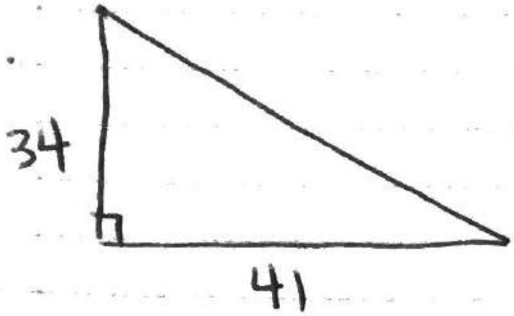
What is the length of the diagonal, in yards, that Tanya runs?

Choose:

- 50
- 60
- 70
- 80

Practice Examples #'s 1 - 4 (Answers)

1.



$34^2 + 41^2 = x^2$

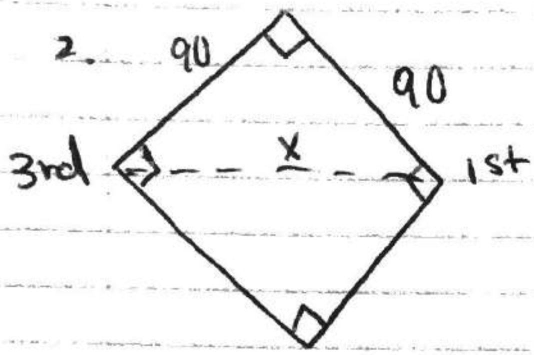
$2837 = x^2$

$\sqrt{2837} = x$

$53.26349594 = x$

[Choice 3 : 53m]

2.



$90^2 + 90^2 = x^2$

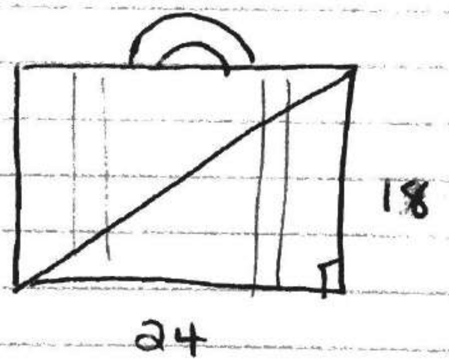
$8100 + 8100 = x^2$

$16,200 = x^2$

$\sqrt{16,200} = x$

$127.3 = x$ [Choice 2]

3.



$18^2 + 24^2 = x^2$

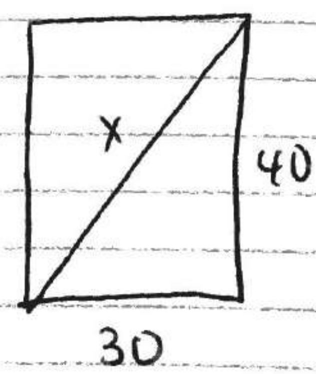
$324 + 576 = x^2$

$900 = x^2$

$\sqrt{900} = x$

$30 = x$ [Choice 4]

4.



$30^2 + 40^2 = x^2$

$900 + 1600 = x^2$

$2500 = x^2$

$\sqrt{2500} = x$

$50 = x$ [Choice 1]

Trigonometry: Solving for a Side

Formulas:

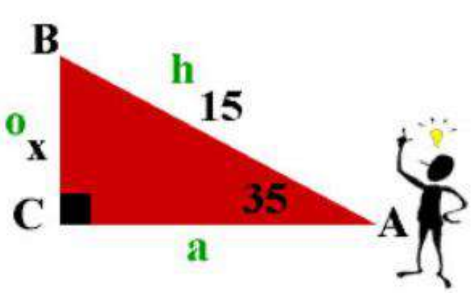
$\sin A = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$	$\cos A = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$	$\tan A = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$
where A represents the angle of reference.		

Basic Trigonometry Rules:

- These formulas ONLY work in a **right triangle**.
- The hypotenuse is always across from the right angle.
- Questions usually ask for an answer *to the nearest* units.
- You will need a scientific or graphing calculator.

How to set up and solve a trigonometry problem when solving for a side of the triangle:

Example: **In right triangle *ABC*, hypotenuse *AB*=15 and angle *A*=35º. Find leg length, *BC*, to the *nearest tenth*.**



$$\sin A = \frac{\text{opposite leg}}{\text{hypotenuse}}$$

$$\sin 35^\circ = \frac{x}{15}$$

$$.5736 = \frac{x}{15}$$

$$x = 8.604 = 8.6$$

ANSWER: 8.6

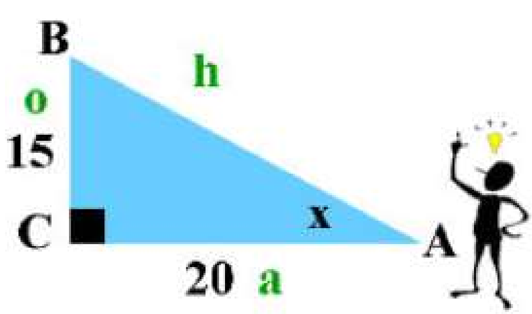
Practice Examples

5.

A ladder 6 feet long leans against a wall and makes an angle of 71° with the ground. Find to the *nearest tenth* of a foot how high up the wall the ladder will reach.

How to set up and solve a trigonometry problem when solving for an angle in the triangle:

Example: In right triangle ABC, leg BC=15 and leg AC=20. Find angle A to the *nearest* degree.



$$\tan A = \frac{\text{opposite leg}}{\text{adjacent leg}}$$

$$\tan x = \frac{15}{20}$$

Now, divide 15 by 20 to change the fraction into a decimal.

$$\tan x = 0.75$$

You now need to find an angle whose tangent is 0.75. To do this, use your scientific or graphing calculator. (On the scientific calculator, enter 0.75. You now need to activate the **$\tan^{-1}$** key (it is located above the **tan** key). To activate this **$\tan^{-1}$** key, press **2nd** (or shift) and then the **tan** key. On the graphing calculator, activate the **$\tan^{-1}$** first, and then enter 0.75.)

$x = 36.87 = 37°$ rounding as directed.

6.

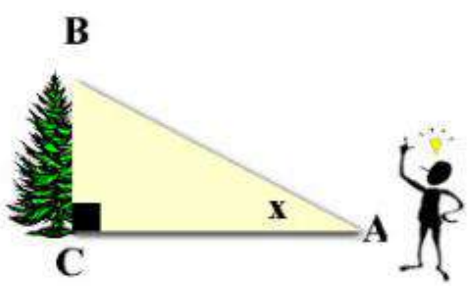
A piece of lumber leans against a wall. The top of this 40 foot piece of lumber touches a point on the wall that is 36 feet above the ground. Find to the *nearest degree* the measure of the angle that the lumber makes with the wall.

In word problems, the formulas remain the same:

$\sin A = \dfrac{\text{opposite leg}}{\text{hypotenuse}}$	$\cos A = \dfrac{\text{adjacent leg}}{\text{hypotenuse}}$	$\tan A = \dfrac{\text{opposite leg}}{\text{adjacent leg}}$

Word problems introduce two new vocabulary terms:

Angle of Elevation

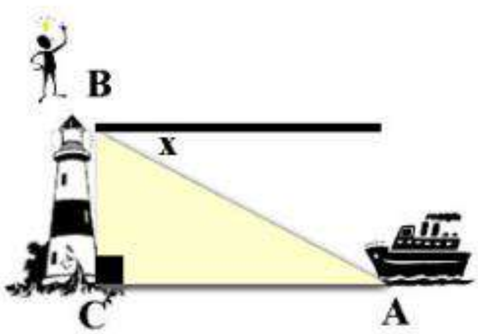


The **angle of elevation** is always measured from the ground up. Think of it like an elevator that only goes up. It is always **INSIDE** the triangle.

In the diagram at the left, *x* marks the angle of elevation of the top of the tree as seen from a point on the ground.

You can think of the angle of elevation in relation to the movement of your eyes. You are looking straight ahead and you must raise (elevate) your eyes to see the top of the tree.

Angle of Depression



The **angle of depression** is always **OUTSIDE** the triangle. It is never inside the triangle.

In the diagram at the left, *x* marks the angle of depression of a boat at sea from the top of a lighthouse.

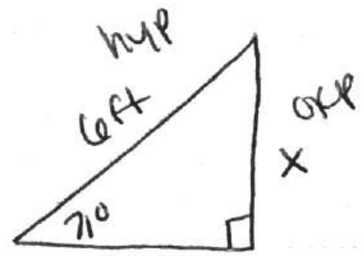
You can think of the angle of depression in relation to the movement of your eyes. You are standing at the top of the lighthouse and you are looking straight ahead. You must lower (depress) your eyes to see the boat in the water.

7.

From a point on the ground 25 feet from the foot of a tree, the angle of elevation of the top of the tree is 32°. Find to the *nearest foot,* the height of the tree.

Practice Examples #'s 5 - 7 (Answers)

5.



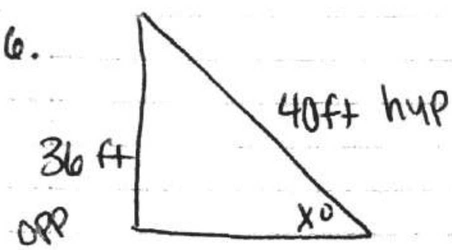
$$\sin 71 = \frac{x}{6}$$

$$\frac{.9455}{1} = \frac{x}{6}$$

$$6(.9455) = x$$

$$5.7 = x$$

6.

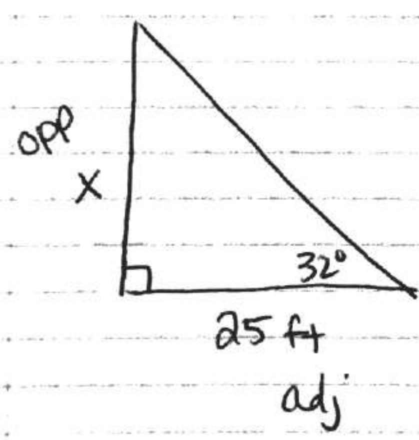


$$\sin x = \frac{36}{40}$$

[2nd] [sin] → $\sin^{-1}(36/40)$

$$x = 64°$$

7.



$$\tan 32 = \frac{x}{25}$$

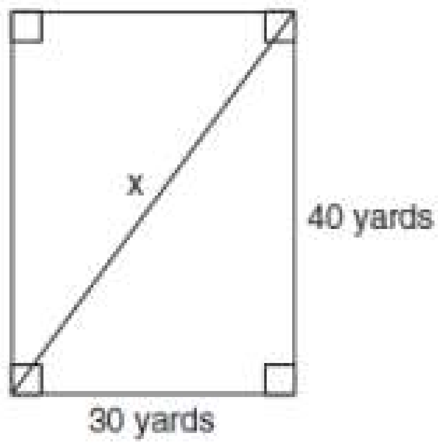
$$\frac{(.6249)}{1} = \frac{x}{25}$$

$$25(.6249) = x$$

$$16 = x$$

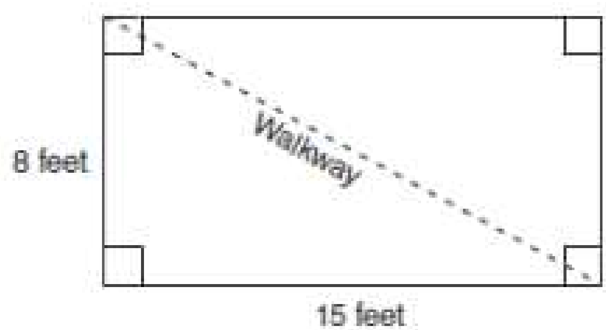
Additional Practice Examples:

1 Tanya runs diagonally across a rectangular field that has a length of 40 yards and a width of 30 yards, as shown in the diagram below.



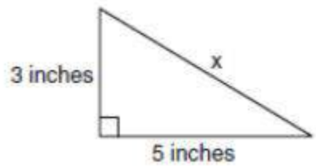
What is the length of the diagonal, in yards, that Tanya runs?
1) 50
2) 60
3) 70
4) 80

2 Nancy's rectangular garden is represented in the diagram below.



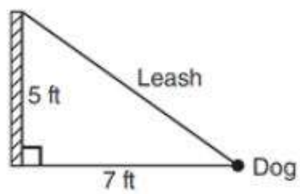
If a diagonal walkway crosses her garden, what is its length, in feet?
1) 17
2) 22
3) $\sqrt{161}$
4) $\sqrt{529}$

3 What is the value of x, in inches, in the right triangle below?



1) $\sqrt{15}$
2) 8
3) $\sqrt{34}$
4) 4

4 The end of a dog's leash is attached to the top of a 5-foot-tall fence post, as shown in the diagram below. The dog is 7 feet away from the base of the fence post.

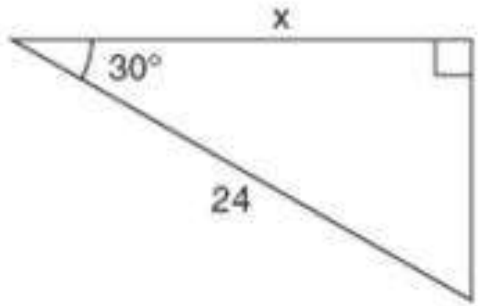


How long is the leash, to the *nearest tenth of a foot*?
1) 4.9
2) 8.6
3) 9.0
4) 12.0

Answers:

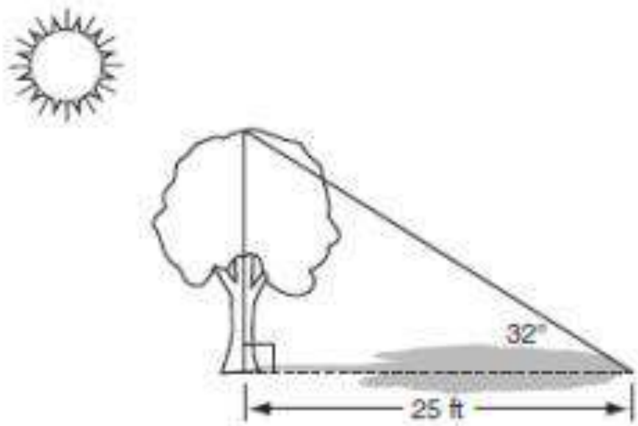
1 ANS: 1 $30^2 + 40^2 = c^2$. 30, 40, 50 is a multiple of 3, 4, 5. $2500 = c^2$ $50 = c$	2 ANS: 1 $8^2 + 15^2 = c^2$ $c^2 = 289$ $c = 17$
3 ANS: 3 $3^2 + 5^2 = x^2$ $34 = x^2$ $\sqrt{34} = x$	4 ANS: 2 $\sqrt{5^2 + 7^2} \approx 8.6$

1 In the right triangle shown in the diagram below, what is the value of x to the *nearest whole number*?



1) 12
2) 14
3) 21
4) 28

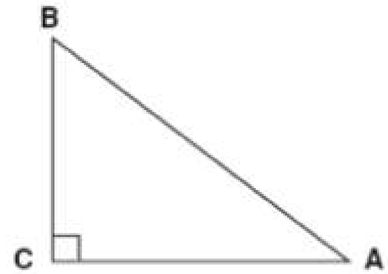
2 A tree casts a 25-foot shadow on a sunny day, as shown in the diagram below.



If the angle of elevation from the tip of the shadow to the top of the tree is 32°, what is the height of the tree to the *nearest tenth of a foot*?

1) 13.2
2) 15.6
3) 21.2
4) 40.0

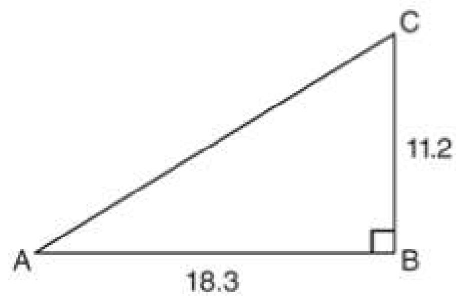
3 In the diagram of $\triangle ABC$ shown below, $BC = 10$ and $AB = 16$.



To the *nearest tenth of a degree*, what is the measure of the largest acute angle in the triangle?

1) 32.0
2) 38.7
3) 51.3
4) 90.0

4 In right triangle ABC shown below, $AB = 18.3$ and $BC = 11.2$.



What is the measure of $\angle A$, to the *nearest tenth of a degree*?

1) 31.5
2) 37.7
3) 52.3
4) 58.5

Answers:

1 ANS: 3 $\cos 30 = \frac{x}{24}$ $x \approx 21$	2 ANS: 2 $\tan 32 = \frac{x}{25}$ $x \approx 15.6$
3 ANS: 3 $\sin A = \frac{10}{16}$ $B = 180 - (90 = 38.7) = 51.3$. A 90° angle is not acute. $A \approx 38.7$	4 ANS: 1

Algebra Lesson Page

Simplifying Radicals

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(For this lesson, the term "radical" will refer only to "square root".)

When working with the simplification of radicals you must remember some basic information about **perfect square** numbers.

You need to remember:

Perfect Squares
4 = 2 x 2
9 = 3 x 3
16 = 4 x 4
25 = 5 x 5
36 = 6 x 6
49 = 7 x 7
64 = 8 x 8
81 = 9 x 9
100 = 10 x 10

Radicals (square roots)
$\sqrt{4} = 2$
$\sqrt{9} = 3$
$\sqrt{16} = 4$
$\sqrt{25} = 5$
$\sqrt{36} = 6$
$\sqrt{49} = 7$
$\sqrt{64} = 8$
$\sqrt{81} = 9$
$\sqrt{100} = 10$

While there are certainly many more perfect squares, the ones appearing in the charts above are the ones most commonly used.

To simplify means to find another expression with the same value. It does not mean to find a decimal approximation.

To simplify (or reduce) a radical:

1. Find the **largest** perfect square which will divide evenly into the number under your radical sign. This means that when you divide, you get no remainders, no decimals, no fractions.

Reduce: $\sqrt{48}$ the **largest** perfect square that divides evenly into 48 is **16.**

If the number under your radical cannot be divided evenly by any of the perfect squares, your radical is already in simplest form and cannot be reduced further.

2. Write the number appearing under your radical as the product (multiplication) of the perfect square and your answer from dividing.

$$\sqrt{48} = \sqrt{16 \cdot 3}$$

3. Give each number in the product its own radical sign.

$$\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3}$$

4. Reduce the "perfect" radical which you have now created.

$$\sqrt{48} = \sqrt{16 \cdot 3} = \sqrt{16} \cdot \sqrt{3} = 4\sqrt{3}$$

5. You now have your answer.

$$\sqrt{48} = 4\sqrt{3}$$

What happens if I do not choose the largest perfect square to start the process?

If instead of choosing 16 as the largest perfect square to start this process,
you choose 4, look what happens.....

$$\sqrt{48} = \sqrt{4 \cdot 12}$$

$$\sqrt{48} = \sqrt{4 \cdot 12} = \sqrt{4} \cdot \sqrt{12} = 2\sqrt{12}$$

Unfortunately, this answer is **not in simplest form.**
The 12 can also be divided by the perfect square (4).

$$2\sqrt{12} = 2\sqrt{4 \cdot 3} = 2\sqrt{4} \cdot \sqrt{3} = 2 \cdot 2\sqrt{3} = 4\sqrt{3}$$

If you do not choose the largest perfect square to start the process, you will have to repeat the process.

Example:

Reduce: $3\sqrt{50}$

Don't let the number in front of the radical distract you.
It is simply "along for the ride" and will be multiplied times our final answer.

The largest perfect square dividing evenly into 50 is 25.

$$3\sqrt{50} = 3\sqrt{25 \cdot 2} = 3\sqrt{25}\sqrt{2}$$

Reduce the "perfect" radical and multiply times the 3 (who is "along for the ride")

$$3\sqrt{25}\sqrt{2} = 3 \cdot 5\sqrt{2} = 15\sqrt{2}$$

Note: The examples shown in these lessons on radicals show **ALL** of the steps in the process. It may **NOT** be necessary for you to list **EVERY** step. As long as you understand the process and can arrive at the correct answer, you are **ALL SET**!!

Evaluate:

1. $\sqrt{169}$

   [A] 169 [B] 13 [C] –13 [D] –169

2. $-\sqrt{64}$

   [A] 64 [B] 8 [C] –64 [D] –8

Simplify:

3. $\sqrt{\dfrac{121}{144}}$

   [A] $\dfrac{13}{14}$ [B] $\dfrac{11}{144}$ [C] $\dfrac{11}{12}$ [D] $\dfrac{11}{72}$

4. $\sqrt{\dfrac{81}{100}}$

5. $-\sqrt{\dfrac{16}{25}}$

6. $\sqrt{0.49}$

   [A] 0.35 [B] 0.7 [C] 0.07 [D] 0.035

Find:

7. $\sqrt{1.21}$

8. $\sqrt{0.36}$

9. Is the statement "$-7 < -\sqrt{38} < -6$" *true* or *false*? Explain your answer.

10. $\sqrt{45}$

11. $\sqrt{75}$

12. $2\sqrt{8}$

13. $-3\sqrt{90}$

14. $\dfrac{1}{3}\sqrt{27}$

To add radicals, **simplify first if possible**, and add "like" radicals.

1. $\sqrt{12}+9\sqrt{\frac{1}{3}}+\sqrt{8}-\sqrt{72}$

$=2\sqrt{3}+3\sqrt{3}+2\sqrt{2}-6\sqrt{2}$

$=5\sqrt{3}-4\sqrt{2}$

2. $\sqrt[3]{8x}+\sqrt[3]{16x}+\sqrt[3]{27x}$

$=2\sqrt[3]{x}+2\sqrt[3]{2x}+3\sqrt[3]{x}$

$=5\sqrt[3]{x}+2\sqrt[3]{2x}$

In #1 above, after simplifying the radicals, it became apparent which radicals could be added	In #2 above, be sure to combine ONLY like radicals. Also be sure to **continue writing the index of 3**. Failing to do so creates an incorrect answer using square root, instead of the correct cube root.

Add or subtract.

1. $9\sqrt{7}+4\sqrt{7}=$ ____ $\sqrt{7}$
2. $-10\sqrt{5}+2\sqrt{5}=$ ____ $\sqrt{5}$
3. $4\sqrt{y}+6\sqrt{y}=$ _____
4. $-2\sqrt{3b}+10\sqrt{3b}=$ ______________
5. $6\sqrt{15}-\sqrt{15}+\sqrt{15}=$ _____
6. $5\sqrt{2}-3\sqrt{2x}-4\sqrt{2}=$ ______________

Simplify each expression.

7. $\sqrt{108}+\sqrt{75}$ ______________
8. $\sqrt{63}+\sqrt{175}+\sqrt{112}$ ______________
9. $\sqrt{28x}+\sqrt{63x}$ ______________
10. $\sqrt{45}+\sqrt{180}$ ______________
11. $\sqrt{52}-\sqrt{1300}$ ______________
12. $5\sqrt{98}-3\sqrt{32}$ ______________
13. $\sqrt{32}+\sqrt{128}$ ______________
14. $\sqrt{147}+6\sqrt{3}$ ______________
15. $\sqrt{168}+\sqrt{42}$ ______________
16. $5\sqrt{17}+17\sqrt{5}$ ______________
17. $6\sqrt{3}+\sqrt{300}$ ______________
18. $-2\sqrt{3b}+\sqrt{27b}$ ______________
19. $4\sqrt{2m}+6\sqrt{3m}-4\sqrt{2m}$ ______________
20. $\sqrt{50m}+\sqrt{72m}$ ______________
21. $\sqrt{16z}+2\sqrt{8z}-3\sqrt{z}$ ______________
22. $\sqrt{216t}+\sqrt{96t}$ ______________
23. $4\sqrt{52x}+\sqrt{117x}-2\sqrt{13}$ ______________
24. $3\sqrt{96k}+2\sqrt{180}$ ______________

Multiplication/Division of Radicals

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(For this lesson, the term "radical" will refer only to "square root".)

When **multiplying** radicals, you must multiply the numbers **OUTSIDE (O)** the radicals **AND** then multiply the numbers **INSIDE (I)** the radicals.

$$O_1\sqrt{I_1}\bullet O_2\sqrt{I_2} = O_1\bullet O_2\bullet\sqrt{I_1\bullet I_2}$$

$$2\sqrt{3}\bullet 4\sqrt{5} = 2\bullet 4\bullet\sqrt{3\bullet 5} = 8\sqrt{15}$$

Example 1:

Multiply and simplify: $2\sqrt{18}\bullet 3\sqrt{8}$

1. Multiply the outside numbers first — $2\bullet 3 = 6$
2. Multiply the inside numbers — $\sqrt{18}\bullet\sqrt{8} = \sqrt{144}$
3. Put steps 1 and 2 together and simplify — $6\sqrt{144} = 6\bullet 12 = 72$
4. **Answer:** — 72

Multiply. Write each product in simplest form.

1. $\sqrt{15}\bullet\sqrt{5}$ __________
2. $\sqrt{42}\bullet\sqrt{12}$ __________
3. $\left(2\sqrt{10}\right)^2$ __________
4. $5\left(\sqrt{5}\right)^2$ __________
5. $3\sqrt{6x}\bullet\sqrt{10x}$ __________
6. $4\sqrt{6x}\bullet\sqrt{12x}$ __________
7. $\sqrt{3}\left(\sqrt{12}+6\right)$ __________
8. $\sqrt{6}\left(\sqrt{10c}-\sqrt{8}\right)$ __________
9. $\left(10+\sqrt{5}\right)\left(4-\sqrt{5}\right)$ __________
10. $\sqrt{7}\left(\sqrt{14}+2\right)$ __________
11. $\sqrt{3}\left(\sqrt{3}-\sqrt{6}\right)$ __________
12. $\left(9-\sqrt{3}\right)\left(4-\sqrt{3}\right)$ __________
13. $\left(4+\sqrt{5}\right)\left(1-\sqrt{5}\right)$ __________
14. $\left(2\sqrt{5}-\sqrt{3}\right)\left(\sqrt{5}-\sqrt{3}\right)$ __________
15. $\left(9-\sqrt{3}\right)^2$ __________

When **dividing** radicals, you must divide the numbers **OUTSIDE (O)** the radicals **AND** then divide the numbers **INSIDE (I)** the radicals.

$$\frac{O_1\sqrt{I_1}}{O_2\sqrt{I_2}} = \frac{O_1}{O_2}\bullet\sqrt{\frac{I_1}{I_2}}$$ such as

… $$\frac{4\sqrt{15}}{2\sqrt{3}} = \frac{4}{2}\bullet\sqrt{\frac{15}{3}} = 2\sqrt{5}$$

Divide and simplify: $\dfrac{-12\sqrt{24}}{3\sqrt{2}}$

1. Divide the outside numbers first. $\dfrac{-12}{3} = -4$

2. Divide the inside numbers. $\dfrac{\sqrt{24}}{\sqrt{2}} = \sqrt{12}$

3. Put steps 1 and 2 together and simplify. $-4\sqrt{12} = -4\sqrt{4\bullet 3} = -4\bullet 2\sqrt{3} = -8\sqrt{3}$

4. **Answer:** $-8\sqrt{3}$

1. $\dfrac{\sqrt{48}}{\sqrt{12}}$

2. $\dfrac{\sqrt{50}}{\sqrt{5}}$

3. $\dfrac{6\sqrt{24}}{\sqrt{6}}$

4. $\dfrac{9\sqrt{90}}{3\sqrt{10}}$

5. $\dfrac{\sqrt{48}+\sqrt{16}}{\sqrt{8}}$

6. $\dfrac{\sqrt{75}-\sqrt{27}}{\sqrt{3}}$

7. $\dfrac{-5\sqrt{40}}{\sqrt{5}}$

8. $\dfrac{8\sqrt{60}}{-2\sqrt{5}}$

Answer Key

Evaluate:

1) B 2) D

Simplify:

3) $\sqrt{\frac{121}{144}} = \frac{\sqrt{121}}{\sqrt{144}} = 11/12$ C

4) $\sqrt{\frac{81}{100}} = \frac{\sqrt{81}}{\sqrt{100}} = \frac{9}{10}$

5) $-\sqrt{\frac{16}{25}} = -\frac{\sqrt{16}}{\sqrt{25}} = \frac{-4}{5}$

6) $\sqrt{0.49} = .7$

Find

7) $\sqrt{121} = 11$

8) $\sqrt{0.36} = .6$

9) $-7 < -\sqrt{38} < -6$

$-\sqrt{49} < -\sqrt{38} < -\sqrt{36}$

True

10) $\sqrt{45} = \sqrt{9}\sqrt{5}$

$= 3\sqrt{5}$

11) $\sqrt{75} = \sqrt{25}\sqrt{3}$

$= 5\sqrt{3}$

12) $2\sqrt{8} = 2\sqrt{4}\sqrt{2}$

$= 2(2)\sqrt{2}$

$= 4\sqrt{2}$

13) $-3\sqrt{90} = -3\sqrt{9}\sqrt{10}$

$= -3(3)\sqrt{10}$

$= -9\sqrt{10}$

14) $\frac{1}{3}\sqrt{27} = \frac{1}{3}\sqrt{9}\sqrt{3}$

$= \frac{1}{3}(3)\sqrt{3}$

$= \sqrt{3}$

Add or subtract

1) $13\sqrt{7}$ 2) $-8\sqrt{5}$ 3) $10\sqrt{y}$ 4) $8\sqrt{3b}$

5) $6\sqrt{15}$ 6) $\sqrt{2} - 3\sqrt{2x}$

7) $\sqrt{108} + \sqrt{75}$

$\sqrt{36}\sqrt{3} + \sqrt{25}\sqrt{3}$

$6\sqrt{3} + 5\sqrt{3}$

$11\sqrt{3}$

8) $\sqrt{63} + \sqrt{175} + \sqrt{112}$

$\sqrt{9}\sqrt{7} + \sqrt{25}\sqrt{7} + \sqrt{16}\sqrt{7}$

$3\sqrt{7} + 5\sqrt{7} + 4\sqrt{7}$

$12\sqrt{7}$

9) $\sqrt{28x} + \sqrt{63x}$

$\sqrt{4}\sqrt{7x} + \sqrt{9}\sqrt{7x}$

$2\sqrt{7x} + 3\sqrt{7x}$

$5\sqrt{7x}$

10) $\sqrt{45} + \sqrt{180}$

$\sqrt{9}\sqrt{5} + \sqrt{36}\sqrt{5}$

$3\sqrt{5} + 6\sqrt{5}$

$9\sqrt{5}$

11) $\sqrt{52} - \sqrt{1300}$

$\sqrt{4}\sqrt{13} - \sqrt{100}\sqrt{13}$

$2\sqrt{13} - 10\sqrt{13}$

$-8\sqrt{13}$

12) $5\sqrt{98} - 3\sqrt{32}$

$5\sqrt{49}\sqrt{2} - 3\sqrt{16}\sqrt{2}$

$5(7)\sqrt{2} - 3(4)\sqrt{2}$

$35\sqrt{2} - 12\sqrt{2}$

$23\sqrt{2}$

13) $\sqrt{32} + \sqrt{128}$

$\sqrt{16}\sqrt{2} + \sqrt{64}\sqrt{2}$

$4\sqrt{2} + 8\sqrt{2}$

$12\sqrt{2}$

14) $\sqrt{147} + 6\sqrt{3}$

$\sqrt{49}\sqrt{3} + 6\sqrt{3}$

$7\sqrt{3} + 6\sqrt{3}$

$13\sqrt{3}$

15) $\sqrt{168} + \sqrt{42}$

$\sqrt{4}\sqrt{42} + \sqrt{42}$

$2\sqrt{42} + \sqrt{42}$

$3\sqrt{42}$

16) $5\sqrt{17} + 17\sqrt{5}$

17.) $6\sqrt{3} + \sqrt{300}$

$6\sqrt{3} + \sqrt{100}\sqrt{3}$

$6\sqrt{3} + 10\sqrt{3}$

$16\sqrt{3}$

19) $4\sqrt{2m} + 6\sqrt{3m} - 4\sqrt{2m}$

$6\sqrt{3m}$

18) $-2\sqrt{3b} + \sqrt{27b}$

$-2\sqrt{3b} + \sqrt{9}\sqrt{3b}$

$-2\sqrt{3b} + 3\sqrt{3b}$

$1\sqrt{3b}$

20) $\sqrt{50m} + \sqrt{72m}$

$\sqrt{50}\sqrt{2m} + \sqrt{36}\sqrt{2m}$

$5\sqrt{2m} + 6\sqrt{2m}$

$11\sqrt{2m}$

21) $\sqrt{16z} + 2\sqrt{8z} - 3\sqrt{z}$

$4\sqrt{z} + 2\sqrt{4}\sqrt{2z} - 3\sqrt{z}$

$4\sqrt{z} + 2(2)\sqrt{2z} - 3\sqrt{z}$

$1\sqrt{z} + 4\sqrt{2z}$

22) $\sqrt{216t} + \sqrt{96t}$

$\sqrt{36}\sqrt{6t} + \sqrt{16}\sqrt{6t}$

$6\sqrt{6t} + 4\sqrt{6t}$

$10\sqrt{6t}$

23) $4\sqrt{52x} + \sqrt{117x} - 2\sqrt{13}$

$4\sqrt{4}\sqrt{13x} + \sqrt{9}\sqrt{13x} - 2\sqrt{13}$

$4(2)\sqrt{13x} + 3\sqrt{13x} - 2\sqrt{13}$

$8\sqrt{13x} + 3\sqrt{13x} - 2\sqrt{13}$

$11\sqrt{13x} - 2\sqrt{13}$

24) $3\sqrt{96k} + 2\sqrt{180}$

$3\sqrt{16}\sqrt{6k} + 2\sqrt{36}\sqrt{5}$

$3(4)\sqrt{6k} + 2(6)\sqrt{5}$

$12\sqrt{6k} + 12\sqrt{5}$

multiply + Simplify

1) $\sqrt{15}\cdot\sqrt{5} = \sqrt{75}$

$\sqrt{25}\sqrt{3}$

$5\sqrt{3}$

2) $\sqrt{42}\cdot\sqrt{12} = \sqrt{504}$

$\sqrt{36}\sqrt{14}$

$6\sqrt{14}$

3) $(2\sqrt{10})^2 = 2\sqrt{10}\cdot 2\sqrt{10}$

$= 4\sqrt{100}$

$= 4(10)$

$= 40$

4) $5(\sqrt{5})^2$

$= 5\cdot\sqrt{5}\cdot\sqrt{5}$

$= 5\cdot\sqrt{25}$

$= 5\cdot 5$

$= 25$

5) $3\sqrt{6x}\cdot\sqrt{10x}$

$3\sqrt{60x^2}$

$3\sqrt{4x^2}\sqrt{15}$

$3(2x)\sqrt{15}$

$6x\sqrt{15}$

6) $4\sqrt{6x}\cdot\sqrt{2x}$

$4\sqrt{72x^2}$

$4\sqrt{36x^2}\sqrt{2}$

$4(6x)\sqrt{2}$

$24x\sqrt{2}$

7) $\sqrt{3}(\sqrt{12}+6)$

$\sqrt{36}+6\sqrt{3}$

$6+6\sqrt{3}$

8) $\sqrt{6}(\sqrt{10c}-\sqrt{8})$

$\sqrt{60c}-\sqrt{48}$

$\sqrt{4}\sqrt{15c}-\sqrt{16}\sqrt{3}$

$(2)\sqrt{15c}-4\sqrt{3}$

$2\sqrt{15c}-4\sqrt{3}$

9) $(10+\sqrt{5})(4-\sqrt{5})$

$40 \div 10\sqrt{5} + 4\sqrt{5} - \sqrt{25}$

$40-10\sqrt{5}+4\sqrt{5}-5$

$35-6\sqrt{5}$

10) $\sqrt{7}(\sqrt{14}+2)$

$\sqrt{98}+2\sqrt{7}$

$\sqrt{49}\sqrt{2}+2\sqrt{7}$

$7\sqrt{2}+2\sqrt{7}$

11) $\sqrt{3}(\sqrt{3}-\sqrt{6})$

$\sqrt{9}-\sqrt{18}$

$3-\sqrt{9}\sqrt{2}$

$3-3\sqrt{2}$

12) $(9-\sqrt{3})(4-\sqrt{3})$

$36-9\sqrt{3}-4\sqrt{3}+\sqrt{9}$

$36-9\sqrt{3}-4\sqrt{3}+3$

$39-13\sqrt{3}$

13) $(4+\sqrt{5})(1-\sqrt{5})$

$4-4\sqrt{5}+\sqrt{5}-\sqrt{25}$

$4-4\sqrt{5}+\sqrt{5}-5$

$-1-3\sqrt{5}$

14) $(2\sqrt{5}-\sqrt{3})(\sqrt{5}-\sqrt{3})$

$2\sqrt{25}-2\sqrt{15}-\sqrt{15}+\sqrt{9}$

$2(5)-2\sqrt{15}-\sqrt{15}+3$

$10-3\sqrt{15}+3$

$13-3\sqrt{15}$

15) $(9-\sqrt{3})^2$

$(9-\sqrt{3})(9-\sqrt{3})$

$81-9\sqrt{3}-9\sqrt{3}+\sqrt{9}$

$81-18\sqrt{3}+3$

$84-18\sqrt{3}$

Divide & Simplify

1) $\frac{\sqrt{48}}{\sqrt{12}}=\sqrt{4}=2$

2) $\frac{\sqrt{50}}{\sqrt{5}}=\sqrt{10}$

3) $\frac{6\sqrt{24}}{\sqrt{6}}$

$=6\sqrt{4}$

$=6(2)$

$=12$

4) $\frac{9\sqrt{90}}{3\sqrt{10}}=3\sqrt{9}$

$=3(3)$

$=9$

5) $\frac{\sqrt{48}+\sqrt{16}}{\sqrt{8}}$

$\sqrt{6}+\sqrt{2}$

6) $\frac{\sqrt{75}-\sqrt{27}}{\sqrt{3}}=\sqrt{25}-\sqrt{9}$

$=5-3$

$=2$

7) $\frac{-5\sqrt{40}}{\sqrt{5}}=-5\sqrt{8}$

$=-5\sqrt{4}\sqrt{2}$

$=-5(2)\sqrt{2}$

$=-10\sqrt{2}$

8) $\frac{8\sqrt{60}}{-2\sqrt{5}}=-4\sqrt{12}$

$=-4\sqrt{4}\sqrt{3}$

$=-4(2)\sqrt{3}$

$=-8\sqrt{3}$