some answers to ∫ packet

7. $\int \frac{\sin x}{\cos^5 x}\,dx$

$-\frac{(\cos x)^{-4}}{-4}+C$

2 8. $\int \frac{(\sqrt{x}-1)^2}{2\sqrt{x}}\,dx$

$\frac{2(\sqrt{x}-1)^3}{3}+C$

9. $\int \sqrt{x^3+x^2}\,(3x^2+2x)\,dx$

$\frac{2}{3}(x^3+x^2)^{3/2}+C$

$\frac{1}{2}$ 10. $\int \frac{2x+1}{(x^2+2x+2)^3}\,dx$

$\frac{1}{2}\frac{(x^2+2x+2)^{-2}}{-2}+C$

11. $\int \cos 2x\sqrt{\sin 2x}\,dx$

$\frac{1}{2}(\sin 2x)^{3/2}\cdot\frac{2}{3}+C$

$\frac{(\sin 2x)^{3/2}}{3}+C$

12. $\int (x+1)\sin(x^2+2x+3)\,dx$

$-\frac{1}{2}\cos(x^2+2x+3)+C$

13. $\int \left(1+\frac{1}{t}\right)^3 \frac{1}{t^2}\,dt$

$-\frac{1}{4}\left(1+\frac{1}{t}\right)^4+C$

14. $\int x^2\sqrt{x^3+1}\,dx$

$\frac{1}{3}(x^3+1)^{3/2}\cdot\frac{2}{3}+C$

$\frac{2}{9}(x^3+1)^{3/2}+C$

$\frac{1}{3}$ 15. $\int \frac{2}{\sqrt{3x-7}}\,dx$

$\frac{2}{3}(3x-7)^{1/2}\cdot 2$

$\frac{4}{3}(3x-7)^{1/2}+C$

2 16. $\int \frac{1}{2\sqrt{x}(\sqrt{x}+1)^2}\,dx$

$-2(\sqrt{x}+1)^{-1}+C$

17. $\int \frac{x}{\sqrt{x+1}}\,dx$

$u=x+1$

$u-1=x$

$(u-1)u^{-1/2}$

$\int u^{1/2}-u^{-1/2}$

$\frac{2}{3}u^{3/2}-2u^{1/2}+C$

$\frac{2}{3}(x+1)^{3/2}-2(x+1)^{1/2}+C$

18. $\int x\sqrt{2x+1}\,dx$

$2x+1=u$

$2x=u-1$

$x=\frac{u-1}{2}$ $\quad dx=\frac{du}{2}$

$\int\left(\frac{u-1}{2}\cdot u^{1/2}\right)\frac{du}{2}$

$\frac{1}{4}\int(u^{3/2}-u^{1/2})\,du$

$\frac{1}{4}\left[\frac{u^{5/2}}{5/2}-\frac{u^{3/2}}{3/2}\right]+C$

$\frac{(2x+1)^{5/2}}{10}-\frac{2(2x+1)^{3/2}}{3}+C$

19. $\int \sqrt{x}\sqrt{x\sqrt{x}+1}\,dx$

$\int \sqrt{x}\,(x^{3/2}+1)^{1/2}\,dx$ $u = x^{3/2}+1$

$du = \frac{3}{2}x^{1/2}\,dx$

$\frac{2}{3}du = x^{1/2}\,dx$

$\frac{2}{3}\int u^{1/2}\,du$

$\frac{2}{3}\,u^{3/2}\cdot\frac{2}{3}+C$

$\frac{4}{9}(x^{3/2}+1)^{3/2}+C$

20. $\int x\tan(x^2)\sec(x^2)\,dx$

$\frac{1}{2}\sec(x^2)+C$

21. $\int (x^2+1)\sqrt{x-2}\,dx$

22. $\int \frac{x^2+2x}{x^2+2x+1}\,dx$

23. $\int \frac{1}{x^2+6x+9}\,dx$

$\int (x+3)^{-2}\,dx$

$-1(x+3)^{-1}+C$

24. $\int \frac{\sec^2 x}{(1+\tan x)^3}\,dx$

$u = 1+\tan x$

$du = \sec^2 x\,dx$

$\int u^{-3}\,du = \frac{u^{-2}}{-2}+C$

$\frac{(1+\tan x)^{-2}}{-2}+C$

$-\frac{1}{3}$ 25. $\int \frac{-3\sin x}{(2+3\cos x)^2}\,dx$

$u = 2+3\cos x$

$du = -3\sin x\,dx$

$+\frac{1}{3}(2+3\cos x)^{-1}+C$

26. $\int x\tan^2(x^2)\sec^2(x^2)\,dx$

$u = \tan(x^2)$

$du = \sec^2(x^2)\cdot 2x\,dx$

$\int u^2\frac{du}{2} = \frac{\tan^3(x^2)}{6}+C$

27. $\int (\tan 2x+\cot 2x)^2\,dx$

28. $\frac{1}{2}\int \frac{2xe^{x^2}}{e^{x^2}+1}\,dx$

$u = e^{x^2}+1$

$du = e^{x^2}\cdot 2x\,dx$

$\frac{1}{2}\ln(e^{x^2}+1)+C$

29. $\int \frac{1}{\sqrt{-x^2+5x-6}}\,dx$

30. $\int \frac{x}{1+x^2}\,dx$

$u = 1+x^2$

$du = 2x\,dx$

$\frac{1}{2}\ln(1+x^2)+C$

31. $\int \frac{4}{5x\sqrt{x^2-3}}dx$

32. $\int \frac{x^2}{1+x^2}dx$

33. $\int xe^{x^2}dx$

$u = x^2$
$du = 2x\,dx$
$\frac{du}{2} = x\,dx$

$\frac{1}{2}e^{x^2} + C$

34. $\int \frac{x}{\sqrt{x-1}}dx$

35. $\int \left(6x + \frac{7}{\sqrt{9-x^2}}\right)dx$

36. $\int x^2\sqrt{x+1}dx$

$u = x+1$
$du = dx$
$u-1 = x$
$(u-1)^2 = x^2$

$\int (u-1)^2 \cdot u^{1/2} =$
$\int (u^2-2u+1)u^{1/2} =$
$\int u^{\frac{5}{2}} - 2u^{3/2} + u^{1/2}\,du =$

$\frac{2}{7}u^{7/2} - 2u^{\frac{3}{2}}\cdot\frac{2}{5} + u^{\frac{3}{2}}\cdot\frac{2}{3} + C$
$\frac{2}{7}(x+1)^{7/2} - \frac{4}{5}(x+1)^{5/2} + \frac{2}{3}(x+1)^{3/2} + C$

37. $\int \left(1+e^{-x}\right)^2 dx$

$\int (1 + 2e^{-x} + e^{-2x})dx$
$x + -2e^{-x} + \frac{-1}{2}e^{-2x} + C$

38. $\int \frac{6\cos x - 2\sin x}{6\sin x + 2\cos x}dx$

$u = 6\sin x + 2\cos x$
$du = 6\cos x - 2\sin x$
$\int \frac{du}{u} = \ln u + C$
$\ln(6\sin x + 2\cos x) + C$

39. $\int \frac{4}{x}\sqrt[3]{(1+2\ln x)^2}dx$

40. $\int \frac{2e^{\tan x}+5}{\cos^2 x}dx$

$\int (2e^{\tan x} + 5)\sec^2 x\,dx$
$u = \tan x$
$du = \sec^2 x$
$\int 2e^{\tan x}\sec^2 x + 5\sec^2 x\,dx$
$2e^{\tan x} + 5\tan x + C$

41. $\int \frac{(1-x^2)^{-1/2}}{3+2\arcsin x}dx$

42. $\int \frac{t^3}{\sqrt{1-t^8}}dt$

$u = t^4$
$du = 4t^3\,dt$
$\frac{du}{4} = t^3\,dt$
$\frac{1}{4}\int \frac{du}{\sqrt{1-u^2}} =$
$\frac{1}{4}\sin^{-1}(t^4) + C$

43. $\int \frac{5-x}{\sqrt{4-5x^2}}dx$