Rational Exponents

- In other words, exponents that are fractions.

Definition of $b^{\frac{1}{n}}$

- For any real number b and any integer n > 1,

$$b^{\frac{1}{n}} = \sqrt[n]{b}$$

- except when b < 0 and n is even

Examples:

1. $36^{\frac{1}{2}} = \sqrt{36} = \boxed{6}$

2. $64^{\frac{1}{3}} = \sqrt[3]{64} = \boxed{4}$

Examples:

3. $49^{-\frac{1}{2}} = \left(\frac{1}{49}\right)^{\frac{1}{2}} = \sqrt{\frac{1}{49}} = \boxed{\frac{1}{7}}$

4. $\left(\frac{1}{8}\right)^{-\frac{1}{3}} = (8)^{\frac{1}{3}} = \sqrt[3]{8} = \boxed{2}$

- Economists refer to inflation as increases in the average cost of purchases. The formula $C = c(1 + r)^n$ can be used to predict the cost of consumer items at some projected time. In this formula C represents the projected cost of the item at the given annual inflation rate, c the present cost of the item and r is the rate of inflation (in decimal form), and n is the number of years for the projection. Suppose a gallon of milk costs $2.69 now. How much would the price increase in 6 months with an inflation rate of 5.3%?

Step 1: Identify the known values

Formula $C = c(1 + r)^n$

$c = \$2.69$

present cost of the item

$r = 0.053$

rate of inflation (in decimal form)

$n = 1/2$

# of years for the projection

Step 2: Find the value for C

Formula $C = c(1 + r)^n$

$$C = 2.69(1 + 0.053)^{1/2}$$

$$C = 2.69(1.053)^{1/2}$$

$$C = \$2.76$$

Answer the question

How much would the price increase?

$2.76-$2.69 = $0.07 or 7¢

Definition of Rational Exponents

- For any nonzero number b and any integers m and n with n > 1,

$$b^{\frac{m}{n}} = \sqrt[n]{b^m} = \left(\sqrt[n]{b}\right)^m$$

- except when b < 0 and n is

NOTE: There are 3 different ways to write a rational exponent

$$27^{\frac{4}{3}} = \sqrt[3]{27^4} = \left(\sqrt[3]{27}\right)^4$$

Examples:

1. $36^{\frac{3}{2}} = \left(\sqrt{36}\right)^3 = (6)^3 = \boxed{216}$

2. $27^{\frac{4}{3}} = \left(\sqrt[3]{27}\right)^4 = (3)^4 = \boxed{81}$

3. $81^{\frac{3}{4}} = \left(\sqrt[4]{81}\right)^3 = (3)^3 = \boxed{27}$

Simplifying Expressions

- No negative exponents
- No fractional exponents in the denominator
- No complex fractions (fraction within a fraction)
- The index of any remaining radical is the least possible number

Examples: Simplify each expression

$$4^{\frac{1}{3}} \cdot a^{\frac{1}{2}} \cdot b^{\frac{5}{6}} = 4^{\frac{2}{6}} \cdot a^{\frac{3}{6}} \cdot b^{\frac{5}{6}}$$

$$= \sqrt[6]{4^2} \cdot \sqrt[6]{a^3} \cdot \sqrt[6]{b^5}$$

$$= \boxed{\sqrt[6]{16a^3b^5}}$$

Rewrite as a radical

Get a common denominator - this is going to be our index

Examples: Simplify each expression

$$x^{\frac{1}{2}} \cdot x^{\frac{3}{4}} \cdot x^{\frac{1}{5}} = x^{\frac{1}{2}+\frac{3}{4}+\frac{1}{5}}$$

Remember we add exponents

$$= x^{\frac{10}{20}+\frac{15}{20}+\frac{4}{20}} = x^{\frac{29}{20}}$$

$$= x^{\frac{20}{20}} \cdot x^{\frac{9}{20}} = \boxed{x\sqrt[20]{x^9}}$$

Examples: Simplify each expression

$$w^{-\frac{4}{5}} = \left(\frac{1}{w}\right)^{\frac{4}{5}} = \frac{1^{\frac{4}{5}}}{w^{\frac{4}{5}}} = \frac{1}{w^{\frac{4}{5}}} \cdot \frac{w^{\frac{1}{5}}}{w^{\frac{1}{5}}}$$

$$= \frac{w^{\frac{1}{5}}}{w^{\frac{4}{5}+\frac{1}{5}}} = \frac{w^{\frac{1}{5}}}{w^{\frac{5}{5}}} = \frac{w^{\frac{1}{5}}}{w} = \boxed{\frac{\sqrt[5]{w}}{w}}$$

To rationalize the denominator we want an integer exponent

Examples: Simplify each expression

$$xy^{-\frac{1}{8}} = x\left(\frac{1}{y}\right)^{\frac{1}{8}} = x \cdot \frac{1}{y^{\frac{1}{8}}} = \frac{x}{y^{\frac{1}{8}}} \cdot \frac{y^{\frac{7}{8}}}{y^{\frac{7}{8}}}$$

$$= \frac{xy^{\frac{7}{8}}}{y} = \boxed{\frac{x\sqrt[8]{y^7}}{y}}$$

To rationalize the denominator we want an integer exponent

Examples: Simplify each expression

$$\frac{\sqrt[10]{32}}{\sqrt[8]{4}} = \frac{32^{\frac{1}{10}}}{4^{\frac{1}{8}}} = \frac{\left(2^{5}\right)^{\frac{1}{10}}}{\left(2^{2}\right)^{\frac{1}{8}}} = \frac{2^{\frac{5}{10}}}{2^{\frac{2}{8}}}$$

$$= \frac{2^{\frac{1}{2}}}{2^{\frac{1}{4}}} = 2^{\frac{1}{2}-\frac{1}{4}} = 2^{\frac{2}{4}-\frac{1}{4}} = 2^{\frac{1}{4}} = \boxed{\sqrt[4]{2}}$$

Examples: Simplify each expression

$$\frac{5^{-\frac{1}{2}}}{2\sqrt{5}} = \frac{5^{-\frac{1}{2}}}{2\cdot 5^{\frac{1}{2}}} = \frac{1}{2}\cdot\frac{5^{-\frac{1}{2}}}{5^{\frac{1}{2}}}$$

$$= \frac{1}{2}\cdot 5^{-\frac{1}{2}-\frac{1}{2}} = \frac{1}{2}\cdot 5^{-1} = \frac{1}{2}\cdot\frac{1}{5} = \boxed{\frac{1}{10}}$$

Examples: Simplify each expression

$$\frac{1}{m^{\frac{1}{2}}-1}\cdot\left(\frac{m^{\frac{1}{2}}+1}{m^{\frac{1}{2}}+1}\right)$$

Multiply by conjugate and use FOIL

$$=\frac{m^{\frac{1}{2}}+1}{m-1}=\boxed{\frac{\sqrt{m}+1}{m-1}}$$

Examples: Simplify each expression

$$6^{\frac{2}{5}} y^{\frac{8}{5}} = 6^{\frac{2}{5}} y^{\frac{5}{5}} y^{\frac{3}{5}} = 6^{\frac{2}{5}} \cdot y \cdot y^{\frac{3}{5}}$$

$$= y \cdot \sqrt[5]{6^2} \cdot \sqrt[5]{y^3} = y\sqrt[5]{6^2 y^3}$$

$$= \boxed{y\sqrt[5]{36y^3}}$$

Examples: Simplify each expression

$$\frac{c^{\frac{2}{3}} - c}{c^{\frac{-1}{2}}} = c^{\frac{1}{2}}\left(c^{\frac{2}{3}} - c\right)$$

$$= c^{\frac{2}{3}} \cdot c^{\frac{1}{2}} - c \cdot c^{\frac{1}{2}} = c^{\frac{7}{6}} - c^{\frac{3}{2}}$$

Examples: Simplify each expression

$$= c^{\frac{7}{6}} - c^{\frac{3}{2}} = c^{\frac{6}{6}} c^{\frac{1}{6}} - c^{\frac{2}{2}} c^{\frac{1}{2}}$$

$$= \boxed{c\sqrt[6]{c} - c\sqrt{c}}$$

Examples: Simplify each expression

$$\left(\frac{2x^{-\frac{2}{2}}}{x^{-\frac{3}{2}}}\right)^{-2} = \left(\frac{2x^{\frac{3}{2}}}{x^{2}}\right)^{-2} = \left(2x^{\frac{3}{2}-2}\right)^{-2}$$

$$= \left(2x^{-\frac{1}{2}}\right)^{-2} = 2^{-2}x^{-\frac{1}{2}\cdot -2} = \frac{1}{2^2}\cdot x = \boxed{\frac{x}{4}}$$