Algebra II

Notes on Rational Expressions

Today you will be doing problems that deal with rational expressions, also known as algebraic fractions. You will need to remember the factoring process we have already done in class. **Read this material and follow the steps and directions exactly.**

Topic 1: Simplify rational expressions

Remember from elementary school, all fractions had to be in lowest terms. Therefore

$$\frac{4}{8}$$
 =

$$\frac{27}{33} =$$

To simplify rational expressions, you must have the factors of the expression so you can cancel them, which is another way of doing the dividing in the above fractions.

Here is an example:

Simplify
$$\frac{x^2 - 9}{x^2 - x - 6}$$
.

You **CANNOT** just cross out parts of the expression that match. You **MUST FACTOR** when possible. So, by factoring you get

Each of the items in parenthesis is a factor. Since there is a factor of (x-3) in both the numerator and the denominator, they will cancel (cross them out), and you are left with the answer

You are finished this problem.

Let's do each of these examples together.

$$1. \quad \frac{x^2 - 3x - 4}{2x - 8} =$$

$$\frac{3x+6}{x^2+4x+4} =$$

Now do these problems.

$$\frac{3x^2 + 2x - 1}{x^2 - 1}$$

$$\frac{3x^2 + 2x - 1}{x^2 - 1}$$
 2.
$$\frac{x^2 + 5x + 6}{x^2 + x - 2}$$
 3.

$$\frac{5x+15}{10x+20}$$

Topic 2: **Multiply and Divide Rational Expressions**

Remember from elementary school, that to multiply fractions, you multiply straight across the numerator and straight across the denominator. Then you can reduce the

answer if possible. So do the problem
$$\frac{5}{24} \cdot \frac{16}{25} =$$

You could have also done the problem by canceling first as shown below.

$$\frac{5}{24} \cdot \frac{16}{25} = \frac{1}{3} \cdot \frac{2}{5} = \frac{2}{15}$$

If you wanted to divide two fractions, you inverted (flipped) the second fraction and did

the multiplication steps. I have done $\frac{4}{9} \div \frac{10}{27}$ using canceling below

$$\frac{4}{9} \div \frac{10}{27} = \frac{4}{9} \cdot \frac{27}{10} = \frac{2}{1} \cdot \frac{3}{5} = \frac{6}{5}$$

I used canceling because to multiple (or divide) two rational expressions, you will **need** to factor, cancel, and then multiply to get the answer.

See how I did this problem:
$$\frac{3x+6}{x-4} \cdot \frac{x^2-16}{x^2+6x+8}$$

$$\frac{3x+6}{x-4} \cdot \frac{x^2-16}{x^2+6x+8} = \frac{3(x+2)}{x-4} \cdot \frac{(x+4)(x-4)}{(x+4)(x+2)} = \frac{3}{1} \cdot \frac{1}{1} = 3$$

Here is a division problem. The only difference is I had to flip the second fraction before

I began the problem.

$$\frac{5x}{x^2 - 25} \div \frac{x^2 - 3x}{5x - 25}$$

$$\frac{5x}{x^2 - 25} \div \frac{x^2 - 3x}{5x - 25} = \frac{5x}{x^2 - 25} \cdot \frac{5x - 25}{x^2 - 3x} = \frac{5x}{(x + 5)(x - 5)} \cdot \frac{5(x - 5)}{x(x - 3)} = \frac{5}{(x + 5)} \cdot \frac{5}{(x - 3)} = \frac{25}{(x + 5)(x - 3)}$$

Now do these problems and check the answers below when you finish.

$$\frac{x^2 - 3x - 4}{x^2 + 2x + 1} \cdot \frac{2x - 2}{4x - 4}$$

$$2. \qquad \frac{5x}{10x - 20} \div \frac{x^2}{x^2 - 4}$$

Now do page 373 - 374 # 1 - 12 odd, 20 - 25 all. Put these in notebook to be checked as homework tomorrow.

Answers Simplifying

1.
$$\frac{3x^2 + 2x - 1}{x^2 - 1} = \frac{(3x - 1)(x + 1)}{(x + 1)(x - 1)} = \frac{(3x - 1)}{(x - 1)}$$

$$\frac{x^2 + 5x + 6}{x^2 + x - 2} = \frac{(x+3)(x+2)}{(x+2)(x-1)} = \frac{(x+3)}{(x-1)}$$

3.
$$\frac{5x+15}{10x+20} = \frac{5(x+3)}{10(x+2)} = \frac{(x+3)}{2(x+2)}$$

Answers Multiply/Divide

$$\frac{x^2 - 3x - 4}{x^2 + 2x + 1} \cdot \frac{2x - 2}{4x - 4} = \frac{(x - 4)(x + 1)}{(x + 1)(x + 1)} \cdot \frac{2(x - 1)}{4(x - 1)} = \frac{(x - 4)}{(x + 1)} \cdot \frac{1}{2} = \frac{(x - 4)}{2(x + 1)}$$

2.
$$\frac{5x}{10x - 20} \div \frac{x^2}{x^2 - 4} = \frac{5x}{10x - 20} \cdot \frac{x^2 - 4}{x^2} = \frac{5x}{10(x - 2)} \cdot \frac{(x - 2)(x + 2)}{x^2} = \frac{1}{2} \cdot \frac{(x + 2)}{x} = \frac{(x + 2)}{2x}$$