What you'll learn about

- Derivative of a Composite Function
- "Outside-Inside" Rule
- Repeated Use of the Chain Rule
- Slopes of Parametrized Curves
- Power Chain Rule

… and why

The chain rule is the most widely used differentiation rule in mathematics.

EQ: How do we apply the Chain Rule?

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Rule 8 The Chain Rule

If f is differentiable at the point $u = g(x)$, and g is differentiable at x, then the composite function $(f \circ g)(x) = f(g(x))$ is differentiable at x, and

$$(f \circ g)'(x) = f'(g(x)) \cdot g'(x)$$

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"Outside-Inside" Rule

It sometimes helps to think about the Chain Rule this way:

If $y = f(g(x))$, then $\frac{dy}{dx} = f'(g(x)) \cdot g'(x)$.

In words, differentiate the "outside" function f and evaluate it at the "inside" function $g(x)$ left alone; then multiply by the derivative of the "inside" function.

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Example **"Outside-Inside" Rule**

Differentiate $\cos\left(3x^4 - 2\right)$ with respect to x.

$$\frac{d}{dx}\cos\underbrace{\left(3x^4 - 2\right)}_{\text{inside}} = -\sin\underbrace{\left(3x^4 - 2\right)}_{\text{inside left alone}} \cdot \underbrace{12x^3}_{\substack{\text{derivative}\\ \text{of inside}}}$$

$$= -12x^3 \sin\left(3x^4 - 2\right)$$

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Example **"Outside-Inside" Rule**

Differentiate $2(1 - x)^2 + (1 - x) + 6$ with respect to x.

$2[\,2(1 - x)](-1) + (-1) = -4(1 - x) - 1 = 4x - 5$

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Example **"Outside-Inside" Rule**

Differentiate $(x^2 + 2x + 5)^6$ with respect to x.

$6[(x^2 + 2x + 5)]^5(2x + 2)$

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EQ: How do we apply the Chain Rule?

Example **Derivatives of Composite Functions**

An object moves along the x-axis so that its velocity at any time $t \geq 0$ is given by $v(t) = \cos(2t + 4)$. Find the acceleration of the object as a function of t.

$$a(t) = v'(t)$$

$$v'(t) = \left(-\sin(2t+4)\right)\left(2\right) = -2\sin(2t+4)$$

EQ: How do we apply the Chain Rule?

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EQ: How do we apply the Chain Rule?

Example **Chain Rule and Product/ Quotient Rule**

Find $\frac{dy}{dx}$ for $y = \left(\sin(3x-2)\right)\left(\cos(2x+3)\right)$

$$y' = \left(\sin(3x-2)\right)\left(-2\sin(2x+3)\right) + \left(\cos(2x+3)\right)\left(3\cos(3x-2)\right)$$
$$= -2\sin(2x+3)\sin(3x-2) + 3\cos(2x+3)\cos(3x-2)$$

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Example **Chain Rule and Product/ Quotient Rule**

Find $\frac{dy}{dx}$ for $y = \frac{\tan\left(x^2 - 5x + 3\right)}{x^3}$

$$y' = \frac{x^3\left(\sec^2\left(x^2 - 5x + 3\right)\right)\left(2x - 5\right) - 3x^2\tan\left(x^2 - 5x + 3\right)}{\left(x^3\right)^2}$$

$$= \frac{\left(2x^4 - 5x^3\right)\left(\sec^2\left(x^2 - 5x + 3\right)\right) - 3x^2\tan\left(x^2 - 5x + 3\right)}{x^6}$$

p. 150 EQ: How do we apply the Chain Rule?

Example **Chain Rule and Product/ Quotient Rule**

Find $\frac{dy}{dx}$ for $g(x) = \left(\frac{x+3}{2x-1}\right)^7$

$$g'(x) = 7\left(\frac{x+3}{2x-1}\right)^6\left(\frac{(2x-1)(1)-(x+3)(2)}{(2x-1)^2}\right)$$

$$= \frac{-49(x+3)^6}{(2x-1)^8}$$

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Example **Chain Rule and Product/ Quotient Rule**

Find $\frac{dy}{dx}$ for $y = x^3\sqrt{3x^2+1}$

$$\frac{dy}{dx} = x^3\left(\frac{1}{2}\left(3x^2+1\right)^{-1/2}(6x)\right) + 3x^2\sqrt{3x^2+1}$$

$$= \frac{3x^4}{\sqrt{3x^2+1}} + 3x^2\sqrt{3x^2+1}$$

p. 150 EQ: How do we apply the Chain Rule?

Example **Repeated Use of the Chain Rule**

Sometimes the chain rule needs to be used more than once to find a derivative.

Find the derivative of $y=\sqrt[3]{\sin x^2}$.

$$y=\sqrt[3]{\sin x^2}=\left(\sin x^2\right)^{\frac{1}{3}}$$

$$\frac{dy}{dx}=\frac{1}{3}\left(\sin x^2\right)^{-\frac{2}{3}}\left(\cos x^2\right)\cdot 2x$$

$$\frac{dy}{dx}=\frac{2}{3}x\left(\sin x^2\right)^{-\frac{2}{3}}\left(\cos x^2\right)$$

p. 150 EQ: How do we apply the Chain Rule?

Example **Repeated Use of the Chain Rule**

Sometimes the chain rule needs to be used more than once to find a derivative.

Find the derivative of $y=\cos^4\left(\frac{x^3}{\sin^2 x}\right)$.

$$y'=4\cos^3\left(\frac{x^3}{\sin^2 x}\right)\left(-\sin\left(\frac{x^3}{\sin^2 x}\right)\right)\left(\frac{\sin^2 x\left(3x^2\right)-x^3\left(2\sin x\right)\left(\cos x\right)}{\left(\sin^2 x\right)^2}\right)$$

$$=-4\cos^3\left(\frac{x^3}{\sin^2 x}\right)\left(\sin\left(\frac{x^3}{\sin^2 x}\right)\right)\left(\frac{3x^2\sin^2 x-2x^3\sin x\cos x}{\sin^4 x}\right)$$

p. 150 EQ: How do we apply the Chain Rule?

Example **Repeated Use of the Chain Rule**

Sometimes the chain rule needs to be used more than once to find a derivative.

Find the derivative of $y=\sqrt{\frac{x^2+1}{x^2-1}}$.

$$y'=\frac{1}{2}\left(\frac{x^2+1}{x^2-1}\right)^{-1/2}\left(\frac{\left(x^2-1\right)2x-2x\left(x^2+1\right)}{\left(x^2-1\right)^2}\right)$$

$$=\frac{1}{2}\left(\frac{x^2+1}{x^2-1}\right)^{-1/2}\left(\frac{-4x}{\left(x^2-1\right)^2}\right)=\frac{-2x}{\left(x^2-1\right)^2}\sqrt{\frac{x^2-1}{x^2+1}}$$

p. 150 EQ: How do we apply the Chain Rule?

Slopes of Parametrized Curves

A parametrized curve $(x(t), y(t))$ is *differentiable at t* if x and y are differentiable at t.

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Finding *dy/dx* Parametrically

If all three derivatives exist and $\frac{dx}{dt} \neq 0$,

$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}}$$

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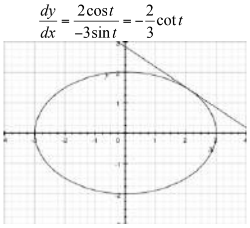
Finding *dy/dx* Parametrically

Find the equation of the line tangent to the curve defined parametrically by $x = 3\cos t$, $y = 2\sin t$ at $t = \pi/4$

$$\frac{dx}{dt} = -3\sin t \quad \frac{dy}{dt} = 2\cos t$$

$$\frac{dy}{dx} = \frac{2\cos t}{-3\sin t} = -\frac{2}{3}\cot t$$



$$x = 3\cos\left(\frac{\pi}{4}\right) = \frac{3\sqrt{2}}{2} \quad y = 2\sin\left(\frac{\pi}{4}\right) = \sqrt{2}$$

$$m = -\frac{2}{3}\cot\left(\frac{\pi}{4}\right) = -\frac{2}{3}$$

$$y - \sqrt{2} = -\frac{2}{3}\left(x - \frac{3\sqrt{2}}{2}\right) \Rightarrow y = -\frac{2}{3}x + 2\sqrt{2}$$

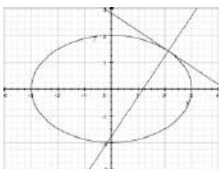
p. 150 EQ: How do we apply the Chain Rule?

Finding *dy/dx* Parametrically

Find the equation of the line normal to the curve defined parametrically by $x = 3\cos t$, $y = 2\sin t$ at $t = \pi/4$

$$\frac{dx}{dt} = -3\sin t \quad \frac{dy}{dt} = 2\cos t$$

$$\frac{dy}{dx} = \frac{2\cos t}{-3\sin t} = -\frac{2}{3}\cot t$$



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$$m = -\frac{2}{3}\cot\left(\frac{\pi}{4}\right) = -\frac{2}{3}$$

$$y - \sqrt{2} = \frac{3}{2}\left(x - \frac{3\sqrt{2}}{2}\right) \Rightarrow y = \frac{3}{2}x - \frac{5\sqrt{2}}{4}$$

p. 151 EQ: How do we apply the Chain Rule?

Power Chain Rule

If f is a differentiable function of u, and u is a differentiable function of x, then substituting $y = f(u)$ into the Chain Rule formula $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ leads to the formula

$$\frac{d}{dx} f(u) = f'(u)\frac{du}{dx}$$

Quick Quiz Sections 3.4 – 3.6

You should solve the following problems without using a graphing calculator.

1. Which of the following gives $\frac{dy}{dx}$ for $y = \sin^4(3x)$?

(A) $4\sin^3(3x)\cos(3x)$

(B) $12\sin^3(3x)\cos(3x)$

(C) $12\sin(3x)\cos(3x)$

(D) $12\sin^3(3x)$

(E) $-12\sin^3(3x)\cos(3x)$

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Quick Quiz Sections 3.4 – 3.6

2. Which of the folllowing gives y'' for $y = \cos x + \tan x$?

(A) $-\cos x + 2\sec^2 x \tan x$

(B) $\cos x + 2\sec^2 x \tan x$

(C) $-\sin x + \sec^2 x$

(D) $-\cos x + \sec^2 x \tan x$

(E) $\cos x + \sec^2 x \tan x$

Quick Quiz Sections 3.4 – 3.6

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(C) $-\sin x + \sec^2 x$

(D) $-\cos x + \sec^2 x \tan x$

(E) $\cos x + \sec^2 x \tan x$

Quick Quiz Sections 3.4 – 3.6

3. Which of the following gives $\frac{dy}{dx}$ for the parametric curve $x = 3\sin t,\ y = 2\cos t$?

(A) $-\frac{3}{2}\cot t$

(B) $\frac{3}{2}\cot t$

(C) $-\frac{2}{3}\tan t$

(D) $\frac{2}{3}\tan t$

(E) $\tan t$

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Quick Quiz Sections 3.4 – 3.6

3. Which of the following gives $\frac{dy}{dx}$ for the parametric curve $x = 3\sin t,\ y = 2\cos t$?

(A) $-\frac{3}{2}\cot t$

(B) $\frac{3}{2}\cot t$

(C) $-\frac{2}{3}\tan t$

(D) $\frac{2}{3}\tan t$

(E) $\tan t$

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