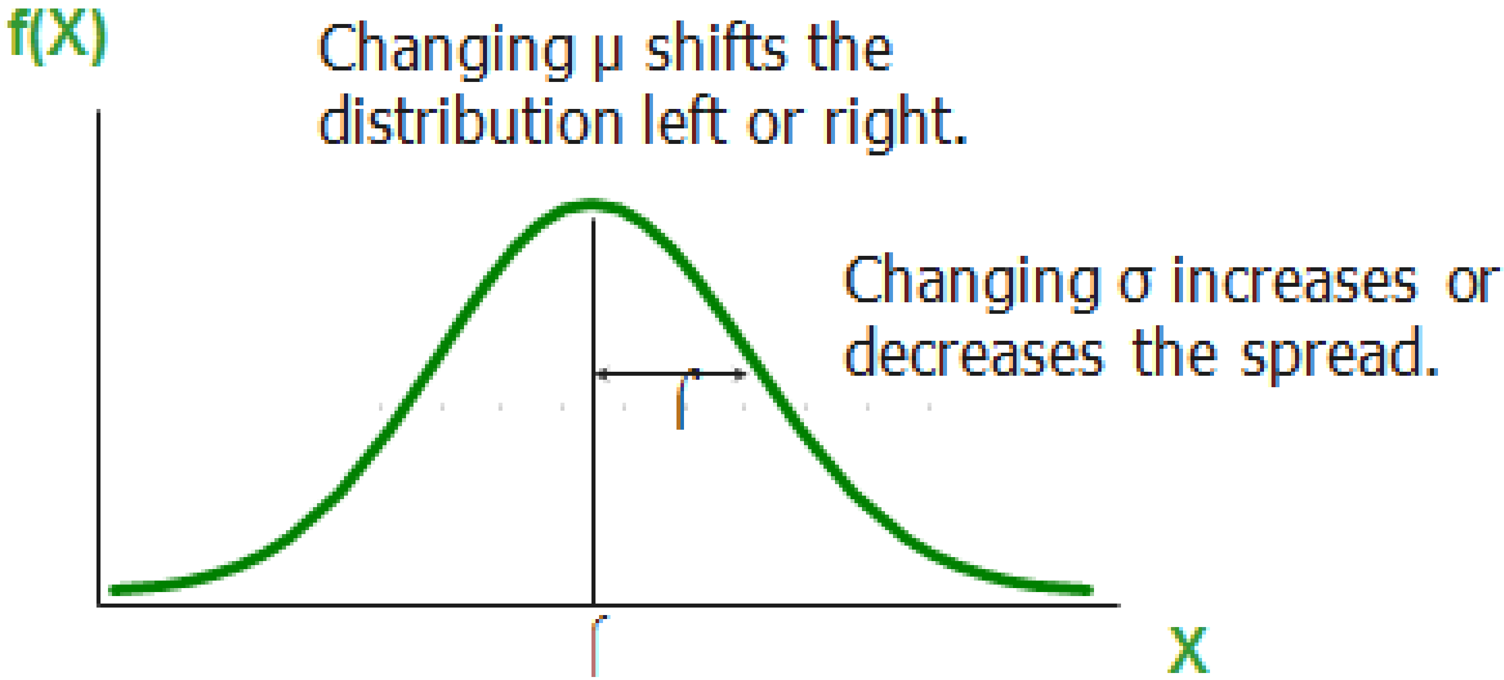
Normal Distribution Curves

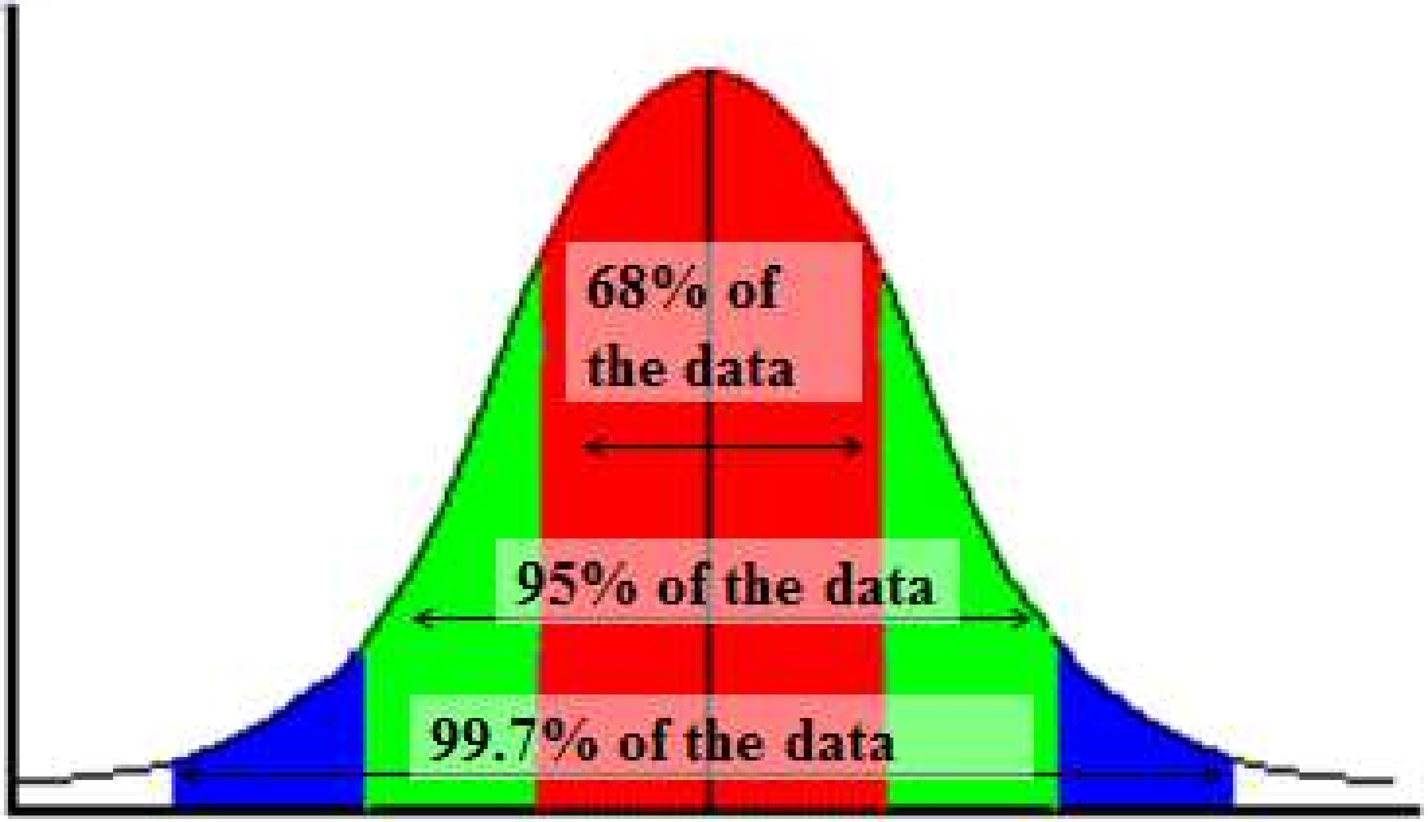
The Normal Distribution



**The beauty of the normal curve:

No matter what μ and σ are, the area between μ-σ and μ+σ is about 68%; the area between μ-2σ and μ+2σ is about 95%; and the area between μ-3σ and μ+3σ is about 99.7%. Almost all values fall within 3 standard deviations.

68-95-99.7 Rule



Example 1

Suppose SAT scores roughly follows a normal distribution in the US population of college- bound students (with range restricted to 200-800), and the average math SAT is 500 with a standard deviation of 50. This would mean that…

68% of students’ scores would fall between ____ & ____,

95% of students’ scores would fall between ____ & ____,

and 99.7% of students’ scores would fall between

____ & ____.

Extension…

What if you wanted to know the math score corresponding to the 90th percentile?

YIKES, Do WHAT?!?

We will come back to this question, let's get some more information…

Click the following link for a Z Scores Chart:

http://www.regentsprep.org/Regents/math/algtrig/ATS7/ZChart.htm

The Standard Normal Distribution (Z)

All normal distributions can be converted into the standard normal curve by subtracting the mean and dividing by the standard deviation:

$$Z = \frac{X - \mu}{\sigma}$$

Somebody calculated all the integrals for the standard normal and put them in a table! So we never have to integrate!

Even better, computers now do all the integration.

Example

- For example: What's the probability of getting a math SAT score of 575 or less, μ=500 and σ=50?

$$Z = \frac{575 - 500}{50} = 1.5$$

- i.e., A score of 575 is 1.5 standard deviations above the mean

$$\therefore P(X \leq 575) = \int_{200}^{575} \frac{1}{(50)\sqrt{2\pi}} \cdot e^{-\frac{1}{2}(\frac{x-500}{50})^2} dx \longrightarrow \int_{-\infty}^{1.5} \frac{1}{\sqrt{2\pi}} \cdot e^{-\frac{1}{2}Z^2} dz$$

Yikes!

But to look up Z= 1.5 in standard normal chart (or enter into SAS)→ no problem! = .9332

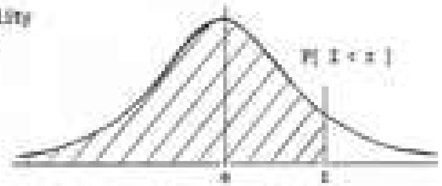
Looking up probabilities in the standard normal table

STANDARD STATISTICAL TABLES

1. Areas under the Normal Distribution

The table gives the cumulative probability up to the standardised normal value z i.e.

$$P[\, Z < z \,] = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} \exp(-\tfrac{1}{2}Z^2)\, dZ$$



z	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5159	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7854
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8804	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	[illegible]	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	[illegible]	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767

What is the area to the left of Z=1.51 in a standard normal curve?

Z=1.51

Z=1.51

Area is 93.45%

http://www.stat.tamu.edu/~lzhou/stat302/standardnormaltable.pdf

Back to the question…

What if you wanted to know the math score corresponding to the 90^{th} percentile?

Click the following link for a Z Scores Chart:

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Practice problem

If birth weights in a population are normally distributed with a mean of 109 oz and a standard deviation of 13 oz,

a. What is the chance of obtaining a birth weight of 141 oz *or heavier* when sampling birth records at random?
b. What is the chance of obtaining a birth weight of 120 *or lighter*?

Example:

The average heating bill for a residential area is \$123 for the month of November with a standard deviation of \$8. If the amounts of the heating bills are normally distributed, find the probability that the average bill for a randomly selected resident is more than \$125.

Example:

Monthly utility bills in a certain city are normal distributed with a mean of $100 and a standard deviation of $12. A utility bill is randomly selected. Find the probability it is between $80 and $115.

Example:

An IQ test has a mean of 100 with a standard deviation of 15. What is the probability that a randomly selected adult has an IQ between 85 and 115?

Ex. Find the z-score corresponding to the 98th percentile.

Ex. Find the z-score corresponding to the 10th percentile.

Ex. Find the z-score corresponding to the .54th percentile.

Example:

Monthly utility bills in certain city are normally distributed with a mean of $100 and a standard deviation of $12. What is the smallest utility bill that can be in the top 10% of the bills?

Example:

Papa Fred's Pizza has found that the mean time to deliver a pizza is 21.2 minutes with a standard deviation of 6.1 minutes. They want to have a guaranteed delivery time. In order to deliver 99% within the guaranteed time you need to find the time represented by the 99^{th} percentile. What is this value?