Newton's Law of Universal Gravitation



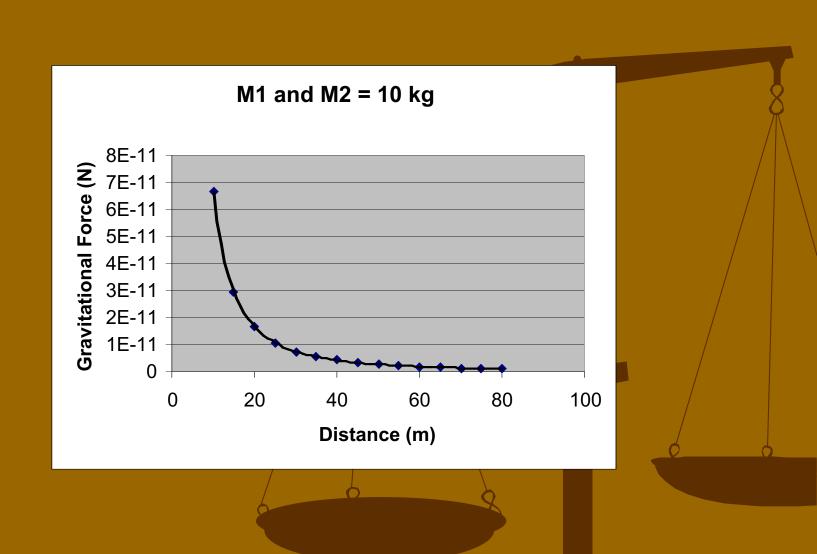
Gravitation

- Every object with mass attracts every other object with mass.
 - Newton realized that the force of attraction between two massive objects:
 - Increases as the mass of the objects increases.
 - Decreases as the distance between the objects increases.

$$\blacksquare F_G = G \frac{M_1 M_2}{r^2}$$

- G = Gravitational Constant
 - $\blacksquare G = 6.67 \times 10^{-11} \text{ N*m}^2/\text{kg}^2$
- $\blacksquare M_1$ and M_2 = the mass of two bodies
- r = the distance between them.

- The LoUG is an inverse-square law:
 - If the distance doubles, the force drops to 1/4.
 - If the distance triples, the force drops to 1/9.
 - Distance x 10 = F_G / 100.



- Jimmy is attracted to Betty. Jimmy's mass is 90.0 kg and Betty's mass is 57.0 kg. If Jim is standing 10.0 meters away from Betty, what is the gravitational force between them?
 - $\blacksquare F_G = GM_1M_2 / r^2$
 - \blacksquare F_G = $(6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(90.0 \text{ kg})(57.0 \text{ kg}) / <math>(10.0 \text{ m})^2$
 - \blacksquare F_G = (3.42x10⁻⁷ Nm²) / (100. m²)
 - \blacksquare F_G = 3.42x10⁻⁹ N = 3.42/nN
 - In standard terms, that's 7.6 ten-billionths of a pound of force.

■ The Moon is attracted to the Earth. The mass of the Earth is 6.0x10²⁴ kg and the mass of the Moon is 7.4x10²² kg. If the Earth and Moon are 345,000 km apart, what is the gravitational force between them?

$$\blacksquare F_G = GM_1M_2 / r^2$$

$$F_{G} = (6.67 \times 10^{-11} \text{ Nm}^{2}/\text{kg}^{2}) \frac{(6.0 \times 10^{24} \text{ kg})(7.4 \times 10^{22} \text{ kg})}{(3.45 \times 10^{8} \text{ m})^{2}}$$

 $\blacksquare F_G = 2.49 \times 10^{20} \text{ N}$

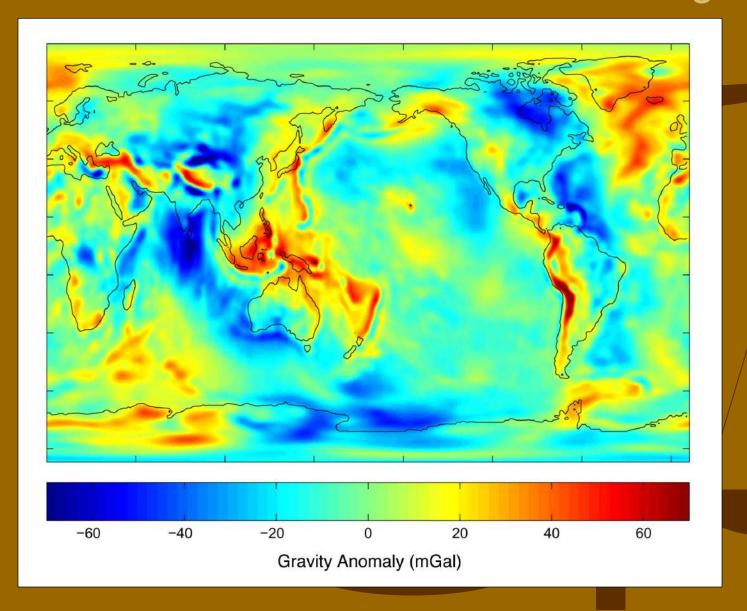
Gravitational Field

- Gravitational field an area of influence surrounding a massive body.
 - Field strength = acceleration due to gravity (g).
- $\blacksquare g = GM / r^2$
 - ■Notice that field strength does not depend on the mass of a second object.
 - $\Box \mathsf{GM_1M_2/r^2} = \mathsf{M_2}g = \mathsf{F_G} = \mathsf{F_w}$
- So gravity causes mass to have weight.

Gravitational Field Strength

- The mass of the Earth is 6.0x10²⁴ kg and its radius is 6378 km. What is the gravitational field strength at Earth's surface?
 - $\square g = GM/r^2$
 - $\Box g = (6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(6.0 \times 10^{24} \text{ kg}) / (6.378 \times 10^6 \text{ m})^2$
 - $\Box g = 9.8 \text{ m/s}^2$
- A planet has a radius of 3500 km and a surface gravity of 3.8 m/s². What is the mass of the planet?
 - \blacksquare (3.8 m/s²) = (6.67x10⁻¹ Nm²/kg²)(M) / (3.5x10⁶ m)²
 - $(3.8 \text{ m/s}^2) = (6.67 \times 10^{-11} \text{ Nm}^2/\text{kg}^2)(\text{M}) / (1.2 \times 10^{13} \text{ m}^2)$
 - \blacksquare (4.6x10¹³ m³/s²) = (6.67x10⁻¹¹ Nm²/kg²)(M)
 - \blacksquare M = 6.9x10²³ kg

Variations in Gravitational Field Strength



Things Newton Didn't Know

- Newton didn't know what *caused* gravity, although he knew that all objects with mass *have* gravity and *respond to* gravity.
- To Newton, gravity was simply a property of objects with mass.
- Newton also couldn't explain how gravity was able to span between objects that weren't touching.
 - ■He didn't like the idea of "action-at-a-distance".

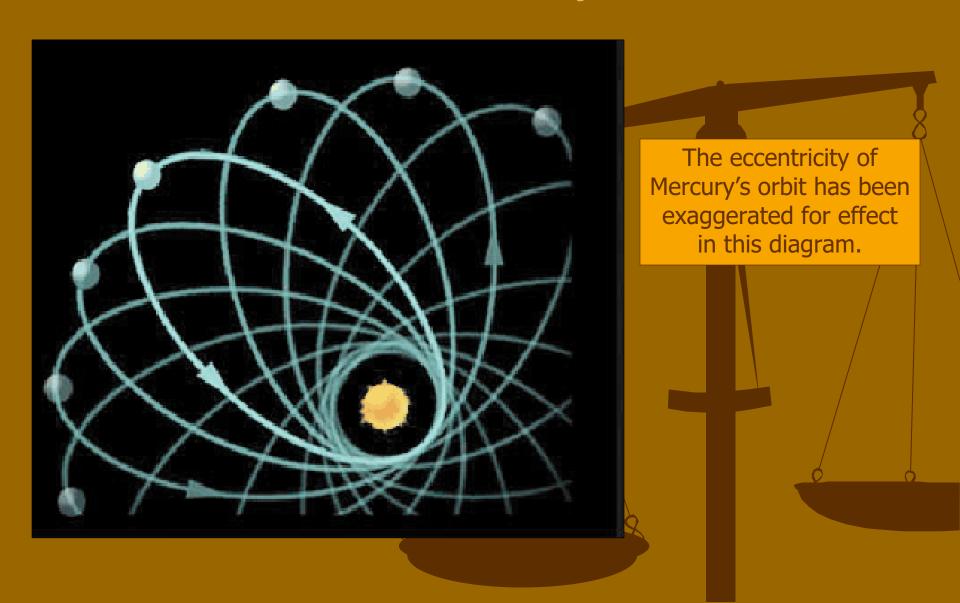
Discovery of Neptune

- Newton's Law of Universal Gravitation did a very good job of predicting the orbits of planets.
 - In fact, the LoUG was used to predict the existence of Neptune.
 - The planet Uranus was not moving as expected.
 - The gravity of the known planets wasn't sufficient to explain the disturbance.
 - Urbain LeVerrier (and others) predicted the existence of an eighth planet and worked out the details of its orbit.
 - Neptune was discovered on September 23, 1846 by Johann Gottfried Galle, only 1° away from where LeVerrier predicted it would be.

Precession of Perihelion

- Newton's Law has some flaws, however:
 - ■It does *not* predict the precession of Mercury's perihelion, nor does it explain it.
 - All planets orbit the Sun in slightly elliptical orbits.
 - Mercury has the most elliptical orbit of any planet in the solar system (if you don't count Pluto).
 - The closest point in Mercury's orbit to the Sun is called the perihelion.
 - Over long periods of time, Mercury's perihelion precedes around the Sun.
 - This effect is not explainable using only Newtonian mechanics.

Precession of Mercury's Perihelion



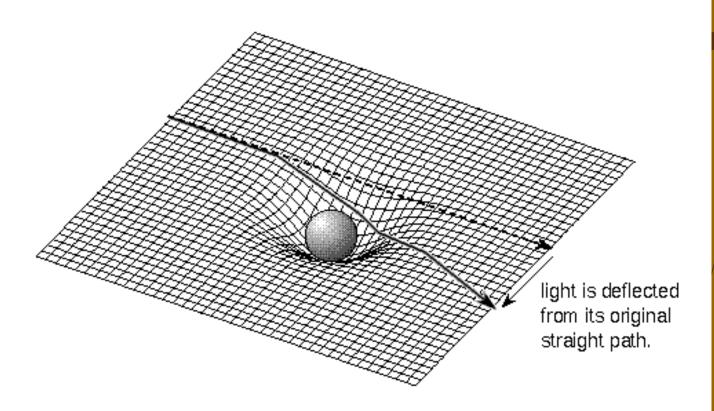
Other Things Newton Didn't Know

- Newton didn't know that gravity bends light.
 - ■This was verified by the solar eclipse experiment you read about earlier this year.
- He also didn't know that gravity slows down time.
 - Clocks near the surface of Earth run slightly slower than clocks higher up.
 - This effect must be accounted for by GPS satellites, which rely on accurate time measurements to calculate your position.

Einstein and Relativity

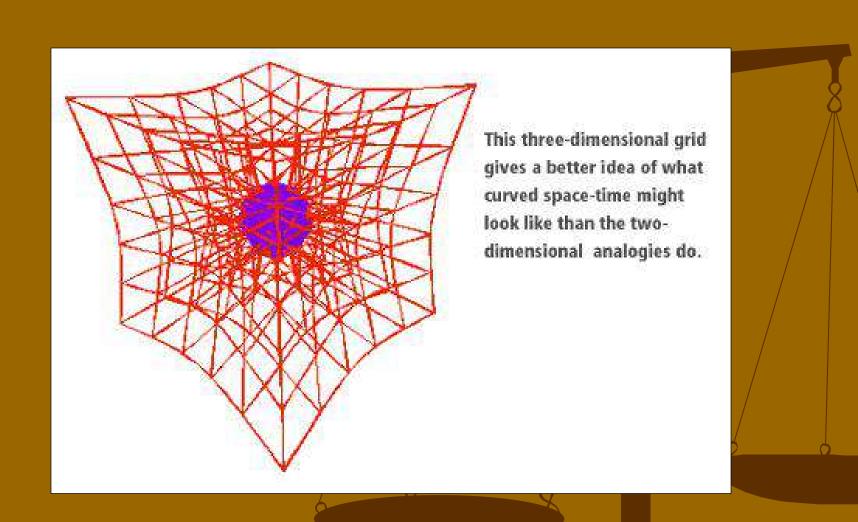
- Einstein's theory of relativity explains many of the things that Newtonian mechanics cannot explain.
 - According to Einstein, massive bodies cause a curvature in space-time.
- Objects moving through this curvature move in *locally* straight paths through curved space-time.
 - ■To any observer *inside* this curved spacetime, the object's motion would appear to be curved by gravity.

Curvature of Space-Time



General Relativity: Light travels along the curved space taking the shortest path between two points. Therefore, light is deflected toward a massive object! The stronger the local gravity is, the greater the light path is bent.

Curvature of Space-Time



Gravity: Not So Simple Anymore

- According to Einstein, gravity isn't technically a force.
 - It's an effect caused by the curvature of space-time by massive bodies.
- Why treat it as a force if/it isn't one?
 - Because in *normal situations*, Newton's LoUG provides an excellent approximation of the behavior of massive bodies.
 - And besides, using the LoUG is a lot simpler than using the theory of relativity, and provides results that are almost as good *in most cases*.
 - So there.