

Particle Motion Problems
AP Calculus

Name:

Answers

1) A particle moves along the x-axis such that its position at any time t where $0 \leq t \leq 5$ is given by the function $x(t) = 2t^3 - 15t^2 + 36t - 22$

- determine the velocity and acceleration functions
- what is the particle's average velocity from $t = 2$ to $t = 4$
- what is the particle's instantaneous velocity at $t = 3$
- when is the particle at rest
- when does the particle move to the right
- what is the total distance traveled by the particle
- what is the particle's maximum velocity
- is the particle moving towards or away from the origin at $t = 1$
- is the particle speeding up or slowing down at $t = 1$

a)
$$v(t) = x'(t) = 6t^2 - 30t + 36$$

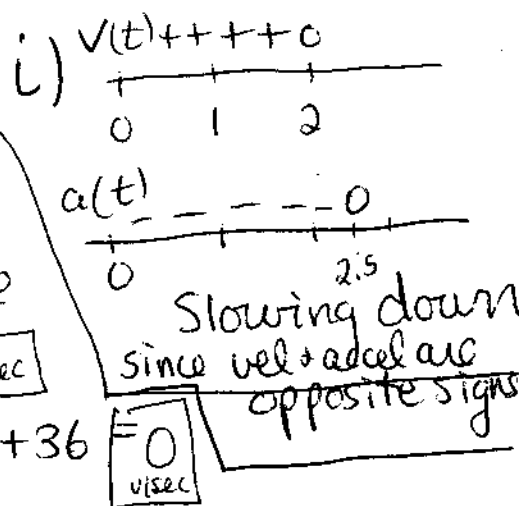
$$a(t) = v'(t) = x''(t) = 12t - 30$$

b) average velocity = $\frac{x(4) - x(2)}{4 - 2} = \frac{10 - 6}{2} = \boxed{2 \text{ u/sec}}$

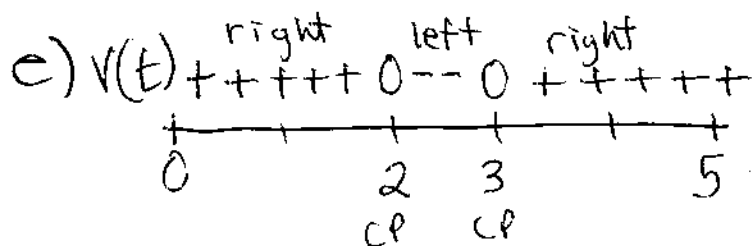
c) instan vel. is $x'(3) = 6(3)^2 - 30(3) + 36 = 0$

d) $v(t) = 0$ $6t^2 - 30t + 36 = 0$
 $6(t^2 - 5t + 6) = 0$
 $6(t - 2)(t - 3) = 0$
 $t = 2, t = 3$

h) $x(1) = 1$
 Particle starts at 1
 + moves right
 away from origin



Particle is at
 rest
 at $t = 2, t = 3$



moves right $(0, 2) \cup (3, 5)$
 moves left $(2, 3)$
 $2 < t < 3$

f) total distance = $|x(0) - x(2)| + |x(2) - x(3)| + |x(3) - x(5)|$
 $= |-22 - 6| + |6 - 5| + |5 - 33| = \boxed{57}$

g) max velocity is at $t = 0 + t = 5$ $v(0) = 6(0)^2 - 30(0) + 36 = 36$
 since $t = 0 + 5$ are abs max
 of $v(t)$ $v(5) = 6(5)^2 - 30(5) + 36 = \boxed{36}$

Try this without a calculator.

2) A particle starts at time $t = 0$ and moves on a number line so that its position at time t seconds is given by $x(t) = (t-2)^3(t-6)$. Show all work that leads to your answers or justify your answer in words.

- Write the particle's velocity function
- When does the particle stop?
- Does the particle change direction at all its stops?
- What is the particle's displacement from $t = [1, 6]$?
- What is the total distance the particle traveled from $t = [1, 6]$?
- Set up an equation that could calculate a time when the particle's instantaneous velocity is equal to its average velocity over the interval $[1, 6]$. Which theorem does this illustrate?

a) $v(t) = x'(t) = 3(t-2)^2(t-6) + (t-2)^3(1)$

b) Particle stops when $v(t) = 0$

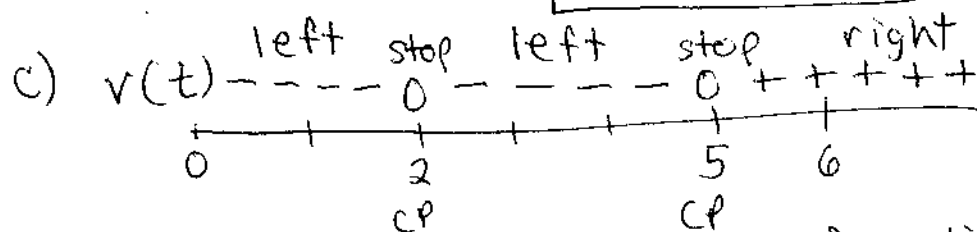
$$(t-2)^2(3(t-6) + (t-2)) = 0$$

$$(t-2)^2(3t-18+t-2) = 0$$

$$(t-2)^2(4t-20) = 0$$

$$\boxed{t=2, t=5}$$

← factor out GCF of $(t-2)$ to solve



No, the particle does not change direction at all its stops! It moves left from 0 sec to 2 sec, then stops, then continues left, then stops, then it turns around & moves right.

d) displacement = $x(6) - x(1) = ((6-2)^3(6-6)) - ((1-2)^3(1-6))$

$$= 0 - 5$$

$$= \boxed{-5}$$

e) total distance =

$$|x(0) - x(5)| + |x(5) - x(6)| = |48 - -27| + |-27 - 0|$$

$$= 75 + 27 = \boxed{102 \text{ units}}$$

f) ave vel from $[1, 6] = \frac{x(6) - x(1)}{6-1} = \frac{-5}{5} = -1$

$-1 = \frac{3(t-2)^2(t-6) + (t-2)^3}{6-1}$ Mean Value Theorem