Particle Motion Problems
AP Calculus

Name: Answers

1) A particle moves along the x-axis such that its position at any time t where $0 \le t \le 5$ is given by the function $x(t) = 2t^3 - 15t^2 + 36t - 22$

a) determine the velocity and acceleration functions
b) what is the particle's average velocity from t = 2 to t = 4
c) what is the particle's instantaneous velocity at t = 3
d) when is the particle at rest
e) when does the particle move to the right
f) what is the total distance traveled by the particle
g) what is the particle's maximum velocity
h) is the particle moving towards or away from the origin at t = 1
i) is the particle speeding up or slowing down at t = 1

a) $v(t) = x'(t) = 6t^2 - 30t + 36$

$a(t) = v'(t) = x''(t) = 12t - 30$

b) average velocity $= \frac{x(4) - x(2)}{4-2} = \frac{10-6}{2} = \boxed{2 \text{ u/sec}}$

c) instan vel. is $x'(3) = 6(3)^2 - 30(3) + 36 = \boxed{0 \text{ u/sec}}$

d) $v(t) = 0$ $\quad 6t^2 - 30t + 36 = 0$

$6(t^2 - 5t + 6) = 0$

$6(t-2)(t-3) = 0$

$t = 2, t = 3$

Particle is at rest at $t = 2, t = 3$

e) $v(t)$: + + + + + + 0 − − 0 + + + + + +

right | left | right

0 | 2 (CP) | 3 (CP) | 5

moves right $(0,2) \cup (3,5)$ $\quad 0<t<2,\ 3<t<5$

moves left $(2,3)$ $\quad 2<t<3$

f) total distance $= |x(0) - x(2)| + |x(2) - x(3)| + |x(3) - x(5)|$

$= |-22 - 6| + |6 - 5| + |5 - 33| = \boxed{57}$

g) max velocity is $\boxed{36}$ at $t = 0$ & $t = 5$ since $t = 0, 5$ are abs max of $v(t)$

$v(0) = 6(0)^2 - 30(0) + 36 = 36$

$v(5) = 6(5)^2 - 30(5) + 36 = \boxed{36}$

h) $x(1) = 1$

Particle starts at 1 & moves right away from origin

i) $v(t)$: + + + + 0 (0, 1, 2)

$a(t)$: − − − − 0 (0, 2.5)

Slowing down since vel & accel are opposite signs

Try this without a calculator.

2) A particle starts at time t = 0 and moves on a number line so that its position at time t seconds is given by $x(t) = (t-2)^3(t-6)$. Show all work that leads to your answers or justify your answer in words.

a) Write the particle's velocity function
b) When does the particle stop?
c) Does the particle change direction at all its stops?
d) What is the particle's displacement from t = [1,6]?
e) What is the total distance the particle traveled from t = [1,6]
f) Set up an equation that could calculate a time when the particle's instantaneous velocity is equal to its average velocity over the interval [1,6]. Which theorem does this illustrate?

a) $v(t) = x'(t) = 3(t-2)^2(t-6) + (t-2)^3 \cdot (1)$

b) Particle stops when $v(t) = 0$

$(t-2)^2(3(t-6)+(t-2)) = 0$ ← factor out GCF of $(t-2)$ to solve

$(t-2)^2(3t-18+t-2) = 0$

$(t-2)^2(4t-20) = 0$

$\boxed{t=2,\ t=5}$

c) $v(t)$: − − − 0 − − − 0 + + + + +

left | stop | left | stop | right

0 | 2 (cp) | 5 (cp) | 6

No, the particle does not change direction at all its stops! It moves left from 0 sec to 2 sec, then stops, then continues left, then stops, then it turns around & moves right.

d) displacement $= x(6) - x(1) = \left((6-2)^3(6-6)\right) - \left((1-2)^3(1-6)\right)$

$= 0 - 5$

$= \boxed{-5}$

e) total distance $= |x(0) - x(5)| + |x(5) - x(6)| = |48 - -27| + |-27 - 0|$

$= 75 + 27 = \boxed{102 \text{ units}}$

f) ave vel from $[1,6] = \frac{x(6)-x(1)}{6-1} = \frac{-5}{5} = -1$

$-1 = 3(t-2)^2(t-6) + (t-2)^3$ [Mean Value Theorem]