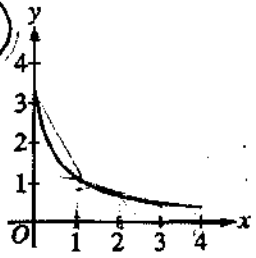
More Fun with Riemann Sums **Name:** Answers

AP Calculus

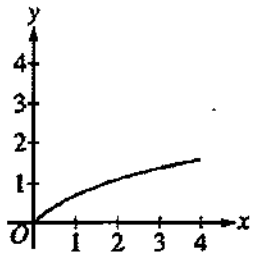
∴ concave up ∴ decreases

1) If a trapezoidal sum overapproximates $\int_0^4 f(x)\,dx$, and a right Riemann sum underapproximates $\int_0^4 f(x)\,dx$, which of the following could be the graph of $y = f(x)$?

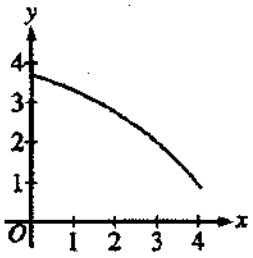
(A)



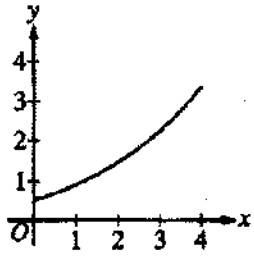
(B)



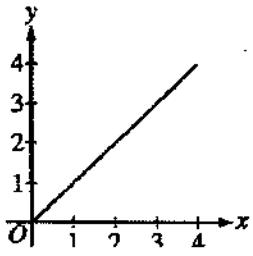
(C)



(D)



(E)

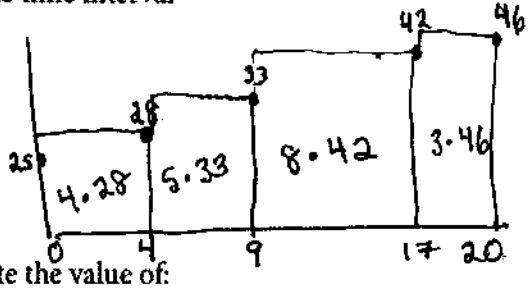


2) The rate at which water is being pumped into a tank is given by the continuous, increasing function $R(t)$. A table of selected values of $R(t)$, for the time interval $0 \le t \le 20$ minutes, is shown below.

Note: Intervals are not the same width!

diff. widths → w=4 .. 5 .. 8 .. 3 ...

t (min)	0	4	9	17	20
$R(t)$ (gal/min)	25	28	33	42	46
	ht	ht	ht	ht	



Units are $\frac{\text{gal}}{\text{min}} \cdot \text{min} =$ gallons

a. Use a right Riemann sum with four subintervals to approximate the value of:

$$\int_0^{20} R(t)\,dt.$$

If it helps, you may draw a picture!

Is your approximation greater or less than the true value? Give a reason for your answer.

Approx area = $4(28) + 5(33) + 8(42) + 3(46) =$ 751 gallons

This is an increasing function, so a right Riemann sum would overestimate the true value.

3) The volume of a spherical hot air balloon expands as the air inside the balloon is heated. The radius of the balloon, in feet, is modeled by a twice-differentiable function r of time t, where t is measured in minutes. For $0 < t < 12$, the graph of r is concave down. The table below gives selected values of the rate of change, $r'(t)$, of the radius of the balloon over the time interval $0 \le t \le 12$. The radius of the balloon is 30 feet when $t = 5$.

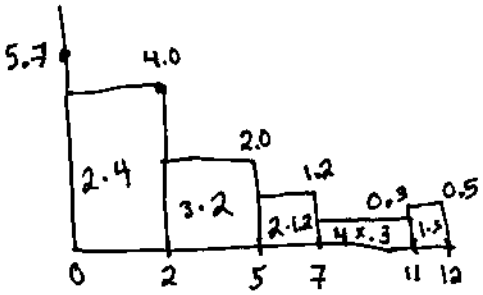
t (minutes)	0	2	5	7	11	12
$r'(t)$ (feet per minute)	5.7	4.0	2.0	1.2	0.3	0.5

Units are $\frac{ft}{min} \cdot \frac{min}{1} = ft.$

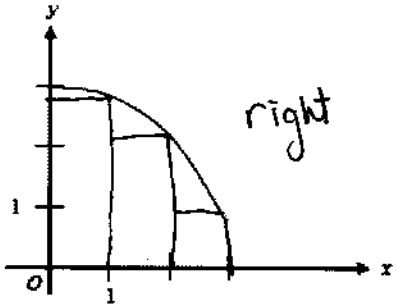
Use a right Riemann sum with the five subintervals indicated by the data in the table to approximate $\int_0^{12} r'(t)dt$. Using correct units, explain the meaning of $\int_0^{12} r'(t)dt$ in terms of the radius of the balloon.

Approx area $R_R = 2 \cdot 4 + 3 \cdot 2 + 2 \cdot 1.2 + 4 \cdot .3 + 1 \cdot .5$

$= \boxed{18.1 \text{ feet}}$



4)



Graph of f

10. The graph of function f is shown above for $0 \le x \le 3$. Of the following, which has the least value?

(A) $\int_1^3 f(x)dx$

(B) Left Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length over

(C) Right Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length under

(D) Midpoint Riemann sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length average

(E) Trapezoidal sum approximation of $\int_1^3 f(x)dx$ with 4 subintervals of equal length close to actual