More Fun with Rates and the FTC
AP Calculus

Name: Answers

The rate at which people enter an amusement park on a given day is modeled by the function E defined by

$$E(t) = \frac{15600}{(t^2 - 24t + 160)}.$$

The rate at which people leave the same amusement park on the same day is modeled by the function L defined by

$$L(t) = \frac{9890}{(t^2 - 38t + 370)}.$$

Both $E(t)$ and $L(t)$ are measured in people per hour and time t is measured in hours after midnight. These functions are valid for $9 \le t \le 23$, the hours during which the park is open. At time $t = 9$, there are no people in the park.

(a) How many people have entered the park by 5:00 P.M. ($t = 17$)? Round answer to the nearest whole number.

(b) The price of admission to the park is \$15 until 5:00 P.M. ($t = 17$). After 5:00 P.M., the price of admission to the park is \$11. How many dollars are collected from admissions to the park on the given day? Round your answer to the nearest whole number.

(c) Let $H(t) = \int_9^t (E(x) - L(x))\,dx$ for $9 \le t \le 23$. The value of $H(17)$ to the nearest whole number is 3725. Find the value of $H'(17)$ and explain the meaning of $H(17)$ and $H'(17)$ in the context of the park.

(d) At what time t, for $9 \le t \le 23$, does the model predict that the number of people in the park is a maximum?

a) $\int_9^{17} E(t) = 6004.270$ $\boxed{\text{about } 6004 \text{ people}}$

b) $\int_{17}^{23} E(t) = 1271.283$

6004 people $\cdot$ \$15 each + 1271 people $\cdot$ \$11 ea = $\boxed{\$104{,}041}$

c) $H'(t) = E(t) - L(t)$

$H'(17) = E(17) - L(17) = -380.281$ people/hour

H(17) is the number of people in the park at 5 PM.

H'(17) is the rate at which the # of people is changing measured in ~~people/hour~~

d) Find max of $H(t)$

$$H(t) = \int_{9}^{t} (E(x) - L(x))\,dx$$

$H'(t) = E(t) - L(t)$ critical point

$E(t) - L(t) = 0$ at $t = 15.795$

	x	$H(t)$
endpt	9	0
c.p.	15.795	3950.680 ← Absolute max
endpt.	23	1.014