THE MONKEY AND THE ZOOKEPER

A monkey spends most of its day hanging from a branch of a tree waiting to be fed by the zookeeper. The zookeeper shoots bananas from a banana cannon.

Unfortunately, the monkey drops from the tree the moment that the banana leaves the muzzle of the cannon and the zookeeper is faced with the dilemma of where to aim the banana cannon in order to feed the monkey.

If the monkey lets go of the tree the moment that the banana is fired, where should he aim the banana cannon?

If the zookeeper aims above the monkey what would be the path of the banana? Would the banana hit the monkey?

Let's find out!

Banana thrown ABOVE the monkey

http://www.physicsclassroom.com/mmedia/vectors/mzg.html

Wrong move!

If the zookeeper aims directly at the monkey what would be the path of the banana? Would the banana hit the monkey?
Let's find out!

Banana thrown AT the monkey

http://www.physicsclassroom.com/mmedia/vectors/mzf.ht

What will happen if the banana is thrown at a lower speed?

Banana thrown slowly at the monkey

http://www.physicsclassroom.com/mmedia/vectors/mzs.html

Happy monkey!

Monkey and Balloon

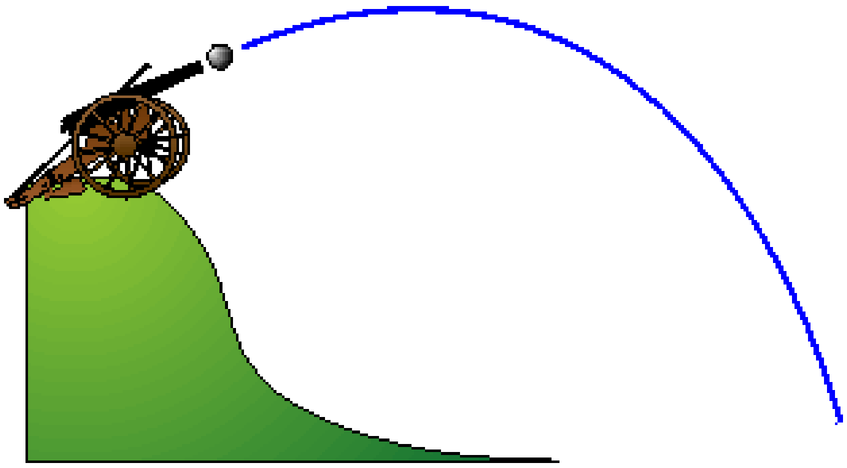
In this simulation the balloon shooter is always aimed directly at the monkey which is hanging from a branch. When you fire the balloon towards the monkey, it lets go of the branch at the same instant. Can you ever miss the monkey? Why?

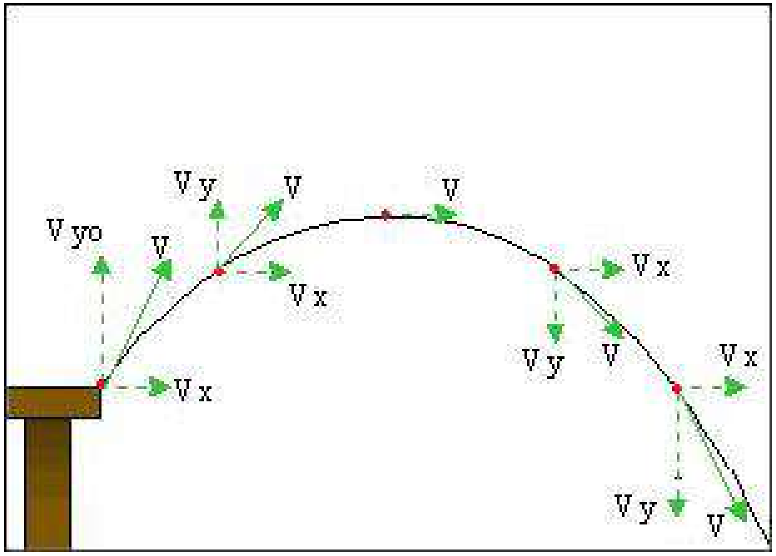
[Please don't shoot the monkey!](http://www.csun.edu/~vceed002/explorsci/dswmedia/monkey.htm)

http://www.csun.edu/~vceed002/explorsci/dswmedia/monkey.htm

PROJECTILE MOTION

An object launched into space without motive power of its own is called a projectile. If we neglect air resistance, the only force acting on a projectile is its weight, which causes its path to deviate from a straight line.

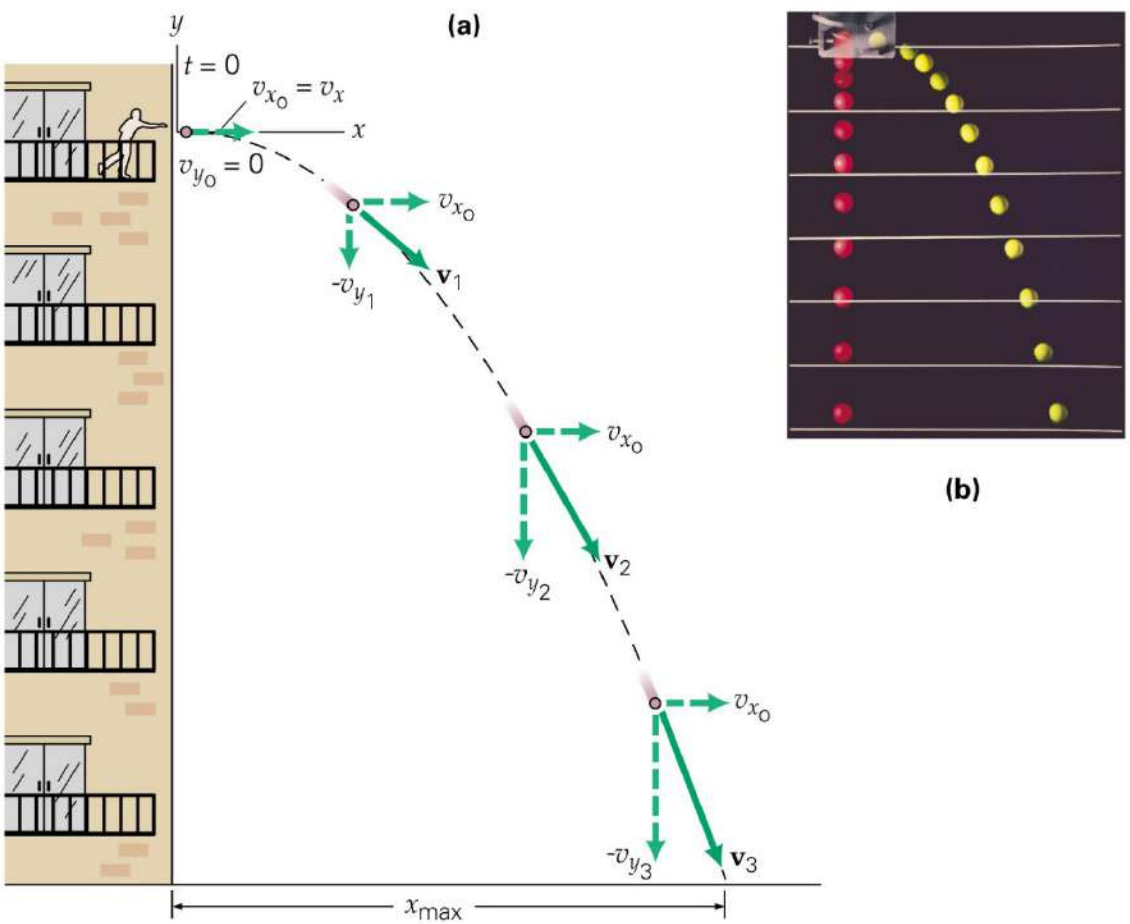




The projectile has a constant horizontal velocity and a vertical velocity that changes uniformly under the influence of the acceleration due to gravity.

HORIZONTAL PROJECTION

If an object is projected horizontally, its motion can best be described by considering its horizontal and vertical motion separately.



In the figure we can see that the vertical velocity and position increase with time as those of a free-falling body. Note that the horizontal distance increases linearly with time, indicating a constant horizontal velocity.

3.1 **An airplane traveling at 80 m/s at an elevation of 250 m drops a box of supplies to skiers stranded in a snowstorm.**

a. **At what horizontal distance from the skiers should the supplies be dropped?**

$v_o = 80$ m/s

$y = 250$ m

$$y = v_{oy}t + \tfrac{1}{2} g t^2$$

$$t = \sqrt{\frac{2y}{t}} = \sqrt{\frac{2(250)}{9.8}} = 7 \text{ s}$$

$$x = v_{ox}t$$

$$= 80(7)$$

$$= 560 \text{ m}$$

b. **Find the magnitude of the velocity of the box as it reaches the ground.**

$v_x = v_{ox} = 80$ **m/s**

and

$v_y = v_{oy} + gt$

$= 9.8\ (7)$

$= 68.6$ **m/s**

$$v_f = \sqrt{(v_x)^2 + (v_y)^2}$$

$$= \sqrt{(80)^2 + (68.6)^2}$$

= **105.4 m/s**

3.2 A person standing on a cliff throws a stone with a horizontal velocity of 15.0 m/s and the stone hits the ground 47.0 m from the base of the cliff. How high is the cliff?

v_o = 15.0 m/s

x = 47.0 m

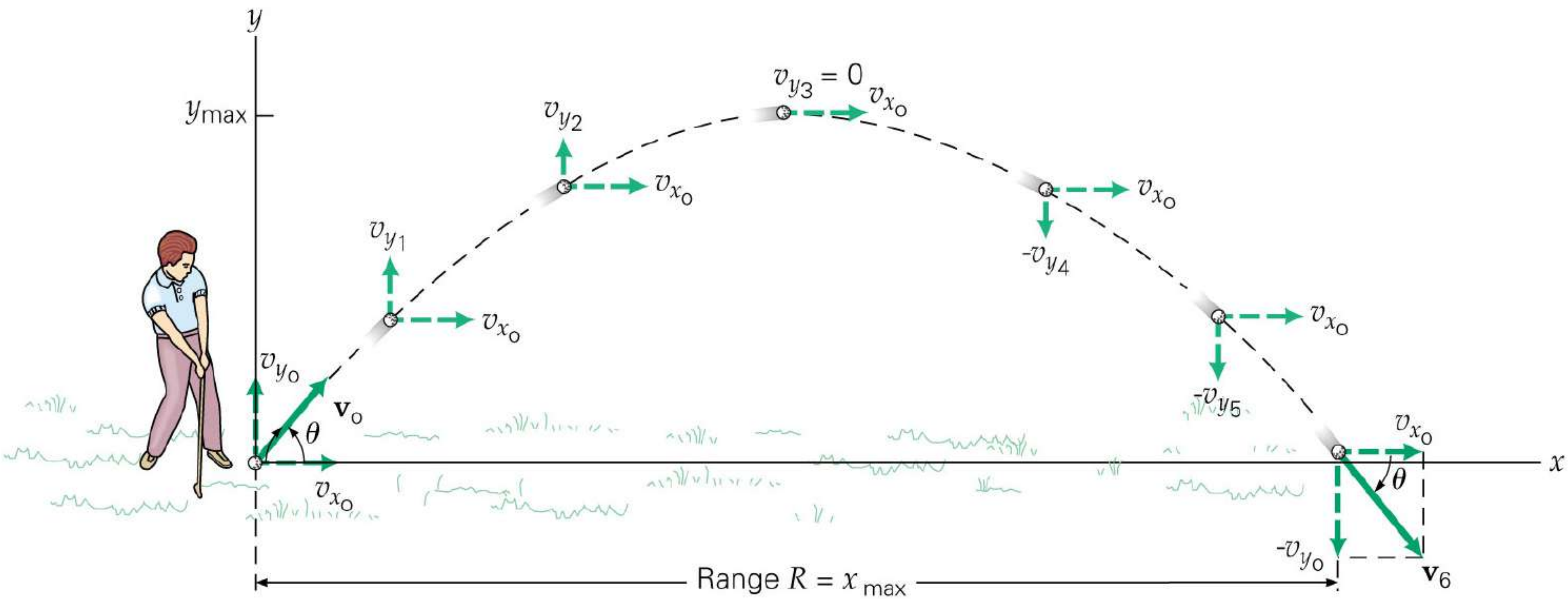
v_y = 0

$$t = \frac{x}{v_x} = \frac{47}{15} = 3.13 \text{ s}$$

$$y = v_{oy}\, t + \tfrac{1}{2} g t^2$$
$$= 0 + \tfrac{1}{2}(9.8)(3.13)^2$$
$$= 48 \text{ m}$$

PROJECTILE MOTION AT AN ANGLE

The more general case of projectile motion occurs when the projectile is fired at an angle.



Problem Solution Strategy:

1. Upward direction is positive. Acceleration due to gravity (g) is downward thus $g = -\ 9.8\ \mathrm{m/s^2}$

2. Resolve the initial velocity v_o into its x and y components:

$v_{ox} = v_o \cos\theta \quad v_{oy} = v_o \sin\theta$

3. The horizontal and vertical components of its *position* at any instant is given by: $x = v_{ox}t \qquad y = v_{oy}\, t + ½gt^2$

4. The horizontal and vertical components of its *velocity* at any instant are given by: $v_x = v_{ox}\ v_y = v_{oy} + gt$

5. The *final* position and velocity can then be obtained from their components.

3.3 An artillery shell is fired with an initial velocity of 100 m/s at an angle of 30° above the horizontal. Find:

a. Its position and velocity after 8 s

v_o = 100 m/s, 30°

t = 8 s

g = - 9.8 m/s^2

$x = v_{ox}\, t$

= 86.6(8)

= 692.8 m

$y = v_{oy}\, t + ½\, g t^2$

= 50(8) + ½ (−9.8)(8)^2

= **86.4 m**

v_{ox} = 100 cos 30°

= **86.6 m/s**

v_{oy} = 100 sin 30°

= 50 m/s

$v_x = v_{ox}$ = 86.6 m/s

$v_y = v_{oy} + gt$

= 50 + (−9.8)(8)

= − 28.4 m/s

b. The time required to reach its maximum height

At the top of the path:

$v_y = 0$

$v_y = v_{oy} + gt$

$$t = \frac{-v_{oy}}{g} = \frac{-50}{-9.8} = 5.1\ \text{s}$$

c. The horizontal distance (range)

Total time

$T = 2t$

$= 2(5.1)$

$= 10.2$ s

$x = v_{ox}\, t$

$= 86.6(10.2)$

$= 883.7$ m

3.4 A plastic ball that is released with a velocity of 15 m/s stays in the air for 2.0 s.

a. At what angle with respect to the horizontal was it released?

$v_o = 15$ m/s

$t = 2$ s

time to maximum height = 1 s

at the top $v_y = 0$

$v_y = v_{oy} + gt$

$$t = \frac{v_o \sin\theta}{g}$$

$$\sin\theta = \frac{tg}{v_o} \qquad \theta = \sin^{-1}\left|\frac{9.8(1)}{15}\right| = 40.8^o$$

b. What was the maximum height achieved by the ball?

$$y = v_{oy}\, t + \tfrac{1}{2} g t^2$$
$$= (15)(\sin 40.8^{\circ})(1) + \tfrac{1}{2}(-9.8)(1)^2$$
$$= \mathbf{4.9\ m}$$

3.5 An arrow was shot at an angle of 55º with respect to the horizontal. The arrow landed at a horizontal distance of 875 m. Find the velocity of the arrow at the top of its path.

θ = 55º
x = 875 m

total time of flight: $t = \dfrac{x}{v_o \cos\theta}$

$$y = v_{oy}\, t + \tfrac{1}{2} g t^2$$

if y = 0

$$0 = t\,(v_{oy} + \tfrac{1}{2} g t)$$

$$v_{oy} = \tfrac{1}{2} g t$$

$$v_o \sin\theta = \tfrac{1}{2} g\, t$$

substituting the time and solving for v_o

$$v_o = \sqrt{\frac{gx}{2\sin\theta\cos\theta}} = \sqrt{\frac{(9.8)(875)}{2(\sin 55^\circ)(\cos 55^\circ)}} = \mathbf{95.5\ m/s}$$

At the top of its path the arrow has v_y = o and

$v_x = v_{ox}$

= 95.5 cos 55º

= 54.8 m/s

3.6 Find the range of a gun which fires a shell with muzzle velocity v_o at an angle θ . What is the maximum range possible?

At top $v_y = 0$

$$v_y = v_{oy} + gt$$
$$= v_o \sin\theta - gt$$

$$t = \frac{v_o \sin\theta}{g}$$

Total time $= 2t$

$$x = v_x t = v_o \cos\theta \left(\frac{2 v_o \sin\theta}{g} \right)$$

$$x = \frac{2 v_o^2}{g} (\sin\theta \cos\theta)$$

sin θ cos θ= ½ sin 2θ

$$x = \frac{2v_o^2}{g}(\sin\theta \cos\theta)$$

$$x = \frac{v_o^2}{g}\sin 2\theta$$

Maximum range is 45° since *2θ* = 90°

b. Find the angle of elevation θ of a gun that fires a shell with muzzle velocity of 125 m/s and hits a target on the same level but 1.55 km distant.

v_o = 125 m/s

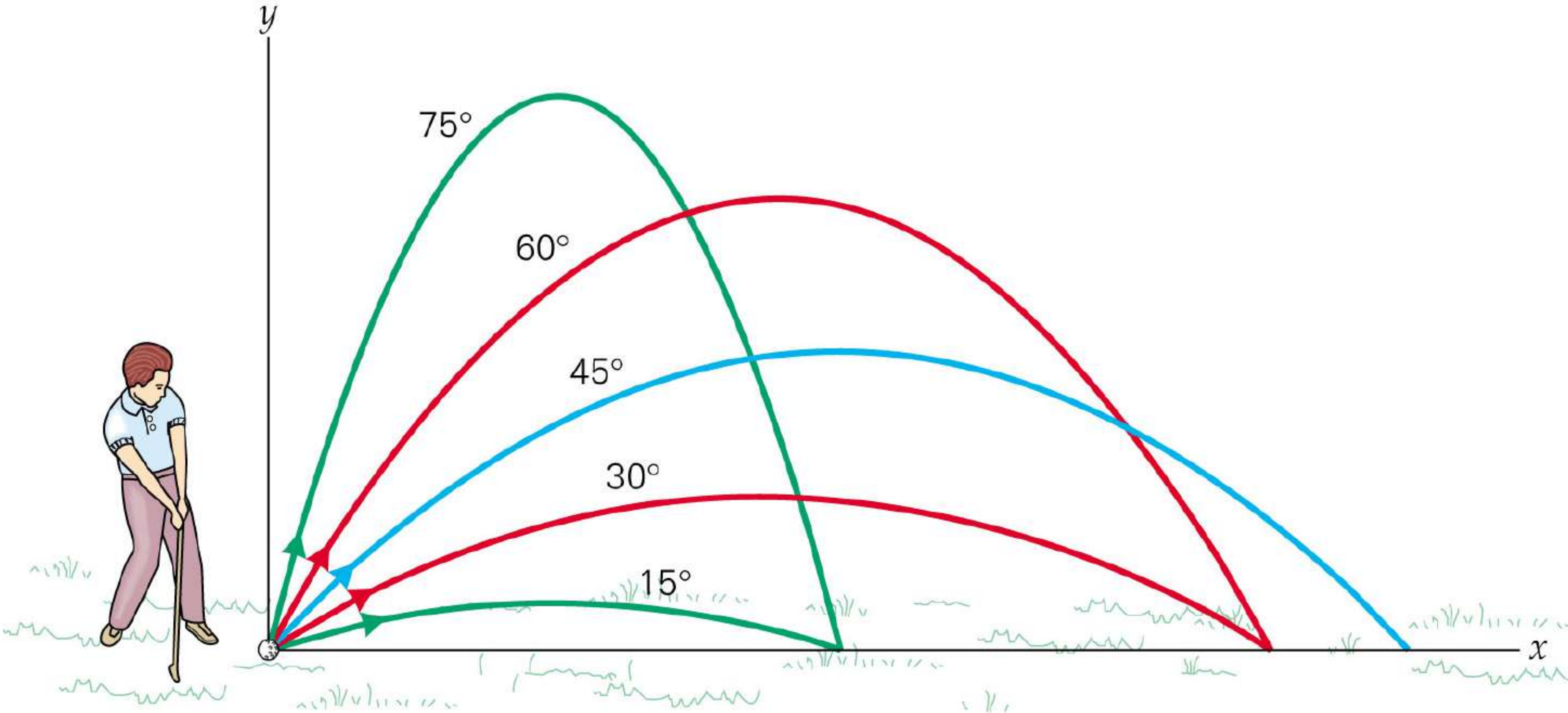
x = 1.55 km

$$x = \frac{v_o^2}{g} \sin 2\theta$$

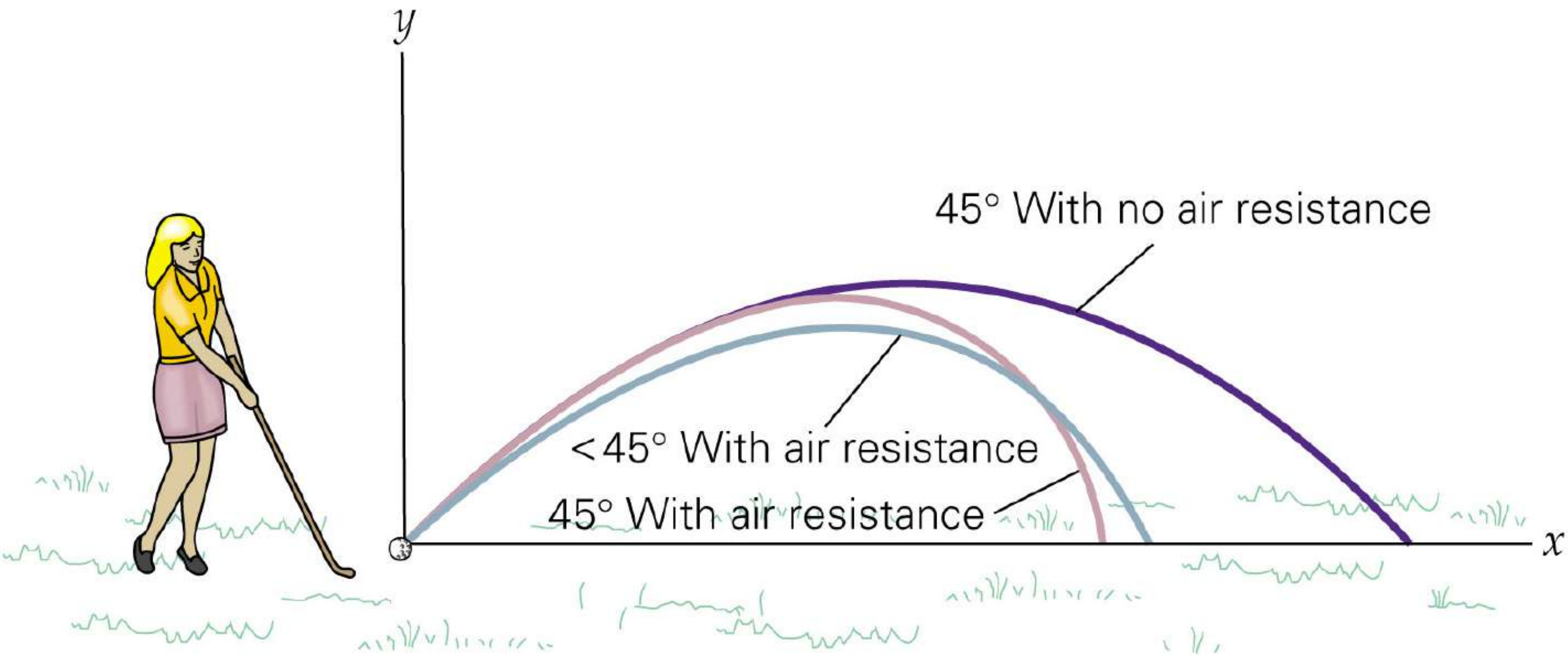
$$\sin 2\theta = \frac{gx}{v_o^2} = \frac{(9.8)(1550)}{(125)^2} = 0.9721$$

$$\sin^{-1}(2\theta) = 76.4$$

$$\theta = 38.2^{\circ}$$



y
75°
60°
45°
30°
15°
x



y
45° With no air resistance
<45° With air resistance
45° With air resistance
x